PREFACE

This book is intended to introduce the student to the theory and design of induction motors.

An attempt has been made to deal with the latest developments in speed and power-factor control, and to incorporate most of the theory connected with the motor and its applications.

While the scope of the book is large, it is hoped that no section of the work is lacking in thoroughness. An endeavour has been made to place the design of induction motors on a firmer scientific basis than it has rested upon heretofore.

The author has largely drawn on his own experience, but he is deeply conscious of his great debt to various writers.

In the course of the work, many technical journals, and especially the *Proceedings of the American Institute of Electrical Engineers*, have been consulted and, in addition, use has been made of the works of Steinmetz.

Due acknowledgment has been made, throughout the book, of the sources from which the information has been drawn.

It is hoped that the book will make a large appeal to engineers and students of the various technical colleges and universities.

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THE INDUCTION MOTOR

CHAPTER I

THE POLYPHASE INDUCTION MOTOR

THE induction motor is the most extensively used of all alternating-current motors. In its simplest form it admits of robust mechanical construction, and its ruggedness and ability to stand abuse make it a most desirable type of industrial motor. It consists essentially of a stationary member called the stator and a rotating member called the rotor. The active part of the stator consists of a core of laminated steel plates of a thickness of 0·5 mm. These plates are slotted on their inner periphery and assembled in a cast-iron or cast steel yoke. In these slots is placed a winding of the requisite number of phases, which may be of the concentric type, or of the mush type, or of the barrel type with diamond-shaped coils. The rotor may be of the squirrel-cage type, consisting of bars placed in the rotor slots and connected at each end by a solid ring of copper or brass, or it may be of the wound type connected to slip-rings on the shaft.

Manner of working of the polyphase induction motor. If polyphase currents of given frequency are supplied to a polyphase stator, a revolving field is produced similar to the armature reaction wave of the polyphase alternator. The fundamental of this field revolves at synchronous speed with respect to the stator winding. With respect to the rotor, it revolves at a speed which is the difference between the synchronous speed and the speed of the rotor. This difference of speed, expressed as a fraction of the synchronous speed, is called the slip.

The rotor winding will have the same number of poles as the stator winding and, if the rotor circuits are closed, currents will be induced in them by the revolving magnetic field.

These currents interacting with the flux will produce a torque tending to rotate the rotor in the direction of rotation of the revolving field. If it were not for frictional losses, synchronous speed would be reached at no load.

Actually at no load the speed of the rotor is slightly less than

synchronous speed; the requisite slip, which is usually of the order of a fraction of 1 per cent, producing sufficient torque to overcome the retarding frictional torque. Under load the slip will increase to such a value as to produce sufficient current in the rotor to produce the torque required by the load and to overcome rotational losses.

The speed of the revolving field depends on the frequency and the number of poles for which the motor is wound. It is entirely

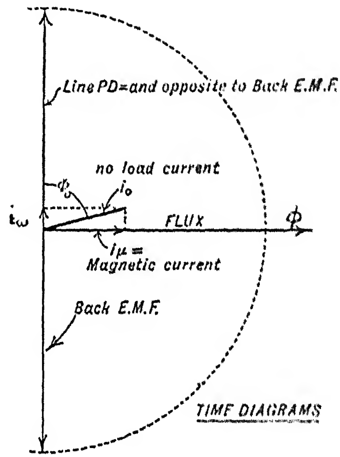


FIG. 1

independent of the number of phases. The only condition which must be satisfied in regard to the number of phases is that the space relations of the windings for the different phases in electrical degrees must be the same as the time-phase relations between the currents they carry.

For a three-phase winding they must be 120 electrical degrees apart; for a four-phase winding they must be 90 electrical degrees apart.

Vector diagram at no load. At no load the motor speed is practically synchronous speed. We may neglect therefore the small current flowing in the rotor. Under this assumption the only current acting in the stator is that which is determined almost entirely by the magneto-motive force required to maintain the flux across the magnetic circuit. This flux is in time-phase with the current in the stator winding, neglecting magnetic hysteresis. The E.M.F. generated in one phase lags 90° behind the flux linking

the winding, and is in phase with the rotating flux cutting the conductors.

Thus when the flux linking a coil is a maximum, the E.M.F. is zero. In Fig. 1 the horizontal line is taken as the vector of flux and magnetizing current. The generated E.M.F. in the stator lags behind the flux vector by 90°. It is marked back-E.M.F. in the diagram. The primary applied P.D. is represented by a line 90° ahead of the flux vector. The angle of lag of the primary current is ϕ_0, and it will be seen that there is a small component of energy current to overcome the iron losses represented by I_w.

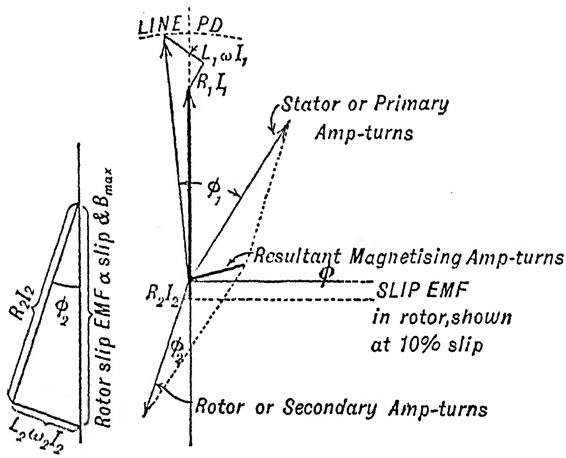


FIG. 2.—VECTOR DIAGRAM ON LOAD

Vector diagram on load. Under full load the motor will have a slip such that sufficient E.M.F. will be generated to produce a torque equal to the retarding load torque. The flux will remain nearly constant, but, owing to the impedance drop in the stator winding, will be about 3 per cent less at full load in a moderate-size motor.

The E.M.F.'s in rotor and stator will have the same relation to the flux as at no load. Now, however, the rotor carries current lagging behind the generated E.M.F. This lag is produced by leakage fluxes, which will be discussed later.

Since the angle of lag of the rotor current is dependent on its frequency, it follows that it will vary from zero at synchronous speed to nearly 90° at standstill.

The flux produced by the rotor current is in phase with the current, and tends to increase the total flux. This latter must, however, remain sensibly constant, and therefore the resultant M.M.F. must remain constant. It follows, therefore, that a current flows in the stator, a component of which offsets the magnetizing action of the rotor current. The total stator current is the vector sum of the no-load current and the component which offsets the rotor current in its magnetizing action.

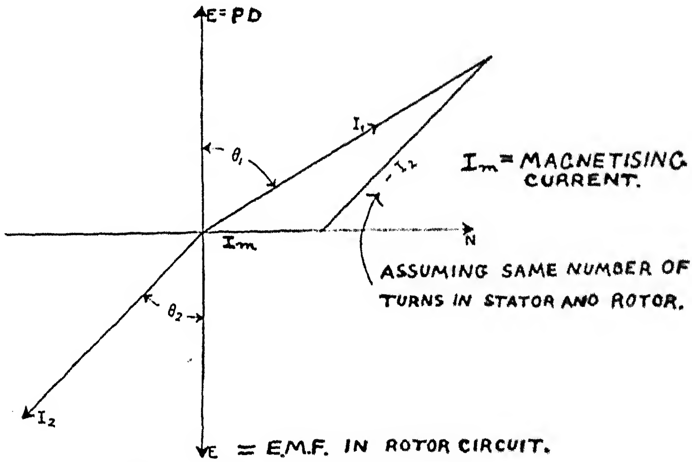


FIG. 3.—VECTOR DIAGRAM FOR STARTING

Vector diagram for starting. When starting, the rotor frequency equals the stator frequency, and the rotor current will lag behind the rotor E.M.F. by an angle whose tangent $= \dfrac{\text{rotor reactance}}{\text{rotor resistance}}$. If the rotor resistance is small, as in the squirrel-cage machine, a large lagging current, probably from 4 to 6 times full-load current, will be drawn from the line. The power factor will be low and will vary from 0·5 to 0·25 or less depending on the size of the machine, the larger value holding for small machines and the lower value for large machines. The torque will be a maximum when the rotor current is in phase with the E.M.F. generated in the motor and therefore 90° behind the flux linking in the winding. With the slip-ring type of motor, with a wound rotor, external resistance is introduced into the rotor circuits at the start. This

results in reduction of the starting current, improvement in power factor, and larger starting torque in virtue of this.

Circle Diagram for induction motor. The operation of the motor can be best understood by considering it as a transformer. If the rotor be held fast, and the resistance in the rotor circuit varied, then the performance of the machine in its general electrical respects is the same as though the slip-rings were short-circuited, and the machine running under load. Consider the rotor in the stationary state, and let resistance be placed in each of its phases.

Let the E.M.F. per phase generated in the rotor at standstill be E_r, and the resistance of the rotor per phase R_r, and let the externally added resistance be R and the rotor inductance be L_r.

Then the rotor current per phase

$$I_R = \frac{E_r}{\sqrt{(R + R_r)^2 + L_r^2\omega^2}} \quad . \quad . \quad . \quad (1)$$

where $\omega = 2\pi f$

$f =$ line frequency.

Now consider the rotor revolving with a slip s under load. The generated E.M.F. in the rotor $= sE_r$ and its reactance per phase $= L_r s\omega$, and

$\therefore$ I_R the rotor current per phase on load

$$= \frac{sE_r}{\sqrt{R_r^2 + s^2L_r^2\omega^2}} \quad . \quad . \quad . \quad . \quad (2)$$

$$= \frac{E_r}{\sqrt{\left(\frac{R_r}{s}\right)^2 + L_r^2\omega^2}} \quad . \quad . \quad . \quad (3)$$

It is evident that, if we so vary the total resistance in each phase of the rotor so as to make $R_r + R = \frac{R_r}{s}$, the current and phase relation will be the same when running as at standstill. This fact enables one to apply the ordinary transformer equations to the motor. Since the electrical characteristics of the motor are not altered by altering the ratio of the number of turns in stator and rotor, it is simplest to assume that the rotor has the same number

of turns as the stator. An ideal transformer having a 1–1 ratio has no effect on the circuit in which it is connected, since it possesses no leakage and has no loss.

Fig. 4 shows the equivalent electrical circuit of a commercial transformer. The local inductance and resistance of primary and secondary are represented by L_p, L_s, R_p, R_s, and R.

The magnetizing current taken is represented by the current that a purely inductive coil L_c would take, and the iron loss is

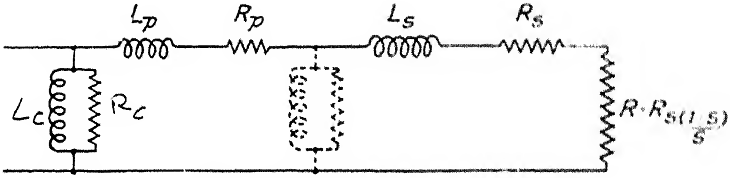


FIG. 4

represented by the loss occurring in the non-inductive resistance R_c. The diagram assumes that the magnetizing current and the iron loss current are taken directly from the mains. This is not strictly correct, since these currents pass through the resistance and inductance of the primary winding. These currents are small, and hence little error is involved. The correct position for L_c and R_c is shown dotted.

We proceed to prove that the locus of the primary and secondary current vectors is a semicircle as the load varies. The current in the circuit beyond L_c and R_c,

$$I_R = \frac{E}{\sqrt{(R_p + R_s + R)^2 + (L_p + L_s)^2\omega^2}} \quad . \quad . \quad (4)$$

This is the rotor current. The stator current is obtained by adding vectorially the current taken by L_c and R_c. The angle of lag of the rotor current is determined by the ratio of reactance to resistance and if this angle is ϕ, then $\sin \phi$

$$= \frac{(L_p + L_s)\omega}{\sqrt{(R_p + R_s + R)^2 + (L_p + L_s)^2\omega^2}} \quad . \quad . \quad (5)$$

$$\therefore I_R = \frac{E \sin \phi}{(L_p + L_s)\omega} \quad . \quad . \quad . \quad . \quad (6)$$

This is the polar equation of a circle having a diameter

$$= \frac{E}{(L_p + L_s)\omega}$$

In Fig. 5, $I_R = OB \sin \phi = \dfrac{E}{(L_p + L_s)\omega} \sin \phi = OC$

OE represents applied E.M.F.

To obtain the primary current, we add vectorially the current taken by L_c and R_c.

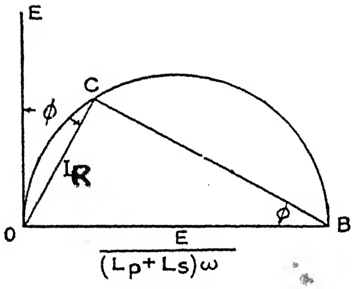


FIG. 5

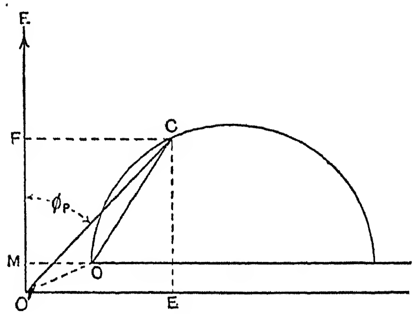


FIG. 6

In Fig. 6, $O'O$ represents the no-load current lagging behind $O'E$ the applied P.D. by the angle $OO'E$, such that $\cos OO'E$

$$= \frac{R_c}{\sqrt{R_c^2 + L_c^2\omega^2}}$$

The component CE represents the power component and CF the wattless component of the primary current. CE represents, to another scale, the power input to the motor. It is clear that maximum power input occurs when C moves to the top of the semicircle and then the angle of lag of the current in the secondary is 45°. Maximum power factor is given by drawing a line from O' tangential to the semicircle. At standstill, $O'P$ (Fig. 7) represents the primary current vector. If the circuit possessed no resistance, $O'P$ would lag behind $O'E$ by 90°. Since it does possess an appreciable resistance, the current lags by an angle $PO'E$ (Fig. 7). The total input to the motor at standstill is absorbed in stator and rotor copper losses and in iron losses. The power component of current HM represents the component to overcome

the no-load losses, which are assumed to remain constant. As the motor is loaded the slip increases, and therefore the frequency of the flux in the rotor increases. The iron loss in the rotor increases, but the speed falls and the friction and windage losses decrease. It is reasonable to assume therefore that the sum of the iron and friction losses is constant at all speeds. *PG* represents the watt component of current corresponding to the rotor loss at standstill,

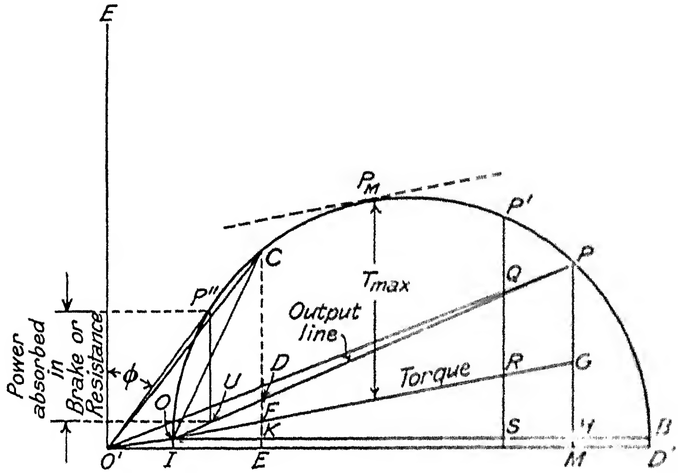


Fig. 7

and *GH* is the watt component of current per phase corresponding to the stator loss per phase.

$O'I$ = magnetizing current per phase ;
OI = watt component of current per phase corresponding to the hysteresis and eddy current losses, and also friction and windage loss and no-load copper loss.

The further construction of the diagram can be simplified by making an approximation which will have but little effect on the results, except at small loads.

The friction and windage losses are supplied by the secondary current I_2. Let the secondary current I_2 be decreased by a component equal to the energy component of current supplying the friction and windage losses, and let this amount be added to the watt component corresponding to the iron losses. Let OI also include the no-load copper loss per phase. Then $O'O$ = no-load current.

CK will represent the motor output plus the secondary copper loss and the increase in primary copper loss caused by the load. As the motor is loaded, C will travel towards the point B and will reach some point such as P when the motor has come to rest. This is the condition existing when the motor is locked under full voltage.

Under this condition, $O'P$ is the primary current and, since the output is zero, PH must represent the secondary copper loss plus the increase in primary copper loss caused by the load.

Now divide PH into two parts, viz. PG and GH, such that PG is proportional to the rotor copper loss and GH is proportional to the increase in primary copper loss caused by the load. Join OG. Then DF is proportional to the rotor copper loss and FK is the increase in primary copper loss due to current $O'C$. OC is the load component of the primary current. It is also the secondary current with a 1–1 ratio of stator and rotor turns. If the ratio of stator turns per phase to rotor turns per phase $= \alpha$, then OC is the secondary current divided by α.

DK, DF, and FK represent respectively the total copper loss, rotor copper loss, and increase in copper loss in the stator due to the load represented by primary current $O'C$.

$$\text{For } \frac{DK}{PH} = \frac{OK}{OH} = \frac{OC \cos COK}{OP \cos POB}$$

$$\text{and } \cos COK = \frac{OC}{OB} \text{ and } \cos POB = \frac{OP}{OB}$$

$$\therefore \frac{DK}{PH} = \frac{OC}{OP} \times \frac{OC}{OB} \times \frac{OB}{OP} = \frac{OC^2}{OP^2}$$

since DK and PH are in the ratio of the squares of the currents OC and OP, DK must be the sum of the rotor copper loss and the loss due to the load component of current in the stator.

It follows therefore that if GH represents the stator copper loss due to the load component of current at short circuit, FK must represent the stator copper loss due to the load component of current with primary current $O'C$.

$$\text{For } \frac{FK}{GH} = \frac{OK}{OH} = \frac{OC^2}{OP^2}$$

Let m = number of phases in the stator,

V = applied voltage per phase

Then $m \times V \times KE =$ no-load losses

$= \text{friction} + \text{windage} + \text{iron} + \text{no-load copper loss} \quad . \quad (7)$

$m \times V \times FK =$ stator copper loss in watts . (8)

$m \times V \times DF =$ rotor copper loss in watts . . (9)

$m \times V \times CE =$ input . . . . . . (10)

$\therefore$ output $= m \times V \times CD$ . . . . . . (11)

and CD is proportional to the output with primary current $O'C$.

The diagram (Fig. 7) refers to one phase only. It is clear therefore that OP is the output line.
OP

$$\text{The efficiency} = \frac{\text{output}}{\text{input}} = \frac{CD}{CE} \quad . \quad . \quad . \quad . \quad (12)$$

Let us investigate how the torque and slip can be represented on the diagram.

Imagine the rotor to be rotated against the magnetic field, which we will suppose to be stationary, with an angular velocity $\omega_1 - \omega_2$ corresponding to the difference of speeds of the field and rotor on load.

Let $\tau =$ torque exerted in kilogramme-metres.

Then $9{\cdot}81\tau(\omega_1 - \omega_2) =$ copper loss in rotor $= 3I_2{}^2R_2$. . (13)

Also $9{\cdot}81\tau\omega_2 =$ output in watts . . . (14)

$\omega_1 =$ synchronous speed in radians per second ;

$\omega_2 =$ actual speed of rotor under load.

$\therefore$ $9{\cdot}81\tau\omega_1 = 3I_2{}^2R_2 +$ output in watts . . . . (15)

for a 3-phase machine, the resistance per rotor phase being R_2

$\therefore$ torque in kilogramme-metres $\times$ synchronous speed in radians

per second $= \dfrac{1}{9{\cdot}81} \times$ input to the rotor . . . (16)

It is convenient to express the value of the torque in synchronous watts or horse-power. By this is meant the output in watts or horse-power that would be developed if the motor ran with the torque which it exerts on load, at synchronous speed.

From the above equation it is clear that the torque in synchronous watts = input to the rotor. We can therefore represent the torque on our diagram (Fig. 7) by CF for a primary current $O'C$. The line OG therefore represents the torque line. It will be noticed that at starting the loss in the rotor, which is the

input to the rotor, is proportional to *PG*. It follows that the starting torque in synchronous watts is equal to the loss in the rotor circuits. Thus if a motor is to start under full-load torque, there will be a loss in the rotor equal to full-load output.

Now the loss in the rotor per phase at standstill

$= I_2^2 R_2$ where $I_2 =$ rotor current in amperes

and $R_2 =$ rotor resistance per phase,

$$\text{where } I_2 = \frac{sE}{\sqrt{R_2^2 + s^2 L_r^2 \omega^2}} \quad . \quad . \quad . \quad (17)$$

$$\therefore I_2^2 R_2 = \frac{s^2 E^2 R_2}{R_2^2 + s^2 L_r^2 \omega^2} \quad . \quad . \quad . \quad . \quad (18)$$

At standstill, $s = 1$

$$\text{and } I_2^2 R_2 = \frac{E^2 R_2}{R_2^2 + L_r^2 \omega^2} \quad . \quad . \quad . \quad . \quad (19)$$

and the starting torque $\propto$ square of voltage per phase of the rotor at standstill and inversely proportional to the square of the rotor impedance at standstill. Since for small values of R_2 this is practically equal to $L_r^2 \omega^2$, it follows that the starting torque is directly proportional to the rotor resistance.

Now torque $\times$ $(\omega_1 - \omega_2)$ = rotor copper loss

and torque $\times$ ω_1 = input to the rotor

$$\frac{\omega_1 - \omega_2}{\omega_1} = \text{slip} \quad . \quad . \quad . \quad . \quad . \quad (20)$$

$$\therefore \text{slip} = \frac{\text{rotor copper loss}}{\text{rotor input}} \quad . \quad . \quad . \quad (21)$$

The percentage slip is the percentage loss of power in the rotor.

In our diagram, (Fig. 7) the slip $= \dfrac{DF}{CF}$ for stator current $O'C$.

Since torque in synchronous watts = input to rotor

$$= \frac{\text{rotor copper loss}}{\text{slip}}$$

$$= \frac{m I_2^2 R_2}{s} \quad . \quad . \quad . \quad (22)$$

where m = number of phases in the rotor

$$\text{and } I_2 = \frac{sE_2}{\sqrt{R_2^2 + s^2L_r^2\omega^2}} \quad . \quad . \quad . \quad . \quad (23)$$

we have torque in synchronous watts

$$= \frac{msE_2}{s\sqrt{R_2^2 + s^2L_r^2\omega^2}} \times I_2 \times R_2 \quad . \quad . \quad (24)$$

$$= \frac{mE_2 \times R_2}{\sqrt{R_2^2 + s^2L_r^2\omega^2}} \times I_2 \quad . \quad . \quad . \quad . \quad (25)$$

$$= mE_2 \cos \phi_2 \times I_2 \quad . \quad . \quad . \quad . \quad (26)$$

$$\text{where } \cos \phi_2 = \frac{R_2}{\sqrt{R_2^2 + s^2L_r^2\omega^2}} \quad . \quad . \quad . \quad . \quad (27)$$

$$\text{Torque in synchronous watts} = \frac{mI_2^2R_2}{s} \quad . \quad . \quad . \quad (28)$$

$$= \frac{msE_2^2 \times R_2}{R_2^2 + s^2L_r^2\omega^2} \quad . \quad . \quad . \quad (29)$$

for small values of slip $s^2L_r^2\omega^2$ is small compared to R_2^2

$$\text{and torque in synchronous watts} = \frac{msE_2^2}{R_2} \quad . \quad . \quad . \quad (30)$$

i.e. inversely proportional to rotor resistance per phase.

The slip at which maximum torque occurs, s_m can be found thus

$$\text{Torque in syn. watts} = \frac{msE_2^2R_2}{R_2^2 + s^2X_2^2} = \tau \quad . \quad . \quad (31)$$

where $X_2 = L_r\omega$

$$\frac{d\tau}{ds} = \frac{mE_2^2R_2\{R_2^2 + s^2X_2^2\} - 2sX_2^2(msE_2^2R_2)}{(R_2^2 + s^2X_2^2)^2} \quad . \quad . \quad (32)$$

$$\text{for max. value } \frac{d\tau}{ds} = O$$

$$\therefore R_2^2 + s^2X_2^2 = 2s^2X_2^2 \quad . \quad . \quad . \quad . \quad . \quad (33)$$

$$\text{i.e. } R_2^2 = s^2X_2^2 \quad . \quad . \quad . \quad . \quad . \quad (34)$$

$$\therefore s^2 = \frac{R_2^2}{X_2^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

$$\therefore s_m = \pm \frac{R_2}{X_2} = \pm \frac{R_2}{L_r\omega} \quad . \quad . \quad . \quad . \quad . \quad (36)$$

This is the value of the slip at maximum torque.

The positive sign corresponds to motor action; the negative sign to generator action.

Substituting this value of the slip in the expression for the torque,

$$\text{we have maximum torque} = \frac{msE_2^2R_2}{R_2^2 + s^2X_2^2} \quad . \quad . \quad (37)$$

$$= \frac{m\dfrac{R_2}{X_2}E_2^2R_2}{R_2^2 + \dfrac{R_2^2}{X_2^2} \times X_2^2} = \frac{mE_2^2R_2^2}{2R_2^2X_2} \quad . \quad . \quad . \quad (38)$$

$$= \frac{mE_2^2}{2X_2} \quad . \quad . \quad . \quad (39)$$

i.e. maximum torque is independent of the value of the rotor resistance, and inversely proportional to the value of the rotor reactance at standstill. Frequently it is desirable to obtain maximum torque at standstill. The value of the resistance to give this effect can be readily found thus—

$$s_m = 1 = \frac{R_2}{X_2}$$

∴ for maximum torque at standstill the rotor resistance must equal the rotor reactance at standstill.

The character of the speed-torque curve can be varied by varying the rotor resistance. This is adopted in the slip-ring or wound rotor-type machine. External resistance is added which is gradually cut out of circuit as the speed increases. Now a large permanent resistance in the rotor circuit means a large slip, a low efficiency, and variable speed under load, and also increased rotor heating.

The torque in synchronous watts = rotor input.

With full-load current in the rotor at starting, we have full-load rotor loss. The slip $= \dfrac{\text{rotor loss}}{\text{rotor input}}$

The starting torque with full-load current expressed as a percentage of full-load torque is numerically equal to the slip. The starting torque expressed as a percentage of full-load torque with a given current

$$I \text{ in the rotor} = \left(\frac{I}{I_f}\right)^2 \times \text{slip at full load} \qquad (40)$$

where I_f = full-load rotor current

$$\text{for slip at full load} = \frac{\text{rotor copper loss at full load}}{\text{rotor input at full load}}$$

$$= \frac{\text{rotor copper loss at full load}}{\text{full-load torque in syn. watts}}$$

$$\begin{array}{c}\text{starting torque with}\\ \text{current } I \text{ in syn. watts}\end{array} = \text{rotor copper loss with current } I$$

$$\therefore \frac{\text{starting torque with current } I}{\text{full-load torque with current } I_f}$$

$$= \frac{\text{rotor copper loss with current } I}{\text{rotor copper loss with current } I_f} \times \text{slip at full load}$$

$$= \left(\frac{I}{I_f}\right)^2 \times \text{slip at full load.}$$

The starting torque can be increased by increasing the current or increasing the slip. A limit is set to the value of the slip from the fact that the efficiency is considerably reduced, the efficiency being less than the speed in percentage of synchronous speed.

A large starting current of low power factor has a bad effect on the voltage regulation of the supply system, and so for high starting torque a compromise must be made. It will perhaps not be out of place to compare the squirrel-cage and wound-rotor types of machines in this respect. Assume that a starting torque of 200 per cent of full-load torque is required.

A squirrel-cage machine would require a slip at full load of 6·6 per cent with a starting current of 5½ times full-load current. To develop full-load torque, it will be necessary to apply 70·7 per cent of full volts at starting. The motor current will be 3·89 full-load motor current. The current in the line will be 2·75 full-load current. Now consider a 20 b.h.p. motor for example. For full-load starting torque, there must be an expenditure of energy in the rotor of 20 b.h.p. If we allow a 4 per cent copper loss in the stator at full load, the percentage loss in the stator, when starting with full-load torque, will be $(3{\cdot}89)^2 \times 4 = 60$ per cent. The iron loss will be about 3 per cent and the total input will be 32·6 h.p. To this must be added the loss in the auto-transformer. With a wound-rotor type of machine, full-load torque is obtained with approximately full-load current. The copper loss in the stator will be 4 per cent and the iron loss will be about 5 per cent. The total input is then 21·8 h.p. The cost of energy used in starting is therefore less.

The current in the case of the squirrel-cage motor at starting is of very low power factor, which is a very serious matter if the size of motor is comparable with the size of generator supplying current to it. It should be mentioned that the iron loss, due to pulsation of the flux in the rotor, increases the torque somewhat.

This refers to the loss which is a maximum at standstill and zero at synchronous speed. Other iron losses, due to pulsation of the flux in the stator and rotor teeth, are present, and these are a

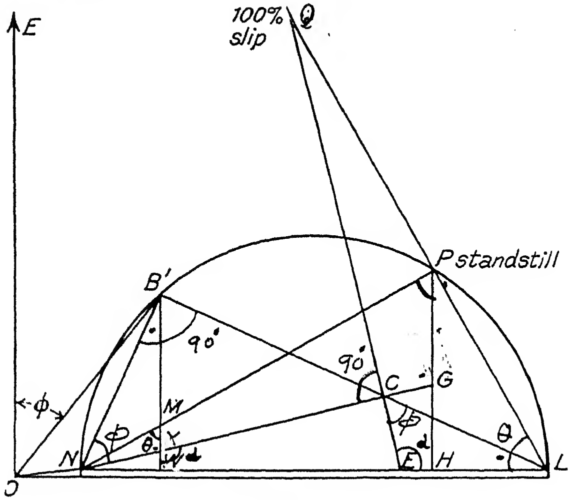


FIG. 8

maximum at synchronous speed and have no effect in increasing the torque at standstill.

The torque of a squirrel-cage machine is much more uniform than that of a wound-rotor machine. The squirrel-cage conductors act powerfully near synchronous speed to reduce flux variation, and so tend to preserve a state of uniform torque. With a wound-rotor machine, the torque varies with varying position of the rotor with respect to the stator, due to varying impedance and varying value of zigzag leakage. It is therefore important to keep the air-gap reluctance as constant as possible, and to keep the flux distribution as constant as possible both in value and in special distribution. This is secured to some extent by the use of fractional pitch windings. It is convenient to be able to read off the slip directly from the diagram.

Join LP (Fig. 8) and produce LP to any distance LQ, and from Q draw QE at right angles to NG, the torque line.

Then the triangles $B'NX$ and LCE are similar.

Also the triangles LQE and MNX are similar.

$$\therefore \frac{B'X}{NX} = \frac{LE}{CE} \text{ and } \frac{LE}{EQ} = \frac{MX}{NX}$$

$$\therefore \quad CE = \frac{MX}{B'X} \times EQ$$

Now the rotor loss = slip × torque

$$\therefore \text{ slip} = \frac{MX}{B'X}, \text{ but this is proportional to } CE$$

$\therefore$ CE is proportional to the slip for the primary current OB'. If the length QE represents 100 per cent slip, then the fraction $\frac{CE}{QE}$ gives directly to the same scale, the percentage slip for the stator current OB'. It is thus clear that, to obtain the slip directly, we must join the points on the circle, representing the different values of primary current, to the point L; then the intersection of these lines with QE gives us the percentage slip directly. For rough preliminary calculations we may neglect the stator copper loss, and then the diagram takes the form shown in Fig. 9.

It is now clear that the torque is represented by OA. Join D to L (the standstill point) and produce DL to meet the vertical through M in E. The triangles DQM and MOA are similar, and triangles DEM and MRA are similar.

$$\therefore \frac{RA}{MA} = \frac{MD}{ME} \text{ and } \frac{MQ}{MD} = \frac{MA}{OA}$$

$$\therefore MQ \times OA = RA \times ME$$

$$\text{and } \therefore MQ = \frac{RA}{OA} \times ME$$

but $RA \propto$ loss in rotor (neglecting stator loss)

$OA \propto$ torque

$$\frac{RA}{OA} = \text{slip}$$

$\therefore$ slip $= \frac{MQ}{ME}$ where ME represents percentage slip at standstill.

Let τ_{max} = maximum torque

s_m = slip at maximum torque

τ = torque at any other slip $s = OA$

then $\dfrac{\tau}{AD} = \dfrac{MQ}{MD} = \dfrac{MQ}{MN} = \dfrac{s}{s_m}$

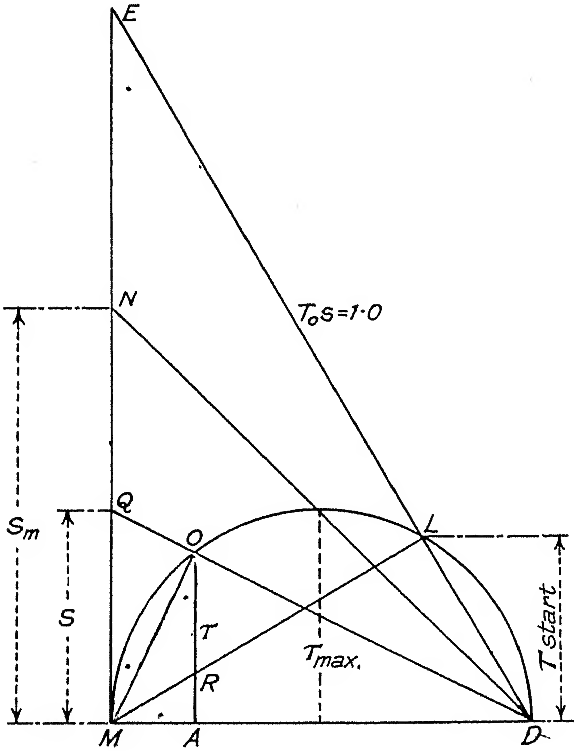


FIG. 9

and $\dfrac{MA}{\tau} = \dfrac{s}{s_m}$ or $MA = \tau \times \dfrac{s}{s_m}$

$$MD = MA + AD = \tau\left\{\frac{s}{s_m} + \frac{s_m}{s}\right\} \quad . \quad . \quad (41)$$

$$\tau_{max} = \tfrac{1}{2}MD$$

$$\therefore \frac{\tau}{\tau_{max}} = \frac{2}{\dfrac{s}{s_m} + \dfrac{s_m}{s}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

At the starting point, $s = 1$

$$\text{and } \frac{\tau_{start}}{\tau_{max}} = \frac{2}{\dfrac{1}{s_m} + s_m} \quad . \quad . \quad . \quad . \quad . \quad (43)$$

These relations are extremely useful; and to illustrate how they can be applied, we shall consider an example or two.

A squirrel-cage motor is to be designed for an overload torque capacity of 150 per cent and 20 per cent margin, with a starting torque of 125 per cent of full-load torque. It is required to find the slip at full load.

$$\text{We have } \frac{\tau}{\tau_{max}} = 0{\cdot}4 \times 0{\cdot}8 = 0{\cdot}32$$

$$\text{and } \frac{\tau_{start}}{\tau_{max}} = 1{\cdot}25 \times 0{\cdot}32 = 0{\cdot}4$$

From the above equations, $s_m = 21\%$ and $s = 3{\cdot}08\%$. Again it is required to find what starting torque may be obtained from a squirrel-cage motor at 100 per cent overload torque and a slip of 2½ per cent at full load.

$$\frac{\tau}{\tau_{max}} = 0{\cdot}5 \times 0{\cdot}9, \text{ allowing 10 per cent margin.}$$

From the equation above, $s_m = 10{\cdot}5$ per cent and

$$\frac{\tau_{start}}{\tau_{max}} = 0{\cdot}208$$

$$\text{i.e. } \tau_{start} = \frac{0{\cdot}208}{\cancel{0{\cdot}405}} \text{ full-load torque}$$

$$= 46 \text{ per cent of full-load torque}$$

The relationship between slip and torque is shown in the following table and graphically in Fig. 10.

SLIP s PER CENT AT TORQUE τ

$\dfrac{\tau_{start}}{\tau_{max}}$	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
$\dfrac{\tau}{\tau_{max}} = 0{\cdot}25$	1·4	2·0	2·7	3·4	4·2	5·5	6·3	8·15	12·8
0·3	1·7	2·33	3·25	4·2	5·0	6·51	7·75	9·75	15·5
0·35	1·98	2·7	3·79	4·9	6·0	7·57	9·0	11·3	18·0
0·40	2·3	3·15	4·4	5·68	6·85	8·83	10·4	13·2	20·8
0·45	2·55	3·53	5·0	6·35	7·85	10·0	11·75	14·8	23·5
0·5	2·96	4·05	5·65	7·2	8·85	11·25	13·3	16·8	26·5
0·55	3·25	4·43	6·25	8·0	10·0	12·5	14·7	18·6	29·5

It is advisable in using these curves to allow a margin of 10 or 15 per cent on the guaranteed overload torque capacity, on account of the neglect of the stator copper loss and ripples on the torque-slip curve.

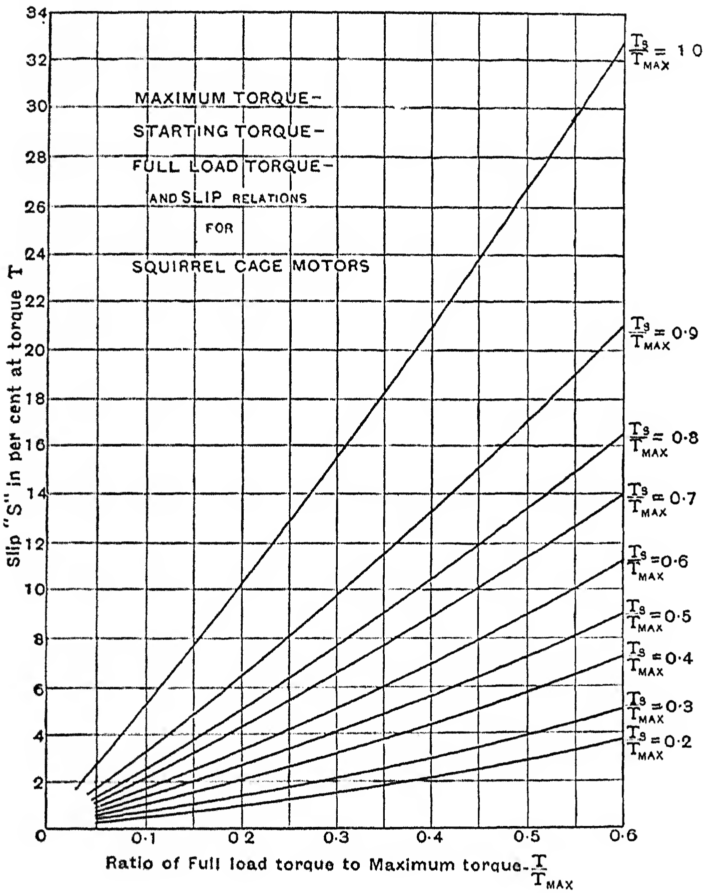


FIG. 10

The efficiency $= \frac{\text{output}}{\text{input}}$. If all losses, except the rotor copper loss, be ignored, then output per phase

$$= E_2 I_2 \cos \phi_2 - I_2^2 R_2 \text{ per phase} \qquad (44)$$

$$\text{and the efficiency } \frac{\text{output}}{\text{input}} = \frac{I^2R_2}{E_2I_2\cos\phi_2} \quad . \quad . \quad (45)$$

$$= 1 - s \quad . \quad . \quad . \quad . \quad (46)$$

$$= \text{speed of motor at full load and slip } s$$

∴ the efficiency is always less than the speed of the machine in per cent of synchronous speed, since there are other losses besides the rotor copper loss.

$$\text{Again, the torque in synchronous watts} = \frac{msE_2^2R_2}{R_2^2 + s^2X_2^2} \quad . \quad (47)$$

$$\therefore \text{ torque in watts} \times \omega_0 = \frac{msE_2^2R_2}{R_2^2 + s^2X_2^2} \quad . \quad . \quad . \quad (48)$$

$$\therefore \text{ torque} = \frac{msE_2^2R_2}{\omega_0(R_2^2 + s^2X_2^2)} \quad . \quad . \quad . \quad . \quad (49)$$

$$\text{Output} = \frac{msE_2^2R_2}{\omega_0(R_2^2 + s^2X_2^2)} \times \omega_0(1 - s) \quad . \quad . \quad (50)$$

$$= \frac{mE_2^2R_2\, s(1 - s)}{R_2^2 + s^2X_2^2} \quad . \quad . \quad . \quad . \quad (51)$$

$$= C \times \frac{s(1 - s)}{R_2^2 + s^2X_2^2} \quad . \quad . \quad . \quad . \quad (52)$$

$$\text{where } C = \text{consts.} = mE_2^2R_2 \quad . \quad . \quad . \quad . \quad (53)$$

To find the slip for maximum output, differentiate the output with respect to the slip and equal to zero.

$$\frac{d(\text{output})}{ds} = \frac{mE_2^2R_2(1 - 2s)\,(R_2^2 + s^2X_2^2)}{(R_2^2 + s^2X_2^2)^2} - \frac{2sX_2^2\{mE_2^2R_2\,s(1 - s)\}}{(R_2^2 + s^2X_2^2)^2} \quad . \quad . \quad (54)$$

$$\therefore\ (1 - 2s)(R_2^2 + s^2X_2^2) = 2s^2X_2^2(1 - s)$$

i.e. $R_2^2 + s^2X_2^2 - 2sR_2^2 - 2s^3X_2^2 = 2s^2X_2^2 - 2s^3X_2^2$

i.e. $R_2^2 - 2sR_2^2 = s^2X_2^2$

i.e. $s^2X_2^2 + 2sR_2^2 - R_2^2 = 0$

i.e. $s = \dfrac{-2R_2^2 \pm \sqrt{4R_2^4 + 4R_2^2X_2^2}}{2X_2^2}$

$$= -\frac{R_2^2}{X_2^2} \pm \sqrt{\frac{R_2^4}{X_2^4} + \frac{R_2^2}{X_2^2}}$$

$$= -\frac{R_2^2}{X_2^2} \pm \frac{R_2}{X_2}\sqrt{\frac{R_2^2}{X_2^2} + 1}$$

$$= -a^2 \pm a\sqrt{a^2 + 1} \quad . \quad . \quad . \quad . \quad . \quad (55)$$

where $a = \frac{R_2}{X_2}$

The plus sign corresponds to motor action; the minus sign to generator action.

The maximum output is obtained by substituting this value of the slip in the expression for the output.

It is seen that maximum output does not occur at the same speed as maximum torque (where $s = \pm a$).

Inserting $s = -a^2 + a\sqrt{a^2 + 1}$ in the expression for the output, we get for a motor

$$\text{output} = \frac{mE_2^2 R_2 s(1 - s)}{R_2^2 + s^2 X_2^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

$$\text{output} = \frac{mE_2^2 a\, s(1 - s)}{X_2 (a^2 + s^2)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

$$s = -a^2 + a\sqrt{a^2 + 1} = a(\sqrt{a^2 + 1} - a)$$

$\therefore$ max. output

$$= \frac{m}{X_2} E_2^2 \cdot \frac{a^2(\sqrt{a^2 + 1} - a)\{1 - a(\sqrt{a^2 + 1} - a)\}}{a^2 + a^2(a^2 + 1 + a^2 - 2a\sqrt{a^2 + 1})} \quad (58)$$

$$= \frac{mE_2^2}{2X_2}(\sqrt{a^2 + 1} - a) \quad . \quad . \quad . \quad . \quad . \quad (59)$$

since $a = \frac{R_2}{X_2}$

$$= \frac{mE_2^2}{2X_2^2}(Z_2 - R_2) \text{ watts} \quad . \quad . \quad . \quad . \quad . \quad (60)$$

Fig. 11 shows the torque-speed curves for various values of rotor resistance.

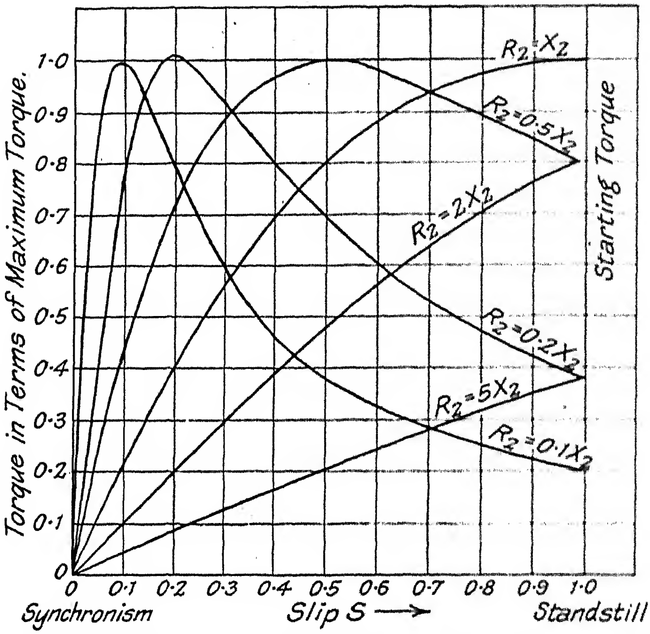


FIG. 11.—TORQUE AS A FUNCTION OF SPEED OR SLIP FOR DIFFERENT VALUES OF ROTOR RESISTANCE
(X_2 = constant)

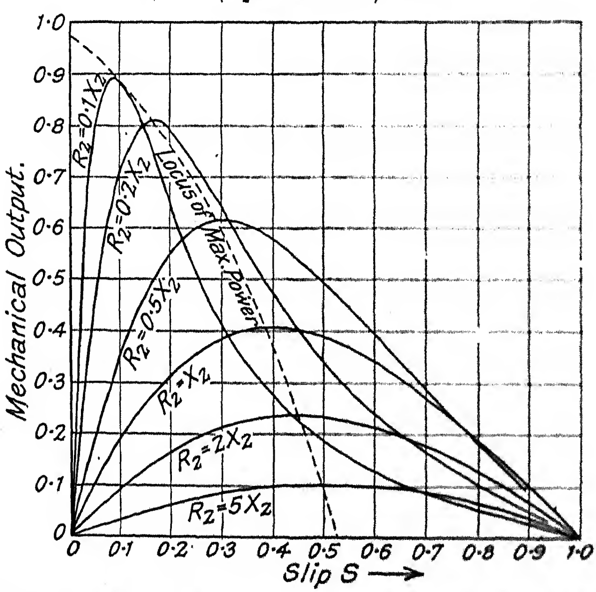


FIG. 12.—MECHANICAL OUTPUT AS A FUNCTION OF SLIP FOR DIFFERENT VALUES OF ROTOR RESISTANCE
(X_2 = constant)

The following table shows the effect of rotor resistance on the maximum output, and also the slip at which maximum output and maximum torque occurs.

$a = \frac{R_2}{X_2}$	MAXIMUM OUTPUT.		MAXIMUM TORQUE.	
	Relative magnitude of max. output $a\sqrt{a^2+1} - a$	Slip at max. output.	Relative magnitude of max. torque.	Slip at τ_{max}
5	0·099	0·495	1	5
2	0·236	0·472	1	2
1	0·414	0·414	1	1
0·5	0·618	0·309	1	0·5
0·2	0·82	0·164	1	0·2
0·1	0·905	0·090	1	0·1

Fig. 12 shows the mechanical output as a function of speed for different values of rotor resistance R_2 ($X_2 =$ constant).

Fig. 13 shows the mechanical output at different speeds as a function of rotor resistance R_2.

It will have been noticed that, in deducing the equations for maximum torque and maximum output, E_2 was assumed constant. E_2 is the voltage induced in the rotor per phase with the rotor stationary, and differs from the counter E.M.F. generated in the stator only on account of the difference in the number of stator and rotor turns. Although the E_2 of a transformer for most purposes may be assumed constant and independent of the load, the E_2 of an induction motor may not so be assumed. On account of the much larger leakage reactance of the induction motor, the induced voltage varies somewhat from no load to full load and beyond full load. For this reason, the E_2 in our equations should be replaced by the impressed voltage V_1, which is constant under ordinary conditions.

An approximate value of V_1 in terms of E_2 may be obtained from the approximate equivalent circuit of Fig. 4.

If $x_1 =$ stator reactance per phase,

$x_2 =$ rotor reactance per phase referred to stator,

$r_1 =$ stator resistance per phase in ohms,

$r_2 =$ rotor resistance per phase in ohms referred to stator by the square of the ratio of turns,

then $$V_1 = I_2\left\{\left(r_1 + jx_1\right) + \left(r_2 + r_2\frac{(1-s)}{s} + jx_2\right)\right\} . \qquad (61)$$

$$sV_1 = I_2\sqrt{(r_1s + r_2)^2 + s^2(x_1 + x_2)^2} \quad . \quad . \quad . \qquad (62)$$

$$sE_2 = I_2\sqrt{r_2^2 + x_2^2s^2} \quad . \quad . \quad . \quad . \quad . \qquad (63)$$

$$\therefore\ E_2^2 = \frac{V_1^2 \times r_2^2 + s^2x_2^2}{(r_1s + r_2)^2 + s^2(x_1 + x_2)^2} \quad . \quad . \quad . \qquad (64)$$

$\therefore$ torque in syn. watts

$$= \frac{mV_1^2 \times sr_2}{(r_1s + r_2)^2 + s^2(x_1 + x_2)^2} \quad . \quad . \quad . \quad . \qquad (65)$$

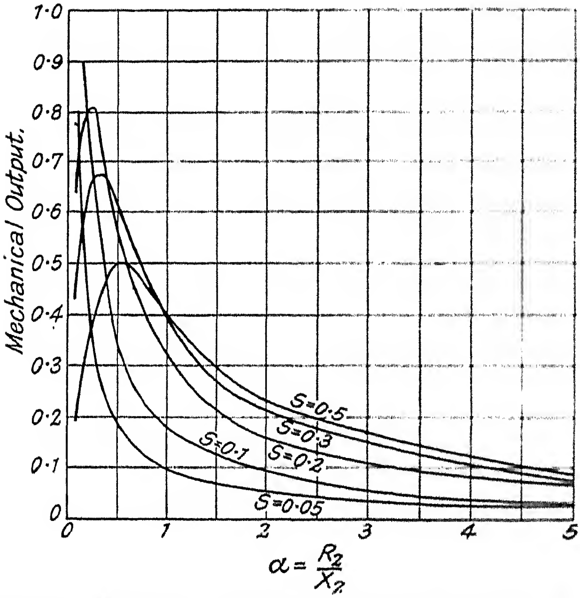


FIG. 13.—MECHANICAL OUTPUT AT DIFFERENT SPEEDS AS A FUNCTION OF ROTOR RESISTANCE R_2. X_2 constant

and since the synchronous speed in radians per second

$$= 2\pi \times \text{revs. per sec.}$$

$$\text{and frequency } f = \frac{\text{poles}}{2} \times \text{revs. per sec.} \quad . \quad . \quad . \qquad (66)$$

$$\therefore\ \omega_o = \frac{2\pi \times 2f}{\text{poles}} = \frac{4\pi f}{p} \quad . \quad . \quad . \quad . \qquad (67)$$

where p = poles

$$\therefore\ \text{torque} = \frac{mV_1^2}{4\pi f} \times \frac{sr_2 \times p}{(r_1s + r_2)^2 + s^2(x_1 + x_2)^2} \qquad (68)$$

If we differentiate this with regard to slip and equate the result to zero, we get slip at maximum torque

$$= \frac{r_2}{\sqrt{r_1^2 + (x_1 + x_2)^2}} \qquad (69)$$

and the maximum torque

$$= \frac{pV_1^2}{4\pi f} \times \frac{1 \times m}{2\{r_1 + \sqrt{r_1^2 + (x_1 + x_2)^2}\}} \qquad (70)$$

The equation we have developed giving the relation between E_2 and V_1 neglects the effect of the no-load current I_n on the primary impedance drop. For a transformer, the effect of I_n on the primary impedance drop is negligible.

With an induction motor, the conditions are different, for the no-load current may be 40 to 80 per cent or more of the full-load current with a machine having a large number of poles. For these reasons, the component I_n of the primary current of the motor cannot be neglected when finding the primary impedance drop without causing serious error.

Since V_1 appears in our equations as a square, any error produced in it by neglecting I_n will be exaggerated. In all equations for torque and power, V_1 should be understood to be the actual impressed voltage minus the part of the stator impedance drop caused by I_n.

No great error will be made by assuming $V_1 - I_n z_1$ vectorially to be equal to $V_1 - I_n z_1$ algebraically and practically to assume the latter $= V_1 - I_n x_1$.

Neither the maximum internal torque nor the slip at which it occurs is independent of the stator resistance, and the same remark applies to maximum power.

In deriving our circle diagram for the motor, we assumed a 1 to 1 ratio of transformation.

If the stator and rotor phases have different numbers of turns, then the rotor reactance and resistance is referred to the stator by the ratio of the squares of the turns in stator and rotor.

Equivalent stator v. rotor quantities. The E.M.F. induced in a phase of T effective turns in series is

$$E = 4{\cdot}44 fT\phi \times 10^{-8} \text{ volts} \qquad (71)$$

At standstill, $$E_2 = 4{\cdot}44 f \times T_2 \times \phi \times 10^{-8} \qquad (72)$$

$$E_1 = 4{\cdot}44 f \times T_1 \times \phi \times 10^{-8} \qquad (73)$$

$$\therefore \frac{E_1}{E_2} = \frac{T_1}{T_2} = \frac{T_s \times B_1}{T_r \times B_2} \qquad (74)$$

where $T_s =$ actual number of turns per phase in the stator,
$B_1 =$ winding factor for the fundamental for the stator,
$T_r =$ actual number of rotor turns per phase,
$B_2 =$ winding factor for the fundamental for the rotor.

Rotor current equivalent to stator current.

Let $I_2 =$ rotor current per phase per circuit,
$I_2' =$ equivalent current in stator per circuit;

then $N_2 T_2 I_2 \times q_2 = N_1 T_1 I_2' \times q_1$ . . (75)

where $N_2 =$ number of phases in the rotor,
$N_1 =$ number of phases in the stator,
$q_2 =$ number of parallel circuits per phase in rotor,
$q_1 =$ number of parallel circuits per phase in stator,
$I_2' =$ equivalent stator current,

$$\therefore \frac{I_2'}{I_2} = \frac{N_2 T_2 q_2}{N_1 T_1 q_1} = \frac{N_2 T_r B_2 \times q_2}{N_1 T_s B_1 \times q_1} \quad . \quad . \quad . \quad (76)$$

$$\frac{\text{Total equivalent stator current}}{\text{Total rotor current}} = \frac{N_2 B_2 T_r}{N_1 B_1 T_s} \quad . \quad . \quad (77)$$

$$\text{If } N_1 = N_2, \text{ this} = \frac{B_2 T_r}{B_1 T_s} \quad . \quad . \quad . \quad (78)$$

$$\text{and} = \frac{T_r}{T_s} \text{ if } B_2 = B_1 \quad . \quad . \quad . \quad (79)$$

Rotor impedance referred to stator impedance.

Let $Z_2' =$ equivalent rotor impedance referred to the stator

$$Z_2' = \frac{E_2'}{I_2'} = \frac{E_2 \times \frac{T_1}{T_2}}{\frac{N_2}{N_1} \times \frac{T_2}{T_1} \times I_2} = \frac{N_1}{N_2}\left(\frac{T_1}{T_2}\right)^2 Z_2$$

$$= \frac{N_1}{N_2}\left(\frac{B_1}{B_2}\right)^2 \times \left(\frac{T_s}{T_r}\right)^2 \times Z_2 \quad . \quad . \quad (80)$$

$$\text{where } Z_2 = \frac{E_2}{I_2} = \text{rotor impedance}$$

If $N_1 = N_2$ and $B_1 = B_2$

$$\text{the above } Z_2' = Z_2 \times \left(\frac{T_s}{T_r}\right)^2 \quad . \quad . \quad . \quad . \quad (81)$$

Similarly we get $R_2' = \frac{N_1}{N_2}\left(\frac{B_1}{B_2}\right)^2 \times \left(\frac{T_s}{T_r}\right)^2 \times R_2.$ 82)

= equivalent resistance

Equivalent reactance $X_2' = \frac{N_1}{N_2}\left(\frac{B_1}{B_2}\right)^2 \times \left(\frac{T_s}{T_r}\right)^2 \times X_2$ (83)

Most of the quantities with which we have dealt can be read directly from the circle diagram. Thus the maximum horse-power is proportional to GH, where GH is the greatest length of line

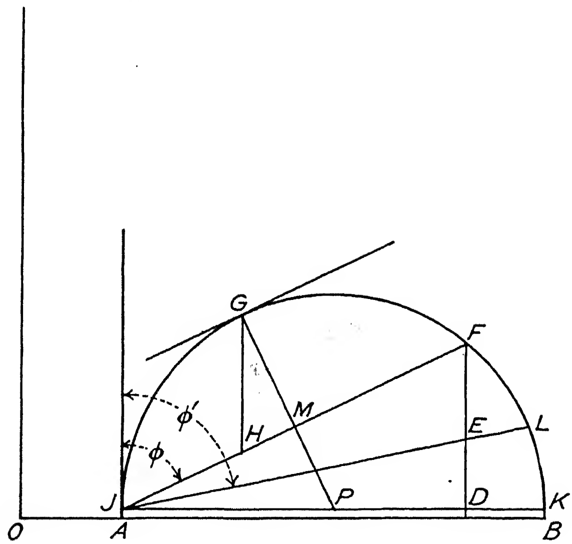


FIG. 14

drawn at right angles to OB, which is intercepted between JF and the circle. If we draw a line tangential to the circle and parallel to JF, this line will touch the circle at G.

$$\text{Then max. h.p.} = \frac{m \times V \times GH}{746} \quad . \quad . \quad . \quad . \quad (84)$$

where m = number of phases in the stator,

V = volts per stator phase.

Now $GH \sin \phi = GM$

$$GH = \frac{GM}{\sin \phi} \quad . \quad . \quad . \quad . \quad . \quad (85)$$

$$= \frac{GP - MP}{\sin\phi} \qquad (86)$$

$$= \frac{GP - JP\cos\phi}{\sin\phi} \qquad (87)$$

$$= \frac{GP\,(1 - \cos\phi)}{\sin\phi} \qquad (88)$$

since $GP = JP$

Now $JF = 2GP \sin\phi \qquad (89)$

$$\therefore\ GP = \frac{JF}{2\sin\phi} \qquad (90)$$

$$\text{and}\ \therefore\ GH = \frac{JF\,(1 - \cos\phi)}{2\sin^2\phi} \qquad (91)$$

$$= \frac{JF\,(1 - \cos\phi)}{2\,(1 - \cos^2\phi)} = \frac{JF}{2\,(1 + \cos\phi)} \qquad (92)$$

$$\text{and max. h.p.} = \frac{mV \times JF}{2\,(1 + \cos\phi) \times 746} \qquad (93)$$

$\cos\phi$ is the power factor at short circuit and $JF = JK \sin\phi$.

In dealing with the motor, JK, the ideal rotor short-circuit current referred to the stator is usually known, or JF is known from short-circuit tests, and therefore the maximum horse-power is known.

In a similar manner the maximum torque is proportional to the maximum vertical intercept between the circle and the torque line JE.

$$\therefore\ \text{max. torque} = \frac{m \times V \times JL}{2\,(1 + \cos\phi')} \qquad (94)$$

$$\text{and } JL = JK \sin\phi' \qquad (95)$$

The dispersion coefficient. If we neglect the no-load energy component of the current, our diagram assumes the very simple form of Fig. 15. The circle diagram gives us the coefficient of magnetic dispersion of the motor. Magnetic leakage exists in both stator and rotor, and the total leakage is the sum of both leakages.

OA = magnetizing current per phase in the stator,

OB = stator short-circuit current per phase, assuming no resistance present and the rotor locked in position.

So long as the same voltage is maintained at the stator terminals, the flux linked with the stator winding is the same in both cases,

neglecting resistance drop. With locked rotor there is no resultant flux through the rotor, and the flux is entirely leakage flux. At synchronism there is no rotor current, and the stator flux is free to enter the rotor.

Hence OB represents the ampere-turns necessary to drive the flux through the leakage paths, while OA represents the ampere-turns required to send the same flux through the useful path and leakage path in parallel.

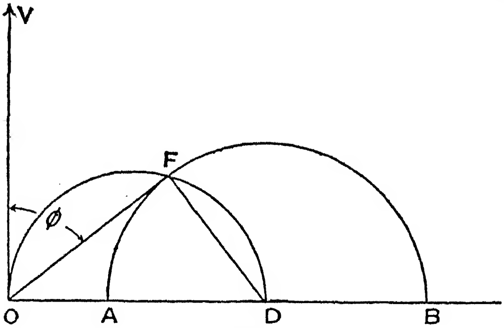


FIG. 15

Thus OB is proportional to the reluctance of the leakage paths, and OA is proportional to the joint reluctance of the useful and leakage paths to the same scale.

$$\therefore \frac{OB}{OA} = \frac{\text{reluctance of leakage paths}}{\text{joint reluctance of useful and leakage paths}}$$

$$= \frac{\text{permeance of leakage path} + \text{permeance of useful path}}{\text{permeance of leakage path}}$$

$$= 1 + \frac{\text{permeance of useful path}}{\text{permeance of leakage path}} \quad . \quad . \quad . \quad . \quad (96)$$

$$\frac{\text{Permeance of useful path}}{\text{Permeance of leakage path}} = \frac{\text{useful flux}}{\text{stray flux}} \quad . \quad . \quad . \quad (97)$$

if the same ampere-turns act upon both paths (as at no load).

$$\text{Hence } \frac{OB}{OA} = 1 + \frac{\text{useful flux}}{\text{stray flux}} \quad . \quad . \quad . \quad . \quad (98)$$

$$\text{Also } \frac{OB}{OA} = \frac{OA + AB}{OA} = 1 + \frac{AB}{OA} \quad . \quad . \quad . \quad (99)$$

$$\therefore \frac{AB}{OA} = \frac{\text{useful flux}}{\text{stray flux}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (100)$$

The leakage factor of any circuit

$$= \frac{\text{total flux}}{\text{useful flux}} = \frac{\text{useful flux} + \text{stray flux}}{\text{useful flux}} \quad (101)$$

$$= 1 + \frac{\text{stray flux}}{\text{useful flux}} \quad . \quad . \quad . \quad . \quad (102)$$

$$= 1 + \frac{OA}{AB} \quad . \quad . \quad . \quad . \quad . \quad (103)$$

The symbol σ has been used to represent another leakage ratio, usually called the dispersion coefficient.

Behn-Eschenberg uses for the dispersion coefficient

$$\sigma = \frac{\text{leakage flux}}{\text{total flux}} = \frac{OA}{OB} \quad . \quad . \quad . \quad . \quad (104)$$

Hobart and others take another coefficient

$$\frac{\text{leakage flux}}{\text{useful flux}} = \frac{OA}{AB} \quad . \quad . \quad . \quad . \quad (105)$$

which we shall call v.

We shall adopt Behn-Eschenberg's definition and call

$$\sigma = \frac{OA}{OB} \quad . \quad . \quad . \quad . \quad (106)$$

$$\text{The leakage factor} = 1 + \frac{OA}{AB}$$

$$= 1 + v \quad . \quad . \quad . \quad (107)$$

$$= 1 + \frac{OA}{OB - OA} = 1 + \frac{\sigma}{1 - \sigma} \quad . \quad . \quad . \quad (108)$$

which gives the relation between the forms usually adopted for the leakage factor and dispersion coefficient.

From our simple diagram, neglecting losses, we have maximum power factor = cos $\angle VOF$, where OF is tangential to the circle

$$= \cos \angle FDO$$

$$= \frac{FD}{OD} = \frac{AB}{2OD}$$

$$= \frac{AB}{2OA + AB} \quad . \quad . \quad . \quad (109)$$

$$= \frac{1}{2v + 1} \quad . \quad . \quad . \quad . \quad (110)$$

$$\text{where } v = \frac{OA}{AB}$$

CHAPTER II

THE SINGLE-PHASE INDUCTION MOTOR

THE single-phase induction motor may be regarded as a special case of the polyphase motor. If one phase of a two-phase motor be opened when the machine is running without load, the motor will continue to run at almost the same speed as before. The only change is in the nature of the hum emitted, and the current per phase will be approximately doubled. The total current and power are nearly unchanged. A voltmeter connected to the idle phase shows that nearly the full-line voltage is present, and further investigation shows that the voltages of the two phases are in quadrature.

Again the application of test coils arranged around the core shows the presence of the same E.M.F. in each, regardless of the position of the coils, and a phase displacement of E.M.F. dependent on the angular displacement of the coils around the periphery. It is clear that a rotating magnetic field is present in the motor.

A simple alternating field which is stationary in space can be resolved into two fields, each of half the amplitude of the stationary wave. These fields rotate with equal velocity in opposite directions.

We will now examine the influence of these two fields on the action of the motor, and, for this purpose, will consider the speed-torque curves.

Fig. 16 shows the speed-torque curves for the two fields. Curve A is the speed-torque curve with short-circuited rotor, and curves B and C give the same relations with resistance connected in the rotor circuit. Curves A', B', and C' give similar relations for the second rotating wave.

The torque on the motor is the resultant of the torques due to the two rotating waves of flux. Thus consider the machine rotating at the fraction of synchronous speed represented by OD. The torque, due to the positive wave, is DF. The negative torque is DE, and the resultant torque is EF. Combining the torque curves A and A', B and B', C and C' in this manner, we get the resultant speed-torque curves given in Fig. 17.

The following facts are brought out clearly by these curves—

(*a*) With a given magnetic field and a given rotor, the rated output cannot be greater than half of that of the same motor

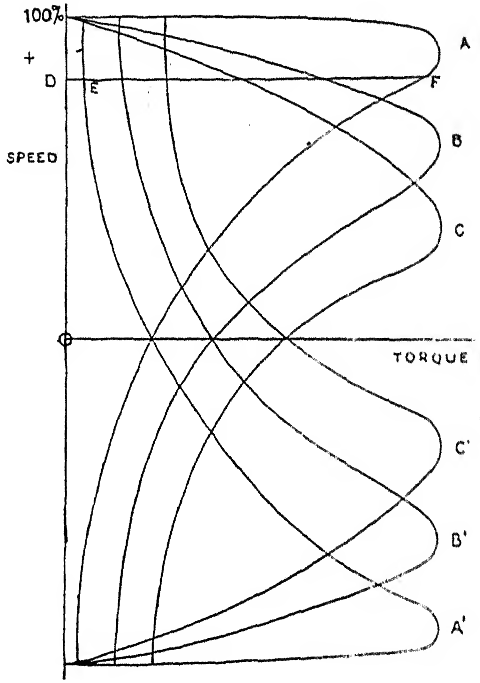


FIG. 16

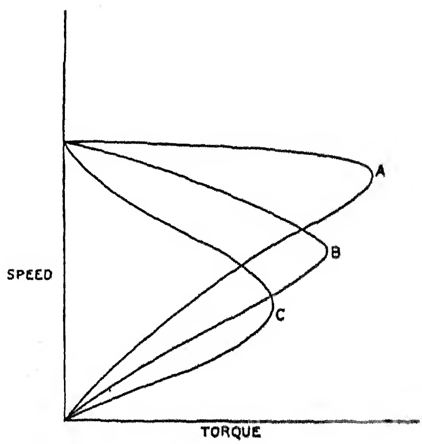


FIG. 17

operated polyphase, since each of the component fields is half the strength of the polyphase field, and further there is a negative torque due to the reverse field.

(*b*) The torque is reduced considerably, and the output very considerably by the introduction of resistance into the rotor circuits. In this respect the single-phase motor differs from the polyphase motor, in which the maximum torque is independent of rotor resistance.

(*c*) The plain single-phase motor has no starting torque. The motor will accelerate to full speed, however, if it is given a start in either direction, provided the load is not too great.

(*d*) The speed at which maximum torque is developed is reduced by the introduction of resistance into the rotor. Thus the introduction of resistance will assist the starting conditions.

Consider first the motor at rest and suppose the rotor winding to consist of isolated coils, each of which is short-circuited on itself. We will assume that a sinusoidal distribution of air-gap flux exists. The instantaneous value of the magnetic induction at a point distant x from the zero value of the flux is given by

$$B = B_0 \sin \omega t \sin \frac{\pi}{\tau} x \quad . \quad . \quad . \quad . \quad . \quad (111)$$

where B_0 = amplitude of the flux density,

$\omega = 2\pi \times$ frequency,

τ = pole pitch.

At a distance y from the origin, the flux density has a value $B_0 \sin \omega t \sin \frac{\pi}{\tau} y$.

Let the length of the conductor be l cm.

The flux in a narrow strip of width dy is

$$B_0 \sin \omega t \sin \frac{\pi}{\tau} y \times dy \times l$$

The total flux in the coil with one side at a point distant x from the zero of the flux wave

$$\phi = \int_x^{x+\tau} B_0 \sin \omega t \sin \frac{\pi}{\tau} yl dy \quad . \quad . \quad . \quad (112)$$

$$= \frac{2\tau}{\pi} l B_0 \sin \omega t \cos \frac{\pi}{\tau} x \quad . \quad . \quad . \quad . \quad (113)$$

and the e.m.f. induced in the coil

$$e = -\frac{d\phi}{dt} = -\frac{2\omega\tau l B_o \cos \omega t \cos\frac{\pi}{\tau}x}{\pi} \quad . \quad . \quad (114)$$

$$\therefore \; e = -E \cos \omega t \cos\frac{\pi}{\tau}x \quad . \quad . \quad . \quad . \quad (115)$$

$$\text{where } E = \frac{2\omega\tau l}{\pi} B_o \quad . \quad . \quad . \quad . \quad . \quad . \quad (116)$$

The instantaneous value of the current is given by

$$i = \frac{-E}{\sqrt{r^2 + L^2\omega^2}} \cos\frac{\pi}{\tau}x \cos(\omega t - \theta) \quad . \quad . \quad (117)$$

$$\text{where } \tan \theta = \frac{L\omega}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (118)$$

r = coil resistance in ohms,

L = coefficient of self-induction in henrys.

It is clear, from the above, that there is a complete lack of uniformity in the value of the current in different coils; i has the greatest value

when $x = o$, and is zero when $x = \frac{\tau}{2}$

Thus the coil in which the induced current is a maximum has its sides in a field of zero value, and the coil in the field of maximum intensity carries no current. Hence neither of these coils experiences any torque. Any coil occupying an intermediate position will be subjected to a definite tangential pull. The force on the two sides of the coil

$$= 2liB \text{ dynes (if } i \text{ is in } CGS \text{ units)} \quad . \quad . \quad . \quad (119)$$

Substituting the values of B and i already found, we have for the total tangential pull on the coil

$$2l\left(\frac{-E}{\sqrt{r^2 + L^2\omega^2}} \cos\frac{\pi}{\tau}x \cos(\omega t - \theta) B_o \sin \omega t \sin\frac{\pi}{\tau}x\right) \quad . \quad (120)$$

$$= -\frac{2lEB_o}{\sqrt{r^2 + L^2\omega^2}} \times \frac{1}{2} \sin\frac{2\pi}{\tau}x \sin \omega t \cos(\omega t - \theta) \quad . \quad (121)$$

$$= -T_o \sin\frac{2\pi}{\tau}x \sin\omega t \cos(\omega t - \theta) \quad . \quad . \quad . \quad . \quad (122)$$

$$\text{where } T_o = \frac{lEB_o}{\sqrt{r^2 + L^2\omega^2}} \quad . \quad . \quad . \quad . \quad . \quad (123)$$

Now, $2 \sin \omega t \cos(\omega t - \theta) = \sin(2\omega t - \theta) + \sin\theta$ . . (124)

$$\therefore \text{ Pull} = -\frac{T_o}{2}\sin\frac{2\pi}{\tau}x\{\sin(2\omega t - \theta) + \sin\theta\} \quad . \quad . \quad (125)$$

The mean value of $\sin(2\omega t - \theta)$ over any complete number of periods vanishes, and the

$$\text{Mean pull} = -\frac{T_o}{2}\sin\frac{2\pi}{\tau}x \sin\theta \quad . \quad . \quad . \quad . \quad . \quad (126)$$

Any coil for which x is less than $\frac{\tau}{2}$ experiences a pull in one direction, and any coil for which x is $< \frac{\tau}{2}$ experiences a pull in the opposite direction. Corresponding to any coil distant x from the origin, there is another coil at a distance of $x + \frac{\tau}{2}$. It follows that the tangential pulls acting on the various coils balance each other, so that the resultant torque of the motor is zero at standstill.

In a similar manner, the torque exerted when the motor is running may be found. As before, one coil is considered, and x is regarded as variable. If v = peripheral velocity of the rotor, then $x = vt$.

The flux linking the coil at any time t is given by

$$\phi = \frac{2\tau l}{\pi}B_o \sin\omega t \cos\frac{\pi}{\tau}vt \quad . \quad . \quad . \quad . \quad . \quad (127)$$

$$\therefore \phi = \frac{\tau l B_o}{\pi}\{\sin(\omega + \omega_1)t + \sin(\omega - \omega_1)t\} \quad . \quad . \quad . \quad (128)$$

$$\text{where } \frac{\pi}{\tau}v = \omega_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (129)$$

$$\text{The E.M.F. induced in the coil} = e = -\frac{d\phi}{dt} \quad . \quad . \quad . \quad (130)$$

$$= -\frac{\tau l B_o}{\pi}\{(\omega + \omega_1)\cos(\omega + \omega_1)t + (\omega - \omega_1)\cos(\omega - \omega_1)t\} \quad (131)$$

The current in the coil at any time $t = i$

$$\text{and } i = -\frac{\tau l B_o}{\pi}\left\{\frac{\omega + \omega_1}{\sqrt{r^2 + L^2(\omega + \omega_1)^2}}\cos\left[(\omega + \omega_1)t - \theta_1\right]\right.$$
$$\left.+\frac{\omega - \omega_1}{\sqrt{r^2 + L^2(\omega - \omega_1)^2}}\cos\left[(\omega - \omega_1)t - \theta_2\right]\right\} \quad . \quad (132)$$

where θ_1 and θ_2 are given by

$$\tan\theta_1 = \frac{(\omega + \omega_1)}{r}L \text{ and } \tan\theta_2 = \frac{(\omega - \omega_1)}{r}L \quad . \quad . \quad (133)$$

$$\therefore\ i = -\frac{\tau l B_o}{\pi L}\{\sin\theta_1\cos\left[(\omega + \omega_1)t - \theta_1\right]$$
$$+ \sin\theta_2\cos\left[(\omega - \omega_1)t - \theta_2\right]\} \quad . \quad (134)$$

The tangential pull on the coil considered

$$= 2liB \quad . \quad . \quad . \quad . \quad . \quad . \quad (135)$$
$$B = B_o\sin\omega t\sin\omega_1 t \quad . \quad . \quad . \quad . \quad (136)$$
$$= \tfrac{1}{2}B_o\{\cos(\omega - \omega_1)t - \cos(\omega + \omega_1)t\} \quad . \quad (137)$$

In finding the mean value of the tangential pull, it is only necessary to consider the product of terms of the same frequency, since the products of terms of different frequency vanishes when integrated over a period.

$$\text{The mean pull} = -\frac{\tau l^2 B_o^2}{2\pi L}(-\sin\theta_1\cos\theta_1 + \sin\theta_2\cos\theta_2) \quad (138)$$

$$= -\frac{\tau l^2 B_o^2}{4\pi L}(\sin 2\theta_2 - \sin 2\theta_1) \quad . \quad . \quad . \quad (139)$$

In order to find the direction of the pull, suppose a positive current sent round the coil, i.e. a current tending to produce a magnetic field in the same direction as the field. Since both field and current are positive, their product is positive, and Fleming's rule shows that in this case the coil will move to the left. But we have taken as the positive direction of motion from left to right. Therefore a negative value of the pull corresponds to the positive direction of motion. If $\sin 2\theta_2 > \sin 2\theta_1$, the motor will exert a driving torque, and an opposing torque if $\sin 2\theta_2 < \sin 2\theta_1$. At standstill $\theta_1 = \theta_2$, and the torque vanishes. If, as is always the case, $\frac{L\omega}{r}$ is a large quantity, then for a small value of ω_1, i.e. a low speed, both θ_1 and θ_2 will be large angles, and $2\theta_1$ and $2\theta_2$

will both exceed 90°, θ_1 being the greater of the two, the motor will exert a driving torque.

If we plot $\sin 2\theta_2 - \sin 2\theta_1$ as a function of the speed, we obtain a curve which shows the variation of torque with speed on the supposition that B_o is constant.

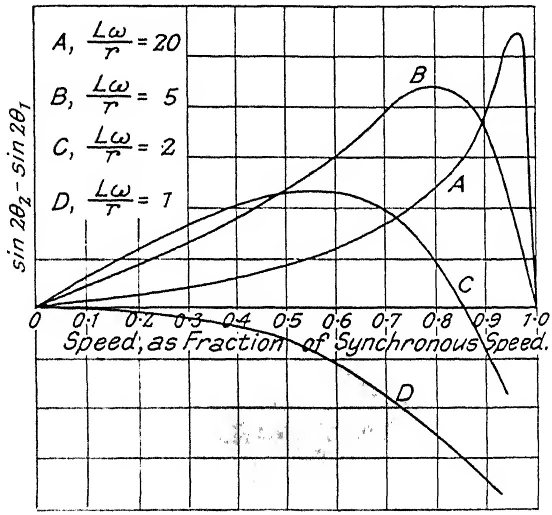


FIG. 18

The exact shape of the curve will depend on the value of

$$\frac{L\omega}{r} = \frac{\text{leakage reactance at full frequency}}{\text{resistance}}$$

Curves A, B, C, D represent 4 curves for 4 values of $\frac{L\omega}{r}$. Curves A and B are typical of ordinary single-phase motors. Curves C and D represent cases where extra resistance is introduced in the rotor circuits.

As previously noted, the maximum torque is rapidly reduced, and when $\frac{L\omega}{r} = 1$, the motor becomes incapable of exerting a driving torque at any speed.

It may be shown that the locus of the primary current vector

for the single-phase motor is a semicircle, and indeed the performance can be deduced from the circle diagram with fair accuracy.

The following method of dealing with the problem is due to Branson. The method of approach is from the transformer theory as distinct from the rotating field theory.

Suppose a two-phase motor running light, and provided with a squirrel-cage rotor.

Phase B produces the flux ϕ_b, and phase A the flux ϕ_a. Now suppose phase A to be disconnected from the line. The flux ϕ_a

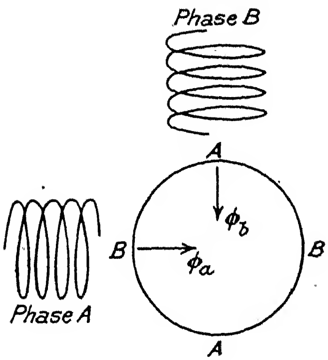


FIG. 19

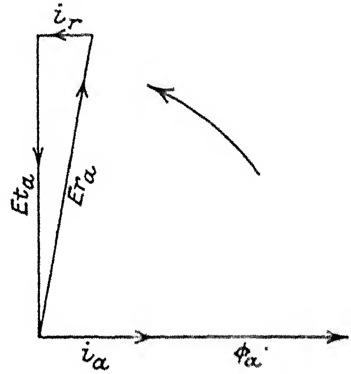


FIG. 20

and the two E.M.F.'s due to it, viz., E_{rb}, the rotational E.M.F., and E_{ta}, the transformer E.M.F. will disappear. It will be noticed that each flux produces two E.M.F.'s: the rotational E.M.F. and pulsational E.M.F. in two circuits at right angles. In the two-phase motor, in the circuit AA, we have a rotational E.M.F., viz., E_{rb} and a transformer E.M.F. E_{ta}, and these are almost in phase opposition. Similarly in circuit BB, we have a rotational E.M.F. E_{ra} and a transformer E.M.F. E_{tb} due to ϕ_b.

Now when phase A is disconnected, the E.M.F.'s due to it, viz., E_{rb} and E_{ta}, will disappear, and the two remaining E.M.F.'s E_{tb} in circuit B and E_{ra} in circuit A will be left unopposed, so that currents will flow in both circuits. Due to E_{ra} a current will flow, and a flux, called the cross-flux, will appear, and a transformer E.M.F. E_{ta} will be induced, which will be approximately in phase opposition to E_{ra}.

The phase relation of the E.M.F.'s in circuit A is shown in the figure above.

With the appearance of the cross-flux, a rotational E.M.F. will again appear in circuit B, and the phase relation of this rotational E.M.F. E_{rb} to the opposing transformer E.M.F. E_{tb} is such as to leave an effective E.M.F. in the circuit so that a current will flow. At synchronous speed the value and phase relation of the effective E.M.F. in circuit B is such as to make the current exactly equal and at right angles to the magnetizing current in circuit A. At

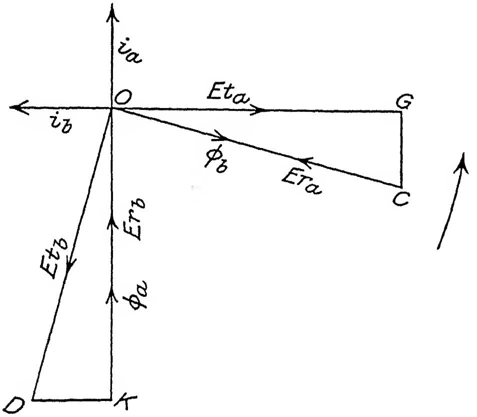


FIG. 21

synchronous speed, transformer and rotational E.M.F.'s due to the same flux must be equal and in phase quadrature.

The main flux ϕ_b induces the transformer E.M.F. E_{tb} represented by OD in Fig. 21, and generates in circuit A the rotational E.M.F. E_{ra} shown as OC. $OD = OC$ at synchronous speed, and COD is a right angle. For similar reasons, OK and OG, which represent the E.M.F.'s due to the cross-flux, must all be equal and at right angles;

$\therefore$ the angle DOK = angle COG, and the triangles COG and DOK are equal.

DK = resistance drop in circuit B,

= CG = resistance drop in circuit A,

$\therefore$ the currents must be equal, and DK and CG are at right angles.

These no-load rotor currents will be called i_a and i_b respectively. Current i_a cannot react directly on the primary, since circuit A in which it flows is not in inductive relation with the primary winding.

Current i_b does react on the primary and, as a consequence of its demagnetizing action, an additional no-load current i_{ms} flows in the primary winding. At a speed slightly above synchronous

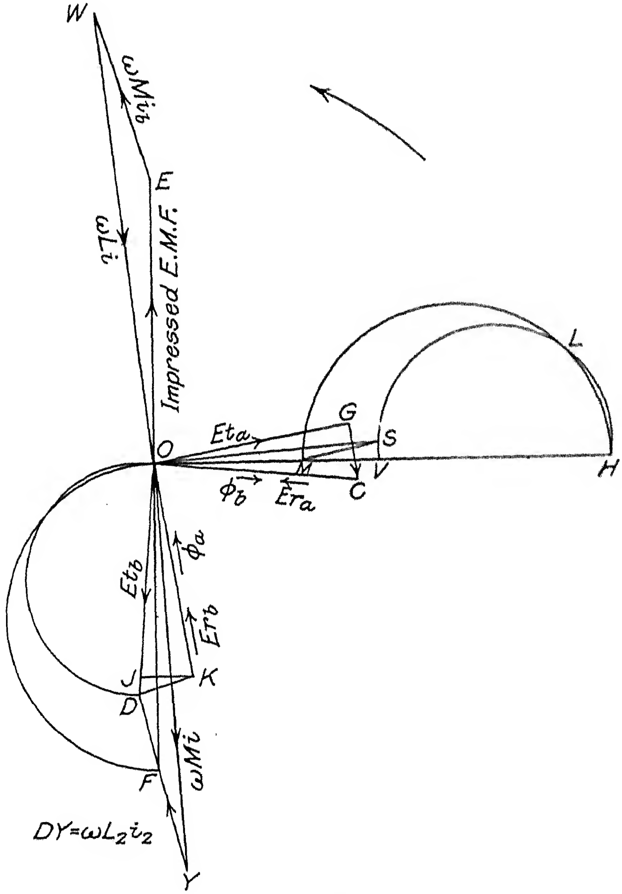


FIG. 22

speed, i_{ms} becomes wattless, and is represented by MV in the diagram (Fig. 22).

Lines OS and MS represent the primary and secondary currents when the motor is running at synchronous speed.

OW, EW, OY, and DY represent the hypothetical E.M.F.'s corresponding to primary and secondary currents.

Since the rotational E.M.F.'s vary with speed, it is evident that.

at slightly above synchronous speed, the value of E_{rb} will be increased to such an extent as to bring DK into exact phase with OM. At this speed, i_{ms} will be represented by MV. The primary

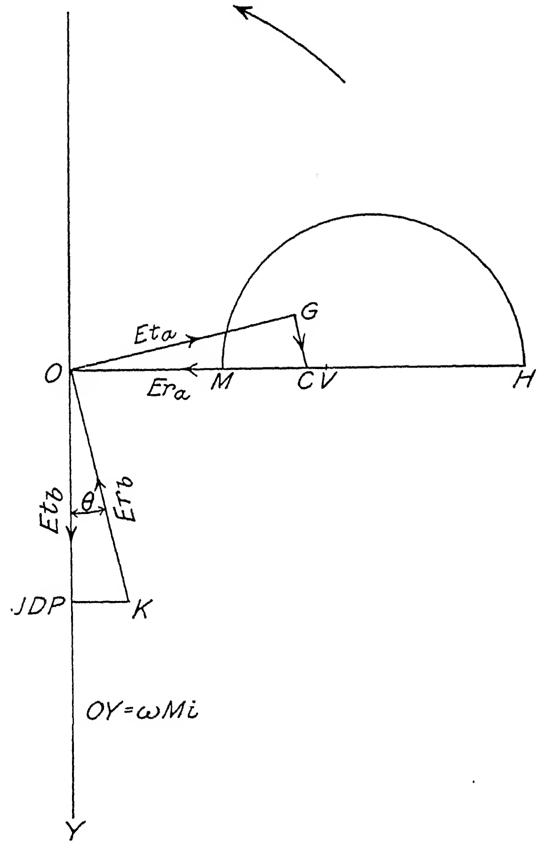


Fig. 23

current will then be wattless and the rotor copper loss will be supplied mechanically. This condition is represented in Fig. 23.

The angle ODK is a right angle, and, at the speed at which i_{ms} becomes wattless, the cross-flux is equal to the main flux.

$$\cos\theta = \frac{OD}{OK} = \frac{OG}{OC} \quad . \quad . \quad . \quad . \quad . \quad (140)$$

$$\therefore \frac{OD}{OG} = \frac{OK}{OC} \text{ i.e. } \frac{E_{tb}}{E_{ta}} = \frac{E_{rb}}{E_{ra}} \quad . \quad . \quad (141)$$

$$\text{Also } \frac{\text{revs. per min.}}{\text{syn. speed}} = \frac{E_{ra}}{E_{tb}} = \frac{E_{rb}}{E_{ta}} \quad . \quad . \quad (142)$$

$$\therefore E_{ra} = \frac{E_{tb} \times E_{rb}}{E_{ta}} \quad . \quad . \quad (143)$$

$$= \frac{E_{rb}^2}{E_{ra}} \quad . \quad . \quad . \quad . \quad (144)$$

$$\therefore E_{ra} = E_{rb} \quad . \quad . \quad . \quad . \quad (145)$$

$$\text{and } \therefore \phi_a = \phi_b \quad . \quad . \quad . \quad . \quad (146)$$

Also, since the two triangles OGC and ODK are equal,

$$DK = GC, \text{ i.e. } i_a = i_b$$

The current i_{ms} is the increase in primary current due to the demagnetizing action of i_b; $i_b = i_a$, and the latter is fixed by the fact that it is the magnetizing current which produces the cross flux ϕ_a. We have to determine the demagnetizing effect on the primary of a rotor current of such a value as to produce in the rotor a flux equal to the main motor flux.

$$\text{Now } E_{tb} = M\omega i - \frac{M\omega i_b}{K_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad (147)$$

M = coefficient of mutual induction of primary and secondary

ω = $2\pi \times$ frequency

$$K_s = \frac{\text{permeance of mutual flux path}}{\text{permeance of mutual and secondary leakage paths in parallel}}$$

L_1 = coefficient of self-induction of primary

L_2 = coefficient of self-induction of secondary

$$K_p = \frac{\text{permeance of mutual path}}{\text{permeance of mutual and primary leakage paths in parallel}}$$

The mutual path refers, of course, to the path of the flux which links both primary and secondary windings.

$$K_p = \frac{M}{L_1}, \quad K_s = \frac{M}{L_2} \quad . \quad . \quad . \quad (148)$$

The rotor magnetizing current required to produce the flux by which the above E.M.F. is generated,

$$i_a = \left(M\omega i - \frac{M\omega i_b}{K_s}\right) \times \frac{K_s}{M\omega} \quad . \quad . \quad . \quad . \quad (149)$$

$$= K_s i - i_b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (150)$$

and since $i_a = i_b$

$$i_b = K_s i - i_b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (151)$$

$$\therefore i_b = \frac{K_s i}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (152)$$

Now the increase of current in the primary due to the demagnetizing effect of a secondary current equals the secondary current multiplied by K_p.

This may be shown thus—

E.M.F. set up by i_b in primary $\equiv M\omega i_b$ . . . . (153)

if x = current which flows in the primary to offset the action of i_b,

$$\text{then } L_1\omega x = \frac{M}{K_p}\omega \times x = M\omega i_b \quad . \quad . \quad . \quad (154)$$

$$\therefore x = K_p i_b \quad . \quad . \quad . \quad . \quad . \quad . \quad (155)$$

$$\therefore i_{ms} = \frac{K_p K_s}{2} i \quad . \quad . \quad . \quad . \quad . \quad (156)$$

$$= \frac{K_p K_s}{2}(i_m + i_{ms}) \quad . \quad . \quad . \quad . \quad (157)$$

$$\therefore i_{ms}\left(1 - \frac{K_p K_s}{2}\right) = \frac{K_p K_s}{2} \times i_m \quad . \quad . \quad . \quad . \quad . \quad (158)$$

$$\therefore i_{ms} = i_m \frac{K_p K_s}{2 - K_p K_s} \quad . \quad . \quad . \quad . \quad . \quad (159)$$

$$\text{i.e. } MV = OM \frac{K_p K_s}{2 - K_p K_s} \quad . \quad . \quad . \quad . \quad . \quad (160)$$

Let i_h = primary current when the secondary is short-circuited

i_{2h} = secondary current under same condition.

Then the secondary E.M.F. due to $i_h = M\omega i_h$ . . (161)

the secondary E.M.F. due to $i_{2h} = \dfrac{M}{K_s}\omega i_{2h}$ . . (162)

Neglecting resistance, these must be equal,

$$\therefore i_{2h} = K_s i_h \quad . \quad . \quad . \quad . \quad (163)$$

The increase in primary current due to the demagnetizing effect of

$$i_{2h} = i_{2h} \times K_p = i_h K_p K_s \quad . \quad . \quad . \quad (164)$$

The total current in the primary on short circuit of secondary

$$= i_h = i_m + i_h K_p K_s \quad . \quad . \quad . \quad . \quad (165)$$

$$\therefore\ i_h = \frac{i_m}{1 - K_p K_s} \quad . \quad . \quad . \quad . \quad . \quad (166)$$

where i_m = magnetizing current at standstill.

$$\therefore\ K_p K_s = 1 - \frac{i_m}{i_h} \quad . \quad . \quad . \quad . \quad . \quad (167)$$

$$2 - K_p K_s = 1 + \frac{i_m}{i_h} \quad . \quad . \quad . \quad . \quad . \quad (168)$$

$$\therefore\ MV = OM \left\{\frac{1 - \sigma}{1 + \sigma}\right\} \text{ where } \sigma = \frac{i_m}{i_h} \quad . \quad (169)$$

$$\sigma = \frac{\text{magnetizing current at standstill}}{\text{short-circuit current at standstill}}$$

Note that MV is independent of the secondary resistance.

We shall now prove that the locus of the extremity of the primary current vector T is a semicircle.

The above diagram shows the currents and the hypothetical E.M.F. vectors.

Thus OM = magnetizing current at standstill

MV = additional wattless current which flows in the stator, or the reflected no-load secondary current in the primary at a speed slightly above synchronous speed

OH = short-circuit current

$OY = M\omega i$ = hypothetical E.M.F. induced in the rotor due to primary current OT and at right angles to OT

$DY = L_2 \omega i_2$ = total self-induced voltage in the secondary due to secondary current i_2

It will be noticed that L_2 is the coefficient of self-induction due to mutual flux plus leakage flux.

$OB = \phi_b$; $OA = \phi_a$

$OG = E_{ta}$; $OC = E_{ra}$; $CG = i_a r_2$

$OK = E_{rb}$; $DK = i_2 r_2$; OE = impressed E.M.F.

$EW = M\omega i_2 =$ E.M.F. induced in primary due to i_2

$OW = L_1\omega i =$ E.M.F. induced to primary due to current i, where $L_1 =$ total coefficient of self-induction due to leakage and mutual flux

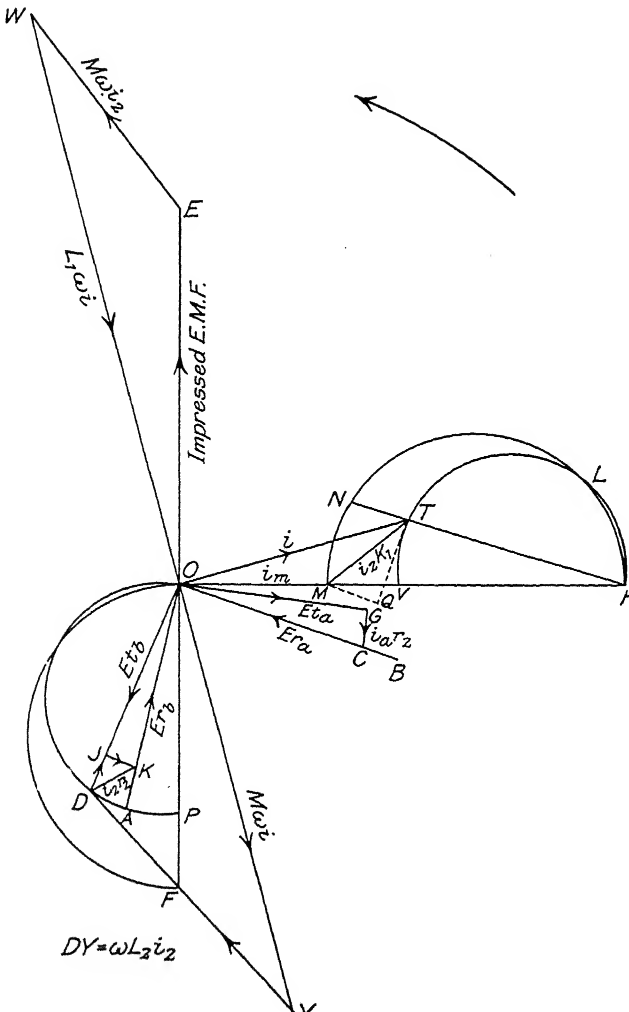


FIG. 24

Triangles FDO and MTH are similar, for each line in FDO is perpendicular to a corresponding line in MTH.

Let $S_e =$ volts per unit length = scale of voltage

$S_1 =$ primary amps. per unit length in the diagram

$=$ primary current scale

$S_2 =$ secondary amps. per unit length $= \dfrac{S_1}{K_p}$

Now $$FD \times S_e = i_2 X_2 \qquad (170)$$

where X_2 = equivalent reactance reduced to the secondary

$$MT \times S_1 = i_2 K_p \qquad (171)$$

$$\therefore MT \times S_1 \times \frac{X_2}{K_p} = i_2 K_p \frac{X_2}{K_p} = i_2 X_2 \qquad (172)$$

$$\therefore FD = MT \times \frac{S_1}{S_e} \times \frac{X_2}{K_p} \qquad (173)$$

Also $FO \times S_e = M\omega i_m$,

for triangles OWE, OYF, and OTM are similar,

$$\text{and } FO = M\omega i \times \frac{OM}{OT} = M\omega i \times \frac{i_m}{i} = M\omega i_m \qquad (174)$$

$$MHS_1 = (OH - OM)\, S_1 = \frac{E}{X_1} - i_m \qquad (175)$$

where X_1 = equivalent leakage reactance reduced to primary,

$$\therefore MHS_1 \frac{X_2}{K_p} = \left(\frac{E}{X_1} - i_m\right) \frac{X_2}{K_p} \qquad (176)$$

$$\text{but } \frac{E}{X_1} = i_h = \frac{i_m}{1 - K_p K_s} \qquad (177)$$

$$\therefore X_1 = \left(\frac{1 - K_p K_s}{i_m}\right) E = \left(\frac{1 - K_p K_s}{i_m}\right) L_1 \omega i_m$$

$$= (1 - K_p K_s) \frac{\omega M}{K_p} \qquad (178)$$

Similarly X_2, the equivalent leakage reactance reduced to secondary $= (1 - K_p K_s) \dfrac{\omega M}{K_s}$

$$\therefore MHS_1 \frac{X_2}{K_p} = \left(\frac{E}{X_1} - i_m\right) \frac{X_2}{K_p} \qquad (179)$$

$$= \frac{EX_2}{K_pX_1} - i_m \frac{X_2}{K_p}$$

$$= \frac{E}{K_s} - \left(\frac{M\omega i_m - K_pK_s M\omega i_m}{K_pK_s}\right)$$

$$= \frac{M\omega i_m}{K_pK_s} - \left(\frac{M\omega i_m - K_pK_s M\omega i_m}{K_pK_s}\right)$$

$$= M\omega i_m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (180)$$

$$\therefore\ FO \times S_e = MHS_1 \frac{X_2}{K_p} \quad . \quad . \quad . \quad . \quad . \quad . \quad (181)$$

Thus $\frac{FD}{MT} = \frac{FO}{MH}$, and FD is at right angles to MT, and FO is at right angles to MH

$\therefore\ \Delta_s MTH$ and FDO are similar

$\therefore\ DO$ is at right angles to TH and equals $TH \frac{S_1X_2}{S_eK_p}$ (182)

In the same way, OP is proportional to VH and

$$= VH \times \frac{S_1X_2}{S_eK_p} \quad (183)$$

DO represents the resultant induced E.M.F. in the rotor.

OB at right angles to DO represents the main rotor flux ϕ_b; OC a part of OB represents the rotational E.M.F. in circuit A; CG and OG represent respectively the resistance drop and transformer induced E.M.F. in circuit A.

OA at right angles to OG represents the cross flux; E_{rb}, the rotational E.M.F. in circuit B, is represented by OK.

The resistance drop DK and E_{rb} are not in phase due to the fact that MT has a component MQ which is 90° out of phase with E_{tb} or, in other words, parallel to TH. At a speed slightly above synchronous speed, the secondary current in circuit B is wattless, and MT coincides with MV. At lower speeds the wattless component of the secondary current is represented by MQ.

The voltage drops due to the two components of the secondary current are

$$DJ \times S_e = TQ \times \frac{S_1}{K_p} r_2 = MN \frac{S_1}{K_p} r_2 \quad . \quad . \quad . \quad (184)$$

and $JKS_e = MQ \frac{S_1}{K_p} r_2 = NT \frac{S_1}{K_p} r_2$ . . (185)

$\angle DOA = \angle GOB$ and OJK is a right angle by construction,

$$\therefore JK \propto OJ$$

At the speed at which MT and NT coincide with MV, the line OJ coincides with OP.

$$\therefore NT \frac{S_1}{K_p} r_2 = MV \frac{S_1}{K_p} r_2 \times \frac{OJ}{OP} \quad . \quad . \quad . \quad . \quad (186)$$

$$\therefore NT = MV \frac{OJ}{OP}$$

$$= MV \left(\frac{OD - DJ}{OP}\right) \quad . \quad . \quad . \quad . \quad (187)$$

$$OD = TH \frac{S_1 X_2}{S_e K_p}; \quad OP = VH \frac{S_1 X_2}{S_e K_p} \quad . \quad . \quad (188)$$

$$\text{and } DJ = NM \frac{S_1 r_2}{S_e K_p} \quad . \quad . \quad . \quad . \quad . \quad (189)$$

$$\therefore NT = MV \left\{ \frac{TH \dfrac{S_1 X_2}{S_e K_p} - MN \dfrac{S_1 r_2}{S_e K_p}}{VH \dfrac{S_1 X_2}{S_e K_p}} \right\}$$

$$= MV \left\{ \frac{TH - MN \dfrac{r_2}{X_2}}{VH} \right\} \quad . \quad . \quad . \quad (190)$$

Let $NT = e$ | Let $D = MH$

$R = TH$ | $K = \dfrac{r_2}{X_2}$

$b = MN$

$C = VH$ | $MV = d$

$$\therefore e = d \times \frac{R - bK}{C} \quad . \quad . \quad . \quad . \quad . \quad (191)$$

$$R = e\frac{c}{d} + bK$$

$$R + e = D \cos \theta \quad . \quad . \quad . \quad . \quad . \quad (192)$$

$$\frac{b}{D} = \sin \theta \text{ and } e = D \cos \theta - R \quad . \quad . \quad (193)$$

$$\therefore R = (D \cos \theta - R) \frac{c}{d} + KD \sin \theta \quad . \quad . \quad (194)$$

$$R + R\frac{c}{d} = D \cos \theta \frac{c}{d} + KD \sin \theta \quad . \quad . \quad . \quad (195)$$

$$R\left(\frac{d+c}{d}\right) = \frac{cD}{d} \cos \theta + KD \sin \theta \quad . \quad . \quad . \quad (196)$$

$$R\frac{D}{d} = \frac{cD}{d} \cos \theta + KD \sin \theta \quad . \quad . \quad . \quad (197)$$

$$\therefore \; R = c \cos \theta + Kd \sin \theta \quad . \quad . \quad . \quad . \quad (198)$$

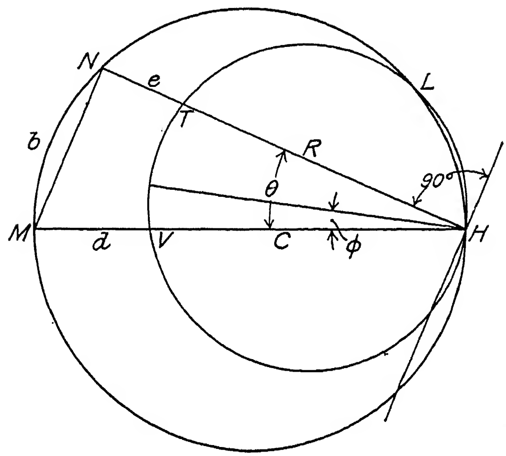


FIG. 25

Let ϕ be an angle whose tangent is $= K\frac{d}{c}$

and M the denominator of a fraction such that $\sin \phi = K\frac{d}{M}$

$$\cos \phi = \frac{\sin \phi}{\tan \phi} = \frac{\frac{Kd}{M}}{\frac{Kd}{c}} = \frac{c}{M} \quad . \quad . \quad . \quad . \quad (199)$$

$$\therefore \; \frac{R}{M} = \frac{c}{M} \cos \theta + \frac{Kd}{M} \sin \theta \quad . \quad . \quad . \quad (200)$$

$$= \cos \phi \, . \cos \theta + \sin \phi \, . \sin \theta$$

$$= \cos (\theta - \phi)$$

$$\therefore \; R = M \cos (\theta - \phi) \; . \quad . \quad . \quad . \quad . \quad (201)$$

Thus the locus of T is a semicircle passing through H.

The centre of this circle is above MH and is at a' distance

$$0{\cdot}5\, MV \frac{r_2}{X_2} \quad . \quad . \quad . \quad . \quad . \quad (202)$$

for diameter $\times$ $0{\cdot}5 \sin\phi = M \,.\, 0{\cdot}5 \sin\phi$

When $\theta = \phi$, M is $= R$ and $\therefore$ to the diameter

$$\therefore a' = 0{\cdot}5\, Kd$$

$$= 0{\cdot}5\, MV \frac{r_2}{X_2} \quad . \quad . \quad . \quad . \quad . \quad (203)$$

Since $\dfrac{\text{rotational E.M.F.}}{\text{transformer E.M.F.}}$ for the same flux $= \dfrac{\text{revs. per min.}}{\text{syn. speed}}$

$$\frac{OC}{OD} = \frac{\text{revs. per min.}}{\text{syn. speed}} \quad . \quad . \quad . \quad . \quad . \quad (204)$$

$$\frac{OK}{OG} = \frac{\text{revs. per min.}}{\text{syn. speed}} \quad . \quad . \quad . \quad . \quad . \quad (205)$$

$$\therefore \frac{\text{R.P.M.}}{\text{syn. speed}} = \sqrt{\frac{OC}{OD} \times \frac{OK}{OG}} \quad . \quad . \quad . \quad . \quad . \quad (206)$$

$$= \sqrt{\frac{TH - MN \dfrac{r_2}{X_2}}{TH}} \quad . \quad . \quad . \quad . \quad . \quad (207)$$

Effect of primary resistance. As pointed out by Branson, this can be taken care of by producing the primary current vector to Z, making OZ of such a length as to represent the resistance drop due to current OT. The vector sum ZE represents the impressed E.M.F. which would be required to make the induced E.M.F. equal to OE.

$$\therefore \frac{OE}{ZE} = \frac{\text{induced E.M.F.}}{\text{impressed E.M.F.}}$$

$$\text{The primary current} = OT \,.\, \times S_1 \times \frac{OE}{ZE} \quad . \quad . \quad . \quad . \quad (208)$$

$$\text{secondary current} = MT \,.\, S_2 \,.\, \frac{OE}{ZE} \quad . \quad . \quad . \quad . \quad . \quad (209)$$

CHAPTER III

STARTING DEVICES FOR SINGLE-PHASE MOTORS

SINCE the single-phase induction motor is not self-starting, it is necessary to supply some starting device. This may take the form of—

(*a*) Starting by hand by a pull on the belt in the case of small motors. In this case the motor will need to be started without load, and the rotor must have a fairly high resistance.

(*b*) Providing the rotor with a commutator winding and converting the machine into a shunt or series motor.

(*c*) By displacing the rotor magnetic axis from that of the stator. Again, a commutator winding is required, and brushes short-circuiting secondary coils in the position of effective torque and open-circuiting them in the position of opposing torque. In other words, the machine is started up as a repulsion motor; and when it is up to speed, the rotor is short-circuited by a centrifugal device and the machine runs as a single-phase induction motor. This method is used fairly extensively for small motors.

(*d*) By providing a starting winding displaced in position by 90° from the running winding and using a phase-splitting device for starting up the motor as a two-phase motor. The starting winding is cut out of circuit when the motor is up to speed.

In some cases the starting winding is designed with a large leakage reactance, and a non-inductive resistance is connected in series with the running winding.

In all cases the idea is to produce two fluxes in time quadrature and in space quadrature. This is partially accomplished by using inductance or capacity in series with one phase and a non-inductive resistance in series with the other.

If ϕ_r = main flux of motor

ϕ_a = flux due to starting winding

ω = space angle of the two fluxes

α = time angle between the two fluxes

then the torque is proportional to $\phi_r\phi_a \sin\omega \sin\alpha$; and in the two-phase motor, with equal fluxes and space and time quadrature relations,

$$\text{torque} \propto \phi_r^2$$

The torque ratio of the starting device

$$= \frac{\phi_a \sin \omega \sin \alpha}{\phi_r} \quad . \quad . \quad . \quad . \quad (210)$$

In general it may be said that single-phase motors are in every way inferior to polyphase machines. Their output for a given

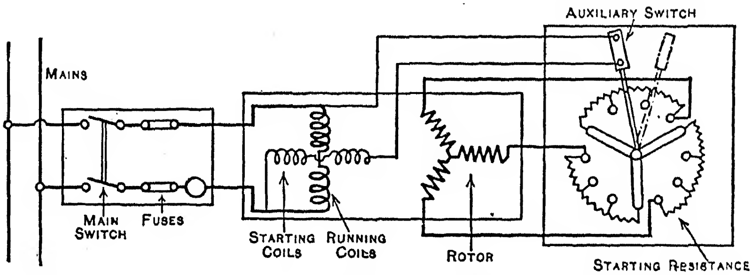


Fig. 26.—Arrangement of Heyland S.P. Motor for Starting on Full Torque

frame is but one-half that of the three-phase motor and their power-factor many per cent lower, especially on machines of low speed. They are regarded with dislike and almost disgust by most electrical manufacturers, and rightly so.

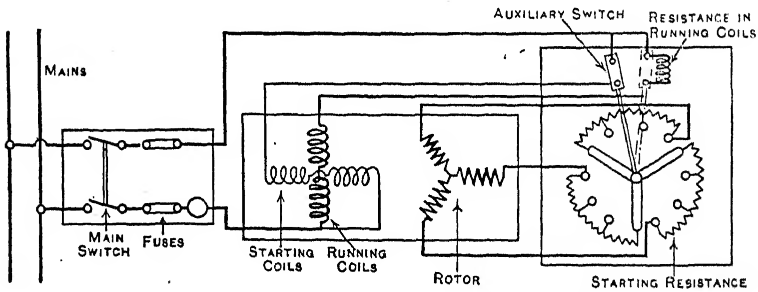


Fig. 27.—Arrangement of Heyland S.P. Motor for Starting on No Load

The induction generator. If the rotor of an induction motor be driven above synchronous speed, the stator being connected to an A.C. supply, the machine will act as a generator and supply power to the mains.

It is obvious that the direction of motion of the conductors relative to the field is reversed, and therefore the E.M.F. and the

current in the conductors are reversed. A component of current in the stator, in the reverse direction, is needed to offset the magnetizing action of the rotor currents as in the transformer. Since the current is reversed, the torque is reversed, and the machine acts as generator.

The locus of the extremity of the primary current vector is still a semicircle, and indeed forms a complete circle with the semicircle of the motor. If f_o is the frequency corresponding to the speed, and f_2 the secondary frequency, and f_1 the frequency of the primary, then $f_o = f_1 + f_2$.

The three powers, viz., primary output, secondary output, and mechanical input are proportional respectively to f_1, f_2, and f_o.

The machine thus converts mechanical power into electric power at two frequencies f_1 and f_2, and is called a synchronous-induction generator, and the machine revolves in synchronism with the sum of the frequencies generated by it.

It is clear from our diagram that this machine requires a reactive lagging current for excitation—in other words, it is not self-exciting—and this excitation must be supplied from synchronous apparatus, or a static condenser may be used. The latter, however, is seldom practicable. Moreover the induction generator can deliver only *leading* current. If a load requires a lagging current, a synchronous machine in parallel with the induction generator must supply this.

It will be seen from Fig. 28 that the induction generator can furnish current at one power factor only for each value of the load, and this power factor is leading. It follows therefore that considerable kVA in synchronous apparatus is required for cases where loads of lagging power-factor exist, such as induction motors, etc. This is the principal objection to its use.

The advantages of the induction generator are that it does not hunt and its absence of synchronizing troubles, and the simplicity of switch-gear. It is simple and rugged, and, when short-circuited, delivers little or no power, because its excitation becomes zero. Its principal use seems to be in the development of small water-powers. It needs no synchronizing, requires no direct-current excitation, and does not fall out of synchronism. It delivers power if there is sufficient water; if not, it merely runs idle as an induction motor. It is evident also, from the diagram, that the maximum output as generator will not be so large as the maximum output as motor. If this output is exceeded, and the torque applied to the machine is maintained, the machine will speed up indefinitely unless the governor of the prime mover acts to limit

the speed. Also the maximum power factor is less as a generator than as motor.

It will be seen that by connecting the stator to a circuit of frequency f_1, the rotor generates a frequency $f_2 = f_o - f_1$, or by connecting the rotor to a circuit of frequency f_2, the stator generates a frequency of $f_1 = f_o - f_2$.

The powers generated in stator and rotor are proportional to their respective frequencies. Either circuit can be primary or

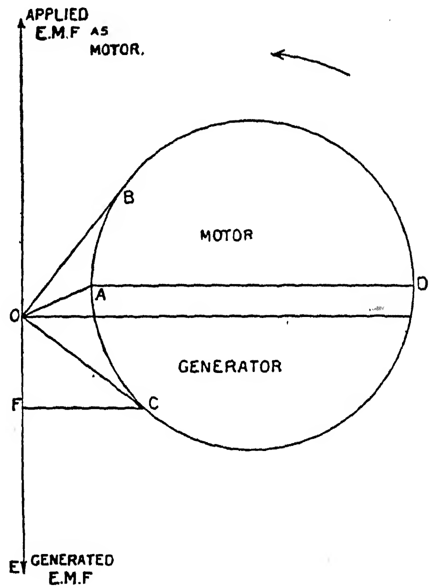


FIG. 28

secondary, and the magnetizing current, required for excitation, can be supplied to the stator circuit at frequency f_1 or to the rotor circuit at frequency f_2. Since the voltage needed for the exciting current is proportional to the frequency, it follows that the volt-amperes required for excitation are small when the low-frequency circuit is used for excitation. Such a machine as two-frequency generator may connect two systems of different frequency, and supply power to both in the ratio of the frequencies.

For example, the rotor may connect to a 25-cycle system and the stator to a 50-cycle system, the frequency of rotation being 75 cycles. If the two frequencies of stator and rotor are equal,

i.e. $f_1 = f_2 = f_{o/2}$, the stator and rotor may be connected in parallel or in series, and the machine generates power at half the frequency of its speed. Such a double synchronous alternator may be useful for prime movers of very high speed, as steam turbines. Such a machine may also be used as a frequency converter. The rotor may be connected to a low-frequency exciter of frequency f_2, say, 5 cycles per second, and driven mechanically at a speed corresponding to 45 cycles. The stator then generates power at 50 cycles per second, or 9 times the impressed frequency, and the electric power input is only $\frac{1}{9}$ of the electric power output, the $\frac{8}{9}$ths remaining being given as mechanical power input. If the rotor winding is connected to a commutator and has a frequency f_2, and is driven at a speed corresponding to frequency f_o, the effective frequency supplied by the commutator brushes to the external circuit is $f_o - f_2 = f_1$ or the stator frequency. Hence the rotor may be connected in parallel with the stator. The rotor E.M.F. then equals the stator E.M.F. Since E.M.F. is proportional to the product of frequency and turns,

$$\therefore\ t_2 f_2 = t_1 f_1 \qquad (211)$$

t_2 = number of effective rotor turns

t_1 = number of effective stator turns

$$\therefore\ \frac{f_1}{f_2} = \frac{t_2}{t_1} \qquad (212)$$

i.e. the frequencies are inversely proportional to the number of effective turns in stator and rotor; and, since

$$f_o = f_1 + f_2 \qquad (213)$$

$$1 + \frac{f_2}{f_1} = 1 + \frac{t_1}{t_2}$$

$$\text{i.e.}\ \frac{f_1 + f_2}{f_1} = \frac{t_2 + t_1}{t_2} \qquad (214)$$

$$\text{i.e.}\ \frac{f_o}{f_1} = \frac{t_2 + t_1}{t_2} \qquad (215)$$

$$\therefore\ f_1 = f_o \frac{t_2}{t_2 + t_1} \qquad (216)$$

That is, the frequency generated in the stator

$$= \text{frequency of rotation} \times \frac{t_2}{t_1 + t_2} \qquad (217)$$

This can be given any value by adjusting the turns in stator and rotor, or by interposing a transformer between stator and rotor.

The powers generated are proportional to the frequencies ; and if t_1 and t_2 are very different, one generates much less power than the other.

Thus by the use of the commutator any desired value for f_1 may be obtained, but it is obvious that economical use of material is possible only when f_1 is approximately $\frac{f_o}{2}$

By shifting the commutator brushes, a component of the rotor current can be made to magnetize, and the machine is a self-exciting A.C. generator.

In the double synchronous alternator in which the stator and rotor have the same number of effective turns and phases, and are connected in series or in parallel with each other, the power components of the stator and rotor currents neutralize each other, neglecting the small hysteresis power current ; but the reactive component of the rotor current in its reaction on the stator is reversed by the reversed direction of relative motion. Thus the synchronous exciter of the machine has to supply the total reactive component of the load in addition to the magnetizing current. It is suitable, therefore, for a load requiring leading current.

CHAPTER IV

THE SYNCHRONOUS-INDUCTION GENERATOR WITH LOW-FREQUENCY EXCITATION

THE magnetic field may revolve in the opposite direction to that of mechanical rotation or in the same direction. In the first case, the exciter will be a low-frequency generator and the induction machine a frequency converter. The voltage regulation in the first case is similar to that of a synchronous alternator.

To maintain constant terminal voltage, the excitation must change with the magnitude and character of the load. At constant excitation of the synchronous exciter, the terminal voltage may be found thus—

Let Z_1 = synchronous impedance of the exciter reduced to full frequency f_1

Z_2 = self-inductive impedance of the rotor reduced to full frequency

Z_3 = self-inductive impedance of the stator

and $$Z = Z_1 + Z_2 + Z_3 = r + jx \qquad (218)$$

where $j = \sqrt{-1}$

$\dot{E}_o$ = nominal generated E.M.F. of the exciter generator, corresponding to the field excitation

$$\dot{I} = i - ji_1 = \text{stator output current} \qquad (219)$$

E = terminal voltage $= e_1$, choosing the terminal voltage as real axis

$$\dot{E}_o = E + Z\dot{I} \qquad (220)$$

$$= e_1 + (r + jx)(i - ji_1) \qquad (221)$$

and $$\therefore\ e_1 = \sqrt{e_o^2 - (xi - ri_1)^2} - (ri + xi_1) \qquad (222)$$

where e_o = absolute value of $\dot{E}_o$

By this equation we may calculate the terminal voltage for various values of the load current, which may be non-inductive, inductive, or capacity in character.

In the second case, the exciter is a synchronous motor, and the induction generator produces power in both stator and rotor circuits. The voltage of the stator is controlled by the voltage

impressed on the rotor circuit. If the impedance drop in the rotor circuit is greater than in the stator, with increasing load, the terminal voltage of the machine rises at constant exciter-field excitation; and if the stator and rotor impedance drops are equal, the machine compounds for constant voltage under the same conditions of excitation. By adjusting the relative values of stator and rotor impedances, automatic rise or fall or constancy of terminal voltage can be produced. The above remarks apply only to non-inductive load.

If the current in the stator differs in phase from the generated E.M.F., the rotor current also differs; but a lagging component of stator current corresponds to a leading component of rotor current, since the stator circuit slips behind, and the rotor circuit is driven ahead of the rotating field and inversely.

The reactive voltage of the lagging current in one circuit is opposite to the reactive voltage of the leading current in the other, and therefore adds, and instead of compounding, regulates in the wrong direction. The automatic compounding of the synchronous induction-generator with low-frequency synchronous-motor excitation fails therefore if the load is inductive.

These cases are very fully investigated by Steinmetz, to whom most of this is due.

Following Steinmetz, we may deduce the equations for this case.

Let $Z_1 = r_1 + jx_1$ = stator self-inductive impedance . (223)

$Z_2 = r_2 + jx_2$ = rotor self-inductive impedance reduced to the stator circuit by the ratio of effective turns and ratio of frequencies . . . . . . (224)

$Z_0 = r_0 + jx_0$ = synchronous impedance of the synchronous motor exciter . . . . . . (225)

$\underset{\cdot}{E}_1$ = terminal volts of stator, chosen as real axis $= e_1$

$\underset{\cdot}{E}_0$ = nominal generated E.M.F. of the synchronous-motor exciter reduced to the stator circuit

$\underset{\cdot}{E}$ = generated E.M.F. of the synchronous-induction generator stator circuit

The actual E.M.F. generated in the rotor circuit, then, is

$$\underset{\cdot}{E} \times \frac{n_2}{n_1} \times \frac{f_2}{f_1}$$

where n_2 = rotor turns per phase
n_1 = stator turns per phase
f_2 = rotor frequency
f_1 = stator frequency

and the actual nominal generated E.M.F. of the synchronous exciter

$$\dot{E}_o' = \dot{E}_o \times \frac{n_2}{n_1} \times \frac{f_2}{f_1} \quad . \quad . \quad . \quad . \quad (226)$$

Let $\dot{I}_1 = i - ji_1$ = stator current . . . . (227)

The rotor current $\dot{I}_2 = i + ji_1$ neglecting the exciting current (228)

If Y = exciting admittance, then the exciting current $\dot{I}_o = \dot{E}Y$ and the total rotor current is $\dot{I}_o + \dot{I}_2$ . . (229)

Then in the rotor circuit

$$\dot{E} = \dot{E}_o + (Z_o + Z_2)\,\dot{I}_2 \quad . \quad . \quad . \quad (230)$$

and in the stator circuit

$$\dot{E} = \dot{E}_1 + Z_1\dot{I}_1 \quad . \quad . \quad . \quad . \quad . \quad (231)$$

$$\therefore \dot{E}_1 = \dot{E}_o + \dot{I}_2(Z_o + Z_2) - \dot{I}_1 Z_1 \quad . \quad . \quad (232)$$

i.e. $\dot{E}_1 = \dot{E}_o + i\,(Z_o + Z_2 - Z_1) + ji_1\,(Z_o + Z_2 + Z_1)$. (233)

let $Z_o + Z_1 + Z_2 = Z_3 = r_3 + jx_3$ . . . . (234)

$Z_o + Z_2 - Z_1 = Z_4 = r_4 + jx_4$ . . . . (235)

then since $\dot{E}_1 = e$

$$\dot{E}_o = (e_1 - r_4 i + x_3 i_1) - j\,(x_4 i + r_3 i_1) \quad . \quad . \quad . \quad (236)$$

or in absolute values

$$e_o^2 = (e_1 - r_4 i + x_3 i_1)^2 + (x_4 i + r_3 i_1)^2 \quad . \quad . \quad . \quad (237)$$

hence $e_1 = \sqrt{e_o^2 - (x_4 i + r_3 i_1)^2} + r_4 i - x_3 i_1$ . . . (238)

The terminal voltage e_1 decreases due to the decrease of the square root, but may increase due to the remaining term.

At no load $i = o$, $i_1 = o$, and $e_1 = e_o$

At non-inductive load $i_1 = o$

and $e_1 = \sqrt{e_o^2 - x_4^2 i^2} + r_4 i$ . . (239)

e_1 first increases from its no-load value e_o, reaches a maximum, and then decreases again.

Since $r_4 = r_o + r_2 - r_1$ . . . . (240)

$x_4 = x_o + x_2 - x_1$ . . . . (241)

When $r_4 = o$ and $x_4 = o$

$$r_1 = r_o + r_2 \quad . \quad . \quad . \quad . \quad . \quad (242)$$

$$x_1 = x_o + x_2 . \quad . \quad . \quad . \quad . \quad (243)$$

and $e_1 = e_o$, or in this case the terminal voltage is constant at all non-inductive loads, at constant exciter excitation.

$$\text{Generally } I_1 = i - ji_1 \quad . \quad . \quad . \quad . \quad . \quad (244)$$

and if i_1 is positive or inductive load, the terminal volts falls with increasing load; while if i_1 is negative, corresponding to capacity load, the terminal voltage rises with increasing load, reaches a maximum value, and then falls again. By changing the impedances, the amount of compounding can be varied. By increasing the resistance r_4, e_1 increases faster than the load, i.e. over-compounding of the machine can be increased by inserting resistance in the rotor circuit.

CHAPTER V

ELECTRIC BRAKING OF INDUCTION MOTORS

In certain classes of service, as for hoists, cranes, rolling mills, etc., means must be provided for stopping the motor quickly. If the power supply to the motor is taken off, the motor will slow down gradually due to the friction load; but the time of retardation is usually too long, and hence braking of some kind must be resorted to. Now an induction motor may be braked by alternating or direct current.

When braking with alternating current, it is necessary to reverse the rotating field of the motor. In a three-phase machine, this is done by reversing two of the primary leads, and in a two-phase motor by reversing two primary leads of one phase.

At the moment of reversal, when the motor is running near synchronous speed, the frequency of the rotor currents is nearly twice the supply frequency, and the secondary voltage will be nearly twice that at standstill. This will necessitate due precautions to be taken in regard to the rotor insulation on very large slip-ring rotors. If the rotor is star-wound, the voltage to earth can be reduced by earthing the neutral point, thus allowing a decrease of insulation to earth. The insulation between phases, however, cannot be reduced, because this voltage does not change by earthing the neutral. On very large motors it becomes more desirable to apply half voltage to the primary, or use direct current for braking, due to the inevitable large voltage between rings at standstill on such machines. When using direct current for braking, the D.C. supply may be connected to the primary in the following ways—

The connections for three-phase machines are shown in figures (*a*) to (*f*), and those for two-phase machines in (*g*) and (*h*).

Figs. (*a*) and (*d*) and (*g*) are generally used, since they involve the simplest switching arrangements. By using some of the other connections, something can be gained in regard to the field form and least power requirements. Before deciding on which type of excitation to use for braking, it is well to investigate which method is better in regard to safety, simplicity, and cost. We will deal first with alternating-current braking, and will develop approximate expressions for the speed-torque and ampere-torque curves.

If the secondary ohmic and inductive resistances are reduced to

the primary by multiplying the resistances by the square of the ratio of primary to secondary turns, the induction motor windings

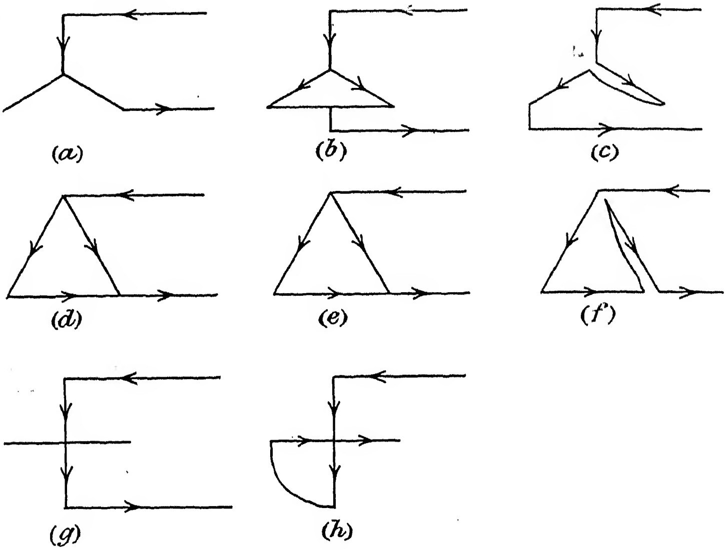


FIG. 29

may be reduced, as already seen, to the equivalent single-phase circuit below. (Fig. 30.)

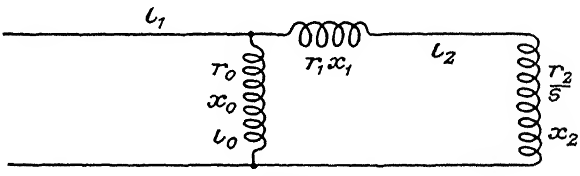


FIG. 30

Let $e_1 =$ primary terminal volts

$e_2 =$ secondary voltage between rings at standstill

$i_1 =$ total primary amps., equivalent to single-phase circuit

for three phases $i_1 = \sqrt{3} \times$ terminal amps.

for two phase $i_1 = 2 \times$ terminal amps.

$i_0 =$ total no-load current

$i_2 =$ total secondary current

r_1 = primary ohmic resistance between terminals divided by 2

r_2 = secondary ohmic resistance (referred to primary) divided by 2

x_1 = primary inductive reactance between terminals divided by 2

x_2 = secondary inductive reactance between terminals (referred to primary) divided by 2

ρ = total watts with motor locked

T = torque in lbs. at 1 ft. radius

s = slip

n_o = revolutions per minute at synchronous speed

Then assuming the primary current, with locked rotor, is nearly in phase with the no-load current, we may write

$$x_1 + x_2 = \frac{\sqrt{[(i_s - i_o)e_1]^2 - \rho^2}}{(i_s - i_o)^2} \quad . \quad . \quad . \quad . \quad (245)$$

i_s = total short-circuit current

$$i_2 = \frac{e_1}{\sqrt{\left(r_1 + \frac{r_2}{s}\right)^2 + (x_1 + x_2)^2}} \quad . \quad . \quad (246)$$

$$\text{Horse-power} = \frac{i_2^2 \times \frac{r_2}{s}(1 - s)}{746} \quad . \quad . \quad . \quad . \quad (247)$$

$$\begin{array}{l}\text{Torque}\\ \text{in lbs.-ft.}\end{array} \quad T = \frac{h.p. \times 5250}{n_o(1 - s)} \quad . \quad . \quad . \quad . \quad . \quad (248)$$

$$\therefore \; T \text{ lbs.-ft.} = \frac{i_2^2 \frac{r_2}{s} \times 5250}{n_o \times 746} \quad . \quad . \quad . \quad . \quad . \quad (249)$$

$$= \frac{i_2^2 \times \frac{r_2}{s}}{n_o} \times 7{\cdot}04 \quad . \quad . \quad . \quad . \quad (250)$$

$$\therefore \; \text{slip } s = \frac{i_2^2 r_2 \times 7{\cdot}04}{n_o \times T} \quad . \quad . \quad . \quad . \quad . \quad (251)$$

By substituting for i_2 the value $\dfrac{e_1}{\sqrt{\left(r_1 + \frac{r_2}{s}\right)^2 + (x_1 + x_2)^2}}$

in the equation for slip and torque above and differentiating, it will be found that the torque is a maximum when

$$r_1 + \frac{r_2}{s} = x_1 + x_2 \qquad (252)$$

The primary current i_1 is obtained by vector addition of the magnetizing current and secondary current (neglecting the no-load watt component).

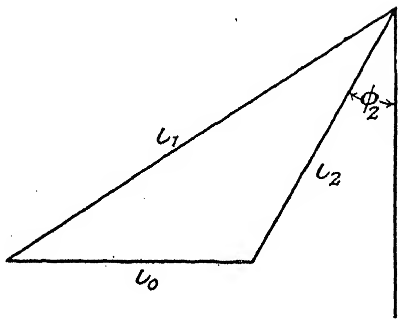


FIG. 31A

$$i_1 = \sqrt{(i_2 \sin\phi_2 + i_o)^2 + (i_2 \cos\phi_2)^2} \qquad (253)$$

and since $\sin\phi_2 = \dfrac{x_1 + x_2}{\sqrt{\left(r_1 + \dfrac{r_2}{s}\right)^2 + (x_1 + x_2)^2}}$ . . (254)

and $\cos\phi_2 = \dfrac{r_1 + \dfrac{r_2}{s}}{\sqrt{\left(r_1 + \dfrac{r_2}{s}\right)^2 + (x_1 + x_2)^2}}$ . . (255)

$$i_1 = \sqrt{\left\{\frac{i_2 (x_1 + x_2)}{\sqrt{\left(r_1 + \frac{r_2}{s}\right)^2 + (x_1 + x_2)^2}} + i_o\right\}^2 + \left\{\frac{i_2 \left(r_1 + \frac{r_2}{s}\right)}{\sqrt{\left(r_1 + \frac{r_2}{s}\right)^2 + (x_1 + x_2)^2}}\right\}^2} \qquad (256)$$

From these equations, the torque-slip and torque-current curves can be drawn for various values of rotor resistance. Now it is clear, from the torque-slip curve equation, that as the slip decreases,

from $s = 2$, corresponding to full speed with the field rotating in the opposite direction of rotation, to $s = 1$, i.e. standstill, if the rotor current and hence the stator current is to remain constant with constant braking torque, then r_2 must decrease in the same ratio as the slip decreases,

$$\text{for } T = \frac{i_2^2 \times \frac{r_2}{s} \times 7\cdot04}{n_o} \quad . \quad . \quad . \quad . \quad . \quad (257)$$

for $T =$ constant and $i_2 =$ constant

$$\frac{r_2}{s} = \text{constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad (258)$$

If a liquid resistance is used, it would be possible to change the resistance in such a manner that the torque and current would remain practically constant during the whole braking period. It is also clear that the primary circuit must be disconnected from the supply as soon as the motor comes to rest, unless reversal of the motor is required.

The following example, and the curves connected with it, is taken from an article by Specht, to whom the above is due.

Motor: 2000 h.p., three-phase, 6600 volts, 25 cycles, 6 poles, 500 r.p.m. synchronous; both primary and secondary star-wound.

The tested values for full volts on primary are—

$i_o =$ 58 amps. total; no-load power $=$ 32 kW

$i_s =$ 1550 amps. with locked rotor

$\rho =$ 1950 kW with locked rotor

$r_1 =$ 0·38 ohm

$e_2 =$ 1700 volts

The resistance per phase of the secondary at 40° C. $=$ 0·026 ohms

$$\therefore\ r_2 \text{ referred to primary} = 0\cdot026 \times \left(\frac{6600}{1700}\right)^2$$

$$= 0\cdot39 \text{ ohms}$$

In the following figure, for the different torque-speed curves

1 to 9, the secondary resistances referred to the primary have the following values—

Curve 1.	$r_2 =$	0·39	without external resistance
,, 2.	$r_2 =$	1·2	including ,, ,,
,, 3.	$r_2 =$	2·4	,, ,, ,,
,, 4.	$r_2 =$	5·0	,, ,, ,,
,, 5.	$r_2 =$	8·0	,, ,, ,,
,, 6.	$r_2 =$	13·0	,, ,, ,,
,, 7.	$r_2 =$	20·0	,, ,, ,,
,, 8.	$r_2 =$	34·0	,, ,, ,,
,, 9.	$r_2 =$	60·0	,, ,, ,,

$$x_1 + x_2 = \frac{\sqrt{[(1550 - 58) \times 6600]^2 - 1950000^2}}{(1550 - 58)^2} = 4{\cdot}37\omega \quad . \quad (259)$$

for $r_2 = 0{\cdot}39$ and $s = 2$

$$i_2 = \frac{6600}{\sqrt{\left(0{\cdot}38 + \frac{0{\cdot}39}{2}\right)^2 + 4{\cdot}37^2}} = 1496 \text{ amps.} \quad . \quad (260)$$

$$T = \frac{1496^2 \times \frac{0{\cdot}39}{2}}{500} \times 7{\cdot}04 = 6100 \text{ lbs.-ft.} \quad . \quad . \quad (261)$$

$[n_0 = 500 \text{ r.p.m.}]$

For various values of the slip from 2 to 0, the torque curve 1 and secondary ampere-curve are determined.

The results are shown in the curves in Fig. 31B.

It will be noticed that, if the motor should be braked with a torque equal to the average full-load torque, and with a current not greatly exceeding full-load current, the resistance should be decreased step by step until the motor stops. This is shown by the heavy zigzag line. This corresponds to the use of a metallic resistance. If a reduced voltage is applied to the stator for braking, the currents vary in the same ratio as the voltage, and the torque varies as the square of the voltage ratio. Therefore, the same curves can be used for reduced voltage as for full voltage, by changing the current scale in the ratio of the voltage and the torque scale in the square of the voltage ratio. In order to brake

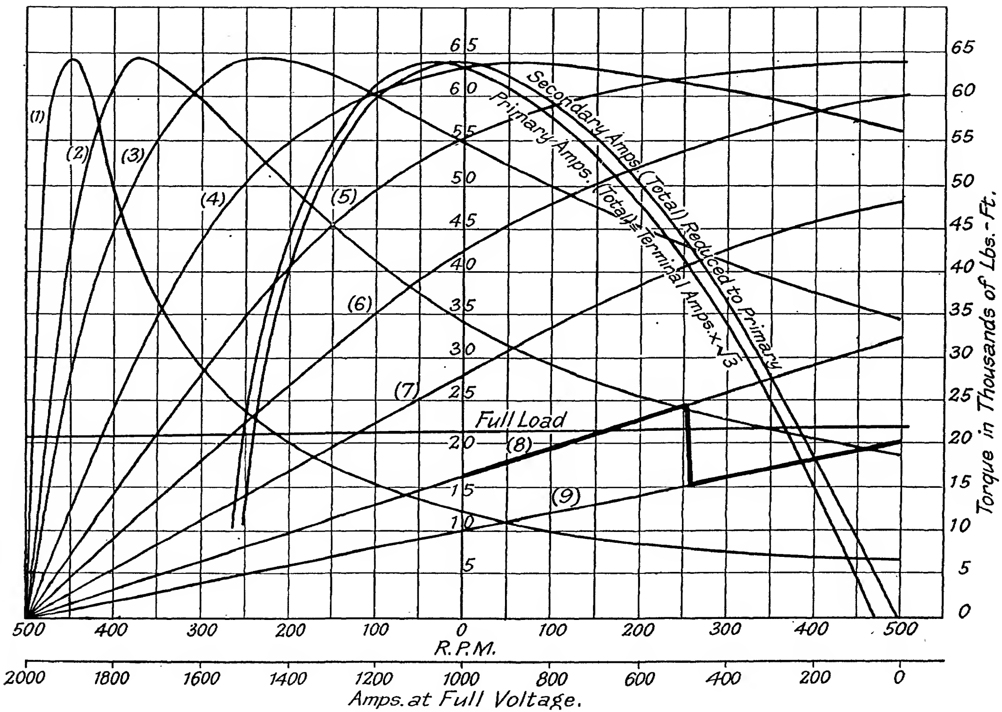


FIG. 31B.—AMPERE SPEED-TORQUE CURVE FOR A.C. EXCITATION

the motor at half-voltage with full-load current and half the full-load torque, the secondary resistance must be half that at full voltage. This torque can only be obtained when the maximum torque is at least twice full-load torque at full voltage, because the maximum torque at half-voltage will be only one quarter of that at full voltage.

In the case of a squirrel-cage motor, one speed-torque curve only can be obtained; and in order to obtain a good torque for braking, without excessive current, a high resistance rotor is necessary.

Such a motor is very inefficient when running normally under load, and should only be applied for very intermittent service for elevators, cranes, or hoists.

Braking by direct current. In the case where one terminal is connected to the negative supply and the remaining two terminals to the positive supply, the current in phase 1 (when at the maximum value) will have double the value of the currents in phases 2 and 3, and will flow in opposite direction. The current in phases 2 and 3 flows from the terminals to the star point (negative direction) and, in phase 1, it will flow from the star point to the terminal (positive direction).

This is just what happens when, in a three-phase system, the current in phase 1 is at its crest value. Let us call the R.M.S. value of the current I, then the momentary current in phase 1 is $\sqrt{2}I$ and, in phases 2 and 3, $\frac{1}{\sqrt{2}}I$

It is clear that the direct current, in this case, which replaces the three-phase excitation, will have the value $\sqrt{2}I$

Let us consider a star-wound motor and connect only two terminals to the supply. Then the current in the third phase is zero, the currents in phases 1 and 2 have the same value, but opposite in direction. This corresponds to the moment in which, in the three-phase machine, the current in phase 3 is zero, whereas the currents in the other two phases are

$$+\sqrt{\frac{3}{2}}I \text{ and } -\sqrt{\frac{3}{2}}I = 1{\cdot}23I \text{ and } -1{\cdot}23I$$

In this case the equivalent direct current will be 23 per cent higher than the measured value of the alternating current.

The watt losses in the first case are, if R represents the resistance of one phase,

$$(1{\cdot}414I)^2R + 2 \times (0{\cdot}707I)^2R = 3I^2R \quad . \quad . \quad (262)$$

and, in the second case, $2(1{\cdot}23I)^2R = 3I^2R$ . . (263)

Therefore the direct-current voltages, applied in cases 1 and 2, are in inverse proportion to the currents.

Exactly the same relation holds good for delta connection. In this case, if the D.C. supply is connected to two terminals only, the direct current has to be 23 per cent greater than the measured alternating current; whereas if one terminal is connected to the positive and the two others to the negative supply, the value of the current is $\sqrt{2}I$

Let r_2 = secondary ohmic resistance measured between terminals and divided by 2

x = inductive reactance at synchronous speed

s = slip = 0 at synchronous speed of motor

= 1 at standstill

n_0 = synchronous revs. per minute

n = revs. per minute of motor when running

e_2 = secondary voltage between terminals at no-load speed

i_2 = *total* secondary current equivalent to single phase

$$\left[\begin{array}{ll} \text{for three-phase } i_2 = \text{terminal amps.} \times \sqrt{3} \\ \text{for two-phase } i_2 = \quad ,, \quad \times 2 \end{array}\right]$$

i_s = total secondary short-circuit current for inductive reactance purely

i_1 = total primary amperes

T = torque in lbs. at 1 ft. radius

i_0 = direct current for exciting

t_1 = number of turns per phase in primary

t_2 = ,, ,, secondary

For the direct-current excitation, an equivalent alternating current $= \dfrac{1}{\sqrt{2}}$ times the direct current can be substituted, and then for synchronous speed, the voltage and short-circuit current in the secondary can be determined by transformation.

The short-circuit current is equal to the equivalent alternating magnetizing current reduced to the secondary turns. The secondary open-circuit voltage is the same as that obtained by the equivalent direct-current excitation when running at synchronous speed. In determining the corresponding alternating short-circuit current, the distribution and the amount of winding

excited by direct current must be considered, i.e. the current must be multiplied by a factor C besides $\frac{1}{\sqrt{2}}$

The factor C for a three-phase winding, of which two phases are excited by direct current, is 1·15.

The total short-circuit current in a three-phase secondary at synchronous speed, which would be obtained if the rotor had inductive reactance only is

$$i_s = \frac{i_o\sqrt{3}}{\sqrt{2}} \times \frac{t_1}{t_2} \times C \quad . \quad . \quad . \quad . \quad . \quad . \quad (264)$$

and the equivalent primary alternating current is

$$i_1 = i_o \times \frac{\sqrt{3}}{\sqrt{2}} \times C \text{ for three-phase primary} \quad . \quad (265)$$

$$i_1 = i_o \frac{2}{\sqrt{2}} \times C \text{ for two-phase primary} \quad . \quad . \quad (266)$$

The corresponding secondary volts e_2 can be read off on the open alternating-current saturation curve.

$$\text{The inductive reactance } x = \frac{e_2}{i_s} \quad . \quad . \quad . \quad . \quad . \quad (267)$$

$$\text{also } i_2 = \frac{e_2}{\sqrt{\left(\frac{r_2}{s}\right)^2 + x^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (268)$$

$$T = \frac{i_2^2 r_2 \times 7{\cdot}04}{n} \quad . \quad . \quad . \quad . \quad . \quad . \quad (269)$$

$$s = \frac{n_o T}{i_2^2 r_2} \times 7{\cdot}04 \quad . \quad . \quad . \quad . \quad . \quad . \quad (270)$$

$$\text{maximum torque occurs when } \frac{r_2}{s} = x$$

We will now compare direct-current braking with alternating-current braking.

The 2000 h.p. motor, to which we have already referred, will be selected for example. Assuming the primary is excited by direct current across two terminals of the star winding, and that this current is 100 amps., then

$$i_s = 100 \times \frac{\sqrt{3}}{\sqrt{2}} \times \frac{6600}{1700} \times 1{\cdot}15 = 550 \text{ amps. total} \quad (271)$$

$$i_1 = 100 \times \frac{\sqrt{3}}{\sqrt{2}} \times 1\cdot15 = 141\cdot7 \text{ amps. total} \qquad (272)$$

From the saturation curve, the corresponding secondary voltage e_2 for $i_1 = 141\cdot7$ amps. is found to be 2420 volts.

$$\text{The inductive reactance} = \frac{2420}{550} = 4\cdot4 \text{ ohms}$$

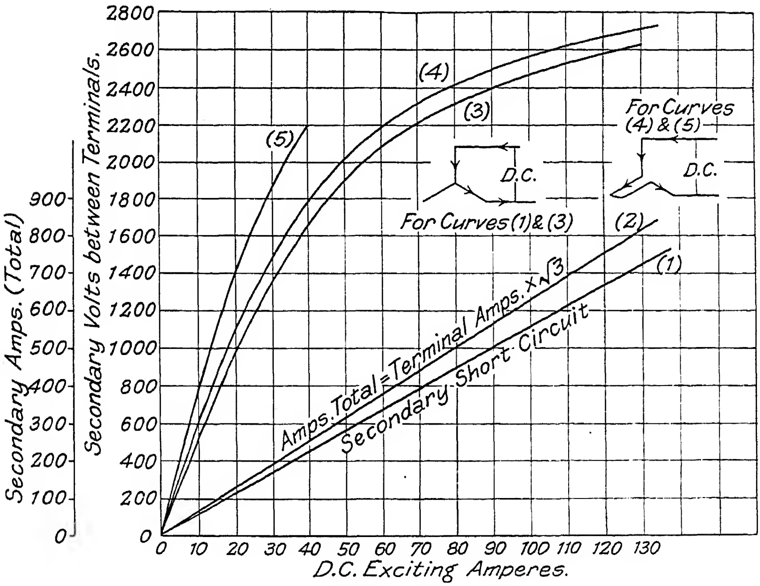


FIG. 32.—SATURATION CURVE FOR A.C. EXCITATION

For various secondary resistances r_2, the following values are selected.

Curve 1. $r_2 =$ 0·026 ohms without external resistance
,, 2. $r_2 =$ 0·5 including external resistance
,, 3. $r_2 = 1\cdot0\omega$,, ,, ,,
,, 4. $r_2 = 2\cdot0\omega$,, ,, ,,
,, 5. $r_2 = 3\cdot0\omega$,, ,, ,,

For $s = 1$ and $r_2 = 0\cdot5$ ohms, the secondary current i_2 and torque T are

$$i_2 = \frac{2420}{\sqrt{\left(\frac{0\cdot5}{1\cdot0}\right)^2 + 4\cdot4^2}} = 548 \text{ amps.} \qquad (273)$$

$$T = \frac{548^2 \times 0{\cdot}5}{500} \times 7{\cdot}04 = 2120 \text{ lbs.-ft.} \quad . \quad . \quad (274)$$

In this way the currents and torques for other slips and resistances may be found. The results are shown in the following figure.

Examination of the curves shows that, without external resistance, the torque at 500 r.p.m. is nearly zero, and increases very

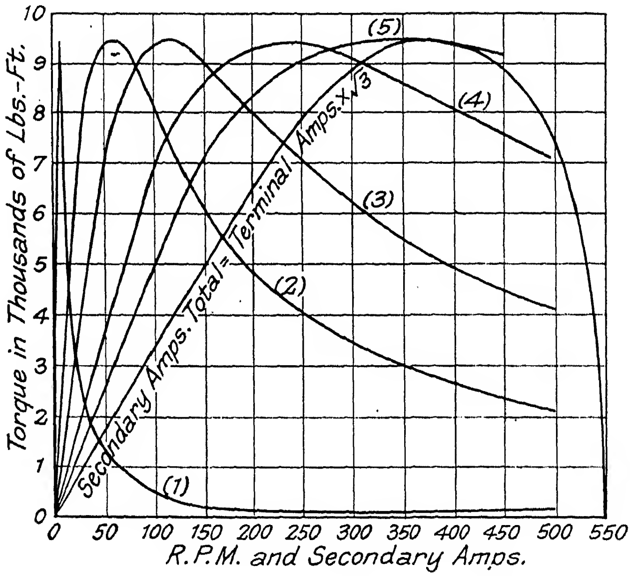


FIG. 33.—AMPERE SPEED-TORQUE CURVE FOR DIRECT-CURRENT EXCITATION OF 100 AMPS.

slowly with decreasing speed, except that below 50 r.p.m. the torque increases much faster. At about 3 r.p.m., the torque reaches the maximum value (9400 lbs.-ft.) and then drops very rapidly to zero value. It is clear that this speed-torque curve is of no practical value; and in order to obtain good braking torque over a wide speed range, it is necessary to insert a fairly large resistance.

Further, the curves show that the maximum torque obtainable with an exciting current of 100 amp. is not even quite half full-load torque, and that the open-circuit secondary voltage at synchronous speed is 42 per cent greater than the voltage at standstill with 6600 volts A.C. on the primary. It would be possible to obtain a greater torque by increasing the exciting current. This

would give, however, a higher secondary voltage and stronger field, and would increase the unbalanced pull and the danger of greater potential rise in case any of the circuits should break. It is pointed out by Specht that the conditions for direct-current braking of this motor are very poor, due to the low value of the no-load current as compared to the full-load current. Motors having a larger ratio in this respect would give more favourable results.

Nevertheless, the best torque which can be obtained by braking with direct current is not greater than full-load torque. The same motor was driven by another motor at synchronous speed, and the primary was excited by direct current and the open-circuit secondary voltage measured. Then the secondary circuit was closed and the short-circuit current was measured. The results are shown in Fig. 32.

Curves 1 and 3 correspond to an excitation of two phases; and curves 2, 4, and 5 correspond to an excitation of three phases.

Analysis of the results shows—

1. The braking torque obtainable by alternating current, even with only half the primary voltage, is, as a rule, considerably greater than with direct current.

2. In braking with alternating current, the line circuit has to be taken off the motor as soon as the motor comes to rest, otherwise the motor will reverse; whereas in braking with direct current, the motor comes to rest only and will not reverse.

3. With alternating current, it is an easy matter to obtain a strong and practically constant braking torque during the whole retardation period; whereas with direct-current excitation, it is difficult to obtain good braking torque near standstill, due to the rapid decrease in torque from maximum to zero.

4. If it is desired to brake the motor with full-load torque by means of direct current, the secondary voltage at synchronous speed will be not far from double voltage; and, further, since the magnetic field has to be much stronger with D.C. current, there is a danger of serious voltage rises due to breaking of any of the circuits.

The only advantage of braking with direct current is the small energy which is needed. Only the I^2R losses of the primary have to be supplied with D.C. excitation; while with A.C. excitation, the full power has to be supplied to the motor which it would require for developing an equal torque at normal operating condition. This section with the curves and examples are taken from the paper by Specht.

CHAPTER VI

INDUCTION MOTOR AS PHASE CONVERTER

THE induction motor can be wound for several different phases. If supplied with two or three-phase current, a rotating flux wave is produced, and this generates E.M.F.'s in the several phases. It is thus possible to use the motor to transform single-phase current to polyphase current. This is of great service where motors are required for driving purposes, and where only a single-phase supply is available. Single-phase motors are more costly, less efficient, and are inferior in every respect to polyphase motors. Three-phase motors might, in such cases, be installed, and a phase-splitting device used to start one motor. When operating near synchronous speed, the voltages between the lines are caused to assume the correct phase relationships, and all the other motors can be operated three-phase. In this connection it may be mentioned that phase converters are used on the Norfolk and Western Railway (U.S.A.). The locomotives, which are of the split-phase type, are driven by three-phase induction motors, which are supplied from a phase converter. The supply is a single-phase one. The converter is virtually a two-phase induction motor, with a squirrel-cage rotor, and is started by a single-phase commutator motor.

The main transformer is shown at T, Fig. 34, and the centre point of the secondary winding C is connected to one of the stator windings D of the phase converter. The other end of this winding is connected to one of the terminals F of the three-phase motor. The other terminals G and H of the motor are connected to the terminals A and B of the transformer, to which are also connected the second stator winding E of the phase converter.

When the phase converter is in operation, an E.M.F. is induced in winding D, which is in quadrature with the E.M.F. at the terminals A and B. By arranging that the E.M.F., induced in D, is $\sqrt{\frac{3}{2}}$, i.e. 0·866 of the E.M.F. across AB, we have, by connecting D to the centre point C of the transformer, the same conditions as exist in the usual three-phase to two-phase transformation. Therefore three-phase currents are obtainable from terminals A, B, and F. As actually made, features must be incorporated into

the design of the phase converter for annulling the inductive effects due to the load current in phase E; while in order to maintain balanced three-phase voltages under load, the tapping C on the transformer must be shifted from the centre point of the winding.

The special feature of the split-phase locomotive is that regenerative braking can be obtained in the same way as in three-phase locomotives. Series connection of the phase converter to the induction motor supplied by it, automatically tends to regulate for equality of the voltages. Thus consider a two-phase converter

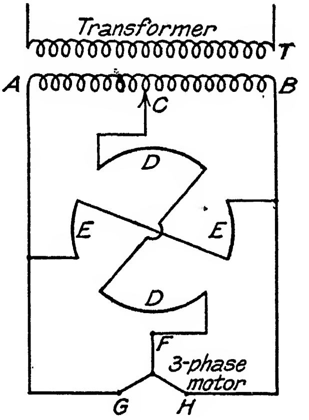


FIG. 34

supplying a two-phase induction motor. One phase of the phase converter is connected in series to one phase of the induction motor driving the locomotive into the circuit of the single-phase supply, and the second phase of the phase converter is connected to the second phase of the induction motor.

At no load, the voltage on the second phase is a maximum. Under load this voltage drops, and also the voltage across the other phase of the motor drops. This raises the voltage on the primary phase of the converter, and this raises the voltage on the second phase, and thus tends to maintain equality of the two voltages impressed on the motor, much more so than if the phase converter were connected in shunt to the induction motor.

Quadrature position of these two-phase voltages can be closely maintained by the use of a series transformer, the primary of which is in series with the supply circuit, the secondary in series with the second phase.

CHAPTER VII

REGULATION AND STABILITY OF THE INDUCTION MOTOR

THE characteristic curves of an induction motor are usually derived for constant impressed voltage. If the voltage at the primary terminals of the transformer is constant, and such as to give the rated motor voltage at full load, it is clear that if there is considerable impedance in the line and transformer supplying the motor, the voltage at no-load will be higher, and the voltage at overload lower than the normal rated voltage.

At no-load, therefore, the flux and exciting current will be increased. At overload, due to fall of the voltage, the short-circuit current will be reduced, and also the maximum torque and maximum output, and also the starting torque of the motor. The magnitude of these effects will obviously depend on the variation of voltage which takes place; and it will be clear, from the expressions for the various quantities mentioned above, that at overloads and in starting it is very important to have as low impedance as possible between the primary terminals of the transformer and the motor terminals.

Variation of frequency. If the supply frequency pulsates so rapidly that the motor speed cannot follow the frequency pulsations, then the actual slip obviously pulsates, and the motor current and torque pulsate between the values corresponding to maximum and minimum slips. The pulsation of current is moderate until synchronism is approached, but becomes large near synchronism.

The average torque drops below the torque corresponding to constant frequency. These effects are shown clearly in the following curve (Fig. 35) for a pulsation of frequency of supply of 2·5 per cent from the average.

Stability of the induction motor. The general character of the torque-slip curve for an induction motor will be seen from the curves in Fig. 11. For small values of the slip, the torque increases proportionately to the slip, and the relation between torque and slip near synchronism is a linear one. The torque curve therefore rises in a straight line near synchronous speed and gradually bends over, increasing to a maximum value and then decreases with decrease of speed to a minimum at standstill. The slip at which maximum torque occurs varies with the rotor resistance. The

higher the rotor resistance, the lower the speed or the greater the slip at which maximum torque occurs. Any value of the torque between the starting torque and maximum torque is possible at

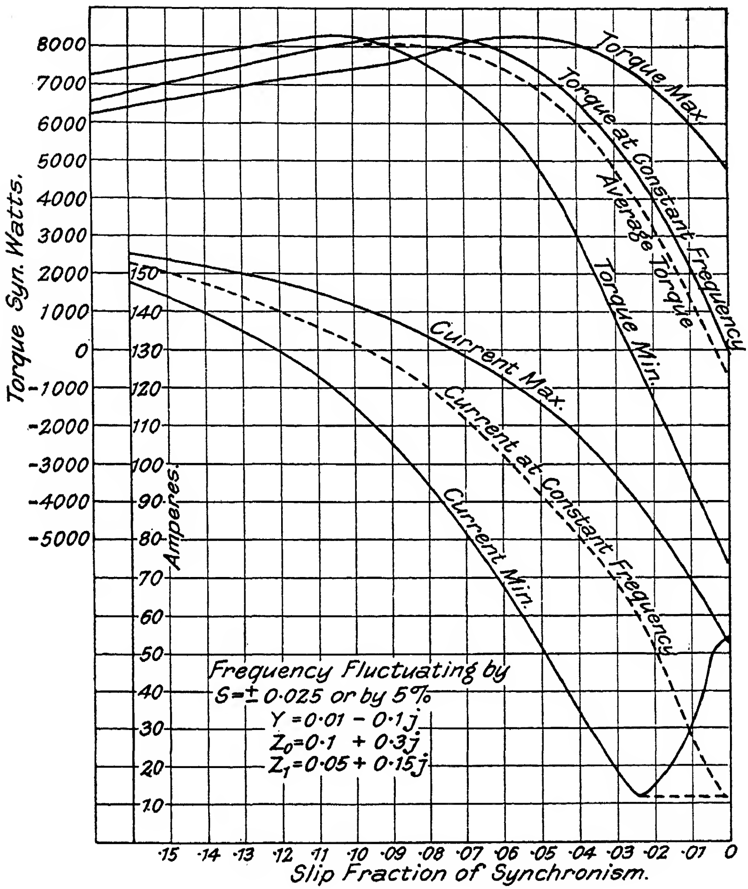


Fig. 35.—Effect of Frequency Pulsation

two different speeds, as is evident by drawing a horizontal line intersecting the slip-torque curve. An induction motor working against a constant torque, as, for example, driving a reciprocating pump against a constant head of water, could run at two speeds. Of these two speeds, however, one is unstable, and this speed is obviously the lower one of the two, for a momentary decrease of

speed, due to a variation in load or voltage, will result in a decrease of torque and speed, and the motor will come to rest. Similarly, an increase in speed, from whatever cause, results in an increase of torque and an increase of speed till the other point is reached on the curve between the maximum torque and synchronous speed. One point therefore is stable only, and that point is on the part of the curve between synchronous speed and the speed of maximum torque.

We may say therefore, that, for loads requiring constant torque, the induction motor is unstable at speeds below the maximum torque point, but stable at speeds above it. This instability is, however, due to the nature of the load. Now, in a motor driving a ship's propeller or a centrifugal pump, the torque will consist of a constant part representing friction, and a part which is proportional to the square of the speed.

The load torque will consist of a parabolic curve whose origin is on the ordinate through zero speed. This parabola may, according to the magnitude of the load, intersect the speed-torque curve on any part of the curve, and in this case no unstable branch of the curve exists; but stability obtains over the entire range, and the motor will run at any such speed within the torque range. Again in the case of a load requiring a torque which consists of a constant portion and a part proportional to the speed, e.g. when driving a D.C. generator with constant excitation connected to a load of constant resistance, several cases may arise. With a moderate load, the motor accelerates up to a speed near synchronous speed, viz., the point on the speed-torque curve which the straight line representing the load torque intersects.

With a very heavy load, the motor runs up to a low speed only, for the straight line will intersect the torque curve at a low value of the speed. The straight line, representing the load torque, may intersect the torque-slip curve in three points, as will be obvious from an inspection of the curve; but of these three points, two only are stable, viz., the low speed and the high speed. The intermediate point is unstable. At the two stable points, a momentary speed increase results in a decrease of torque below that required by the load and thus limits itself, and inversely a decrease of speed increases the torque beyond that required by the load and thus restores the speed.

At the intermediate speed, the motor is unstable, and a momentary speed increase causes the motor to accelerate up to the higher speed stable point; a momentary decrease of speed causes the motor to slow down to the lower stable point.

Stability is a function of the character of the load in its relation to the speed-torque curve. If the change of motor torque with change of speed is less than the change of torque required by the load, the condition is stable, otherwise it is unstable. It is thus clear that under certain classes of load, the motor may fail to accelerate the load to full speed.

This is especially the case with single-phase induction motors, in which there may be a marked depression in the torque-slip curve between standstill and maximum torque.

Again, the stability of the motor is affected by the generator regulation, and by the rapidity with which the voltage of the generator is changed with varying load, and the rapidity with which the motor speed can change as affected by the mechanical momentum of the motor and its load.

The stability coefficient of the motor, as defined by Steinmetz

$$= \frac{1}{D}\frac{dD}{di} = K_s$$

where D = torque of the motor

i = current

If K_s is positive, an increase of current, caused by an increase of slip or decrease of speed, increases the torque D and therefore checks the decrease of speed and the motor is stable. Inversely, if K_s is negative, the motor is unstable. If the regulation of the generator to constancy of voltage requires a finite time, however short, the maximum output of the motor is thereby reduced, the more so the more rapidly the motor speed can change. At constant slip s, the motor torque is proportional to the square of the impressed E.M.F. e^2. If by a variation of slip, caused by a fluctuation of the load, the motor current varies by di, if the terminal voltage e remains constant, the motor torque D varies by the fraction $K_s = \frac{1}{D}\frac{dD}{di}$. If, however, the variation of current causes a variation of the impressed E.M.F. e, the motor torque, being proportional to e^2, changes still further, i.e. by the fraction

$$K_r = \frac{1}{e^2}\frac{de^2}{di} = \frac{2}{e}\frac{de}{di}$$

and the total change of motor torque resulting from a change of current $di = K_s + K_r$

Since K_r is negative, the voltage decreasing with increasing current, the stability coefficient of the motor is reduced.

$$K_r = \frac{2}{e}\frac{de}{di}$$

represents the torque change due to momentary voltage change, and is a characteristic of the supply system. It depends on the motor only in so far as $\frac{de}{di}$ depends upon the power factor of the load.

CHAPTER VIII

THE NATURE OF THE REVOLVING FIELD IN THE POLYPHASE MOTOR

THE character of the revolving field in the polyphase induction motor is determined by the shape of the wave of impressed voltage, and also by the distribution of the winding along the periphery of the air-gap, and also by the effects of saturation. Two classes of harmonics are thus possible, viz., time harmonics of the voltage wave, which are of higher frequency, but which have the same number of motor poles, and space harmonics, i.e. harmonics due to the spacial distribution of the windings, which are of fundamental frequency, but of a higher number of poles.

In addition, but of relatively small importance, higher space harmonics of higher time harmonics exist. Now each of the harmonics present in the flux wave will give rise to corresponding currents in the rotor winding, and will affect the character of the torque-slip curve.

In general, it may be said that the harmonics due to the spacial distribution of the winding are the most important, since this distribution differs greatly from the sinusoidal type, whereas the voltage wave in modern alternators, especially of the non-salient pole type, differs little from the sine shape. Most of the trouble due to crawling, especially in pole-changing motors, is attributable to the shape of the M.M.F. curve due to bad distribution. We will deal first with the harmonics due to the shape of the voltage wave, and will deal first with the three-phase motor. Let the three voltages be represented by the following equations—

$$\begin{aligned} E_a = {} & e_1 \cos \omega t + e_3 \cos (3\omega t - \beta_3) \\ & + e_5 \cos (5\omega t - \beta_5) + e_7 \cos (7\omega t - \beta_7) \\ & + e_9 \cos (9\omega t - \beta_9), \text{ etc.} \qquad (275) \end{aligned}$$

$$\begin{aligned} E_b = {} & e_1 \cos\left(\omega t - \frac{2\pi}{3}\right) + e_3 \cos\left(3\omega t - \frac{6\pi}{3} - \beta_3\right) \\ & + e_5 \cos\left(5\omega t - \frac{10\pi}{3} - \beta_5\right) \\ & + e_7 \cos\left(7\omega t - \frac{14\pi}{3} - \beta_7\right) \\ & + e_9 \cos\left(9\omega t - \frac{18\pi}{3} - \beta_9\right) \text{ etc.} \end{aligned}$$

$$= e_1 \cos\left(\omega t - \frac{2\pi}{3}\right) + e_3 \cos(3\omega t - \beta_3)$$
$$+ e_5 \cos\left(5\omega t - \beta_5 + \frac{2\pi}{3}\right)$$
$$+ e_7 \cos\left(7\omega t - \beta_7 - \frac{2\pi}{3}\right)$$
$$+ e_9 \cos(9\omega t - \beta_9) \quad . \quad . \quad . \quad . \quad (276)$$

$$E_c = e_1 \cos\left(\omega t - \frac{4\pi}{3}\right) + e_3 \cos(3\omega t - \beta_3)$$
$$+ e_5 \cos\left(5\omega t - \beta_5 + \frac{4\pi}{3}\right)$$
$$+ e_7 \cos\left(7\omega t - \beta_7 - \frac{4\pi}{3}\right)$$
$$+ e_9 \cos(9\omega t - \beta_9) \quad . \quad . \quad . \quad . \quad (277)$$

It is thus seen that voltages of the different frequencies below are impressed on the motor phases

$e_1 \cos \omega t$, $e_1 \cos\left(\omega t - \frac{2\pi}{3}\right)$, $e_1 \cos\left(\omega t - \frac{4\pi}{3}\right)$—fundamental

$e_3 \cos(3\omega t - \beta_3)$, $e_3 \cos(3\omega t - \beta_3)$, $e_3 \cos(3\omega t - \beta_3)$
—3rd harmonic

$e_5 \cos(5\omega t - \beta_5)$, $e_5 \cos\left(5\omega t - \beta_5 + \frac{2\pi}{3}\right)$, $e_5 \cos\left(5\omega t - \beta_5 + \frac{4\pi}{3}\right)$
—5th harmonic

$e_7 \cos(7\omega t - \beta_7)$, $e_7 \cos\left(7\omega t - \beta_7 - \frac{2\pi}{3}\right)$, $e_7 \cos\left(7\omega t - \beta_7 - \frac{4\pi}{3}\right)$
—7th harmonic

$e_9 \cos(9\omega t - \beta_9)$, $e_9 \cos(9\omega t - \beta_9)$, $e_9 \cos(9\omega t - \beta_9)$
—9th harmonic

From the above it will be seen that the third harmonics are in phase with each other or single-phase voltages. This also applies to its multiples, viz., the ninth, etc. Though they have no phase rotation, they may exert torque when the motor speeds up as in the single-phase motor. The fifth harmonic gives backward phase rotation, and thus give negative torque; while the seventh harmonic has the same phase rotation as the fundamental, and thus gives forward torque up to its synchronous speed, which is $\frac{1}{7}$th of the synchronous speed of the fundamental. Beyond its synchronous speed, it gives a retarding or generator torque.

In the true three-phase winding, with a phase speed of $\frac{2\pi}{3}$, the triple-frequency current flows as a single-phase current in opposite directions in the upper and lower layers, and there can be no third harmonic flux in the true three-phase winding. This is not the

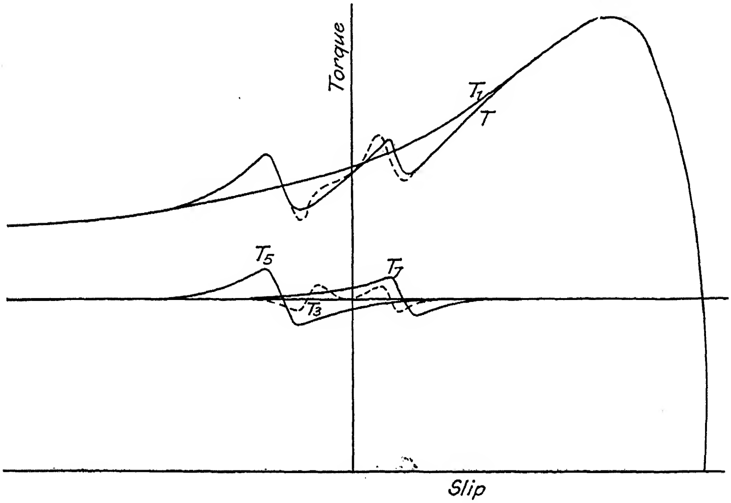


FIG. 36.—THREE-PHASE INDUCTION MOTOR: COMPONENT HARMONICS AND RESULTANT TORQUE

case, however, in the usual three-phase winding for induction motors, in which the shorter phase spread of $\frac{\pi}{3}$ is used.

In this case the third harmonic current flows in one direction for $\frac{\pi}{3}$ and in the other direction for the next $\frac{\pi}{3}$, and a single-phase flux exists which has three times as many poles as the fundamental flux.

All these component voltages or harmonics produce corresponding torques. They are distinguished from the space harmonics by having the same number of poles as the fundamental, with the exception of the third harmonic just mentioned.

The above diagram, taken from Steinmetz, to whom most of this section is due, shows the torque curves due to the various harmonics.

Again, in the quarter-phase motor, let the applied E.M.F.'s be—

$$E_a = e_1 \cos \omega t + e_3 \cos (3\omega t - \beta_3) + e_5 \cos (5\omega t - \beta_5) + e_7 \cos (7\omega t - \beta_7) + e_9 \cos (9\omega t - \beta_9), \text{ etc.} \quad (278)$$

$$E_b = e_1 \cos\left(\omega t - \frac{\pi}{2}\right) + e_3 \cos\left(3\omega t - \beta_3 + \frac{\pi}{2}\right)$$
$$+ e_5 \cos\left(5\omega t - \beta_5 - \frac{\pi}{2}\right)$$
$$+ e_7 \cos\left(7\omega t - \beta_7 + \frac{\pi}{2}\right)$$
$$+ e_9 \cos\left(9\omega t - \beta_9 - \frac{\pi}{2}\right) \text{ etc.} \qquad (279)$$

As in the three-phase case, each of the harmonics produces a flux and torque, and the effective torque is that due to the superposition of the component torques. It is clear that the third harmonic rotates in a backward direction and produces a torque in opposite direction to that of the fundamental, and would give its zero torque at one-third of the synchronous speed of the fundamental in the negative direction.

For backward rotation above one-third of synchronous speed, the torque is a generator torque. The fifth harmonic rotates in the same direction as the fundamental, and assists the torque due to the fundamental. It reaches its synchronous speed at one-fifth of the synchronous speed of the fundamental, and above this speed the torque becomes a generator or retarding torque. The seventh harmonic has backward phase rotation and so gives negative torque. The ninth harmonic gives a forward or driving torque up to a speed of one-ninth of synchronous speed of the fundamental, and above this speed negative generator torque. The results are shown in the following curve, which is taken from Steinmetz. (Fig. 37.)

Quarter-phase Motor

Order of harmonic	1	3	5	7	9	11	13	etc.
Phase rotation	+	−	+	−	+	−	+	
Synchronous speed S	$+1$	$-\frac{1}{3}$	$+\frac{1}{5}$	$-\frac{1}{7}$	$+\frac{1}{9}$	$-\frac{1}{11}$	$+\frac{1}{13}$	
Torque positive $S =$ up to . . . . . otherwise negative	$+1$	−	$+\frac{1}{5}$	−	$+\frac{1}{9}$	−	$+\frac{1}{13}$	

It will be seen from Fig. 37 that, with large harmonics, the torque curve is very irregular and shows depressions or dead points at low speeds.

Harmonics due to distribution of the winding. The component torque curves, due to the harmonics of spacial distribution of magnetizing force and flux in the gap, have the same character as those due to the time harmonics in the voltage wave, and can be represented by diagrams similar to Figs. 36 and 37. As is well known, according to Fourier's theorem, any periodic function

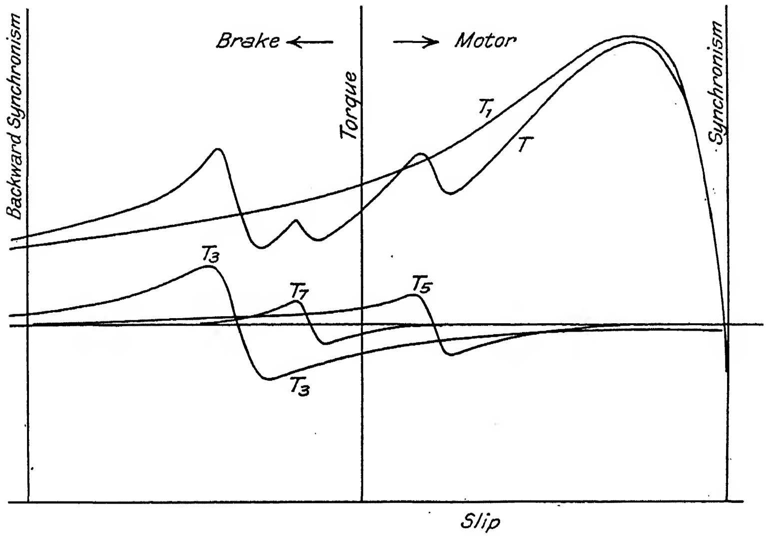


FIG. 37.—QUARTER-PHASE INDUCTION MOTOR: COMPONENT HARMONICS AND RESULTANT TORQUE

which is single valued can be analysed into a series of sine curves. So with the usual types of winding used on induction motors, it is possible to analyse the M.M.F. curve into a fundamental and higher harmonics.

We will consider, first, one phase of a three-phase winding where the turns are laid in one unskewed slot per pole per phase.

The M.M.F. curve due to the coil will be a rectangle, and if y = maximum height of the rectangle, the maximum amplitude of the nth harmonic

$$A_n = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} y \sin n\alpha \, d\alpha$$

$$= \frac{4y}{n\pi} \text{ for } n = \text{odd integer} \quad . \quad . \quad . \quad . \quad (280)$$

In all cases where the M.M.F. curve is symmetrical, an expression for A_n of the form $\frac{4y}{n\pi} X$ is obtained.

X may be called the reduction factor, for if the maximum amplitude of any harmonic in the case of a concentrated winding is multiplied by this factor, the maximum amplitude of the corresponding harmonic in the distributed case is obtained.

For a winding normally distributed in two unskewed slots per pole per phase in the three-phase case,

$$A_n = \frac{4y}{\pi} \int_{\frac{\pi}{12}}^{\frac{\pi}{2}} \sin na \, da$$

$$= \frac{4y}{n\pi} \cos \frac{n\pi}{12} \quad [n = \text{odd}] \quad . \quad . \quad . \quad . \quad (281)$$

For a three-phase winding, in which the winding is uniformly distributed,

$$A_n = \frac{4y}{\pi} \left[\frac{6}{\pi} \int_a^{\frac{\pi}{6}} a \sin na \, da + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin na \, da \right]$$

$$= \frac{4y}{n\pi} \times \frac{6}{n\pi} \sin \frac{n\pi}{6} \quad . \quad . \quad . \quad . \quad . \quad (282)$$

If ψ = angle in electrical degrees between successive slots
m = number of slots per pole per phase
n = order of the harmonic

$$\text{the reduction factor} = \frac{\sin \frac{1}{2}m\, n\psi}{m \sin \frac{n\psi}{2}} \quad . \quad . \quad . \quad . \quad . \quad (283)$$

It will be noticed that this is the distribution factor for the nth harmonic.

The reduction factors for windings of normal distribution in unskewed slots are given in the table at foot of the opposite page.

It will be noticed that some of the reduction factors are negative. It will be observed from the table above that certain harmonics have reduction factors numerically equal to that of the fundamental.

$$\text{For this to be so } \frac{\sin \frac{m\psi}{2}}{m \sin \frac{\psi}{2}} = \frac{\sin \frac{nm\psi}{2}}{m \sin \frac{n\psi}{2}} \quad . \quad . \quad . \quad . \quad (284)$$

$$\text{and } n = 6Km \pm 1$$

where K is an integer.

It will be noticed that the more the winding is distributed, the smaller the reduction factors become. In cases where the speed is low and the number of poles is large, frequently not more than two slots per pole per phase can be used. It is clear that in this case, especially with a squirrel-cage rotor, trouble is likely to occur due to the large value of the harmonics. In this case, one can adopt the method of skewing the slots through a full slot pitch.

In this case, the harmonics are practically eliminated.

Again, harmonics may be reduced and certain harmonics eliminated by the use of fractional pitch windings. If ε = deficiency in pitch from the full span, the coil span factor for the nth harmonic $= \cos n\dfrac{\varepsilon}{2}$, and it is clear that this will vanish when $n\dfrac{\varepsilon}{2} = \dfrac{\pi}{2}$

$$\text{i.e. when } \varepsilon = \frac{\pi}{n}$$

Therefore we may eliminate any harmonic of order n by making the span of the coil to differ from full pitch by $\dfrac{\pi}{n}$

Frequently in pole-changing motors, an M.M.F. curve on the smaller number of poles is obtained, which gives rise to large harmonics in the flux wave. Such motors, usually of the squirrel-cage type, may give trouble due to crawling. It is advisable in such cases to calculate the magnitude of the harmonics and to select a proper slot ratio in the rotor which will nullify such effects. At starting, the torque curve may be so depressed in value, due to these superposed harmonics, that it is insufficient to accelerate the

REDUCTION FACTORS FOR THREE-PHASE WINDINGS

Order of Harmonic.	Slots per pole per phase.					
	2	3	4	5	6	∞
Fundamental . .	0·966	0·96	0·958	0·957	0·957	0·955
5th harmonic . .	0·259	0·217	0·203	0·200	0·197	0·191
7th ,, . .	−0·259	−0·177	−0·158	−0·149	−0·145	−0·136
11th ,, . .	−0·966	−0·177	−0·126	−0·110	−0·102	−0·087
13th ,, . .	0·966	0·217	0·126	0·102	0·092	0·073

motor against the friction and load torques. If we denote the M.M.F. distribution of one phase by

$$F = F_o \{\cos \omega + a_3 \cos 3\omega + a_5 \cos 5\omega + a_7 \cos 7\omega, \text{ etc.}\} \quad . \quad . \quad . \quad . \quad (285)$$

and the corresponding flux distribution by

$$\phi = \phi_o(\sin \omega + b_3 \sin 3\omega + b_5 \sin 5\omega + b_7 \sin 7\omega + \ . \ . \ . \ ) \quad . \quad . \quad . \quad . \quad (286)$$

the values of the space harmonics for different numbers of phases and different coil spans are shown in the table, on page 89, due to Steinmetz—

q = number of phases

p = pitch deficiency

Q = quarter-phase winding

S = six-phase winding

T = three-phase winding

The two classes of harmonics and their characteristics are shown in the following table. (From Steinmetz.)—

		1	3	5	7	9	11	13	15	17
Order of harmonic		1	3	5	7	9	11	13	15	17
Quarter-phase motor—										
Phase rotation		+	−	+	−	+	−	+	−	+
Synchronous speed		$+1$	$-\frac{1}{3}$	$+\frac{1}{5}$	$-\frac{1}{7}$	$+\frac{1}{9}$	$-\frac{1}{11}$	$+\frac{1}{13}$	$-\frac{1}{15}$	$+\frac{1}{17}$
Time harmonics	Frequency	f	$3f$	$5f$	$7f$	$9f$	$11f$	$13f$	$15f$	$17f$
	No. of poles	p	p	p	p	p	p	p	p	p
Space harmonics	Frequency	f	f	f	f	f	f	f	f	f
	No. of poles	p	$3p$	$5p$	$7p$	$9p$	$11p$	$13p$	$15p$	$17p$
Three-phase motor—										
Phase rotation		+	0	−	+	0	−	+	0	−
Synchronous speed		$+1$	$(\pm \frac{1}{3})$	$-\frac{1}{5}$	$+\frac{1}{7}$	$(\pm \frac{1}{27})$	$-\frac{1}{11}$	$+\frac{1}{13}$	$(\pm \frac{1}{15})$	$-\frac{1}{17}$
Time harmonics	Frequency	f	$3f$	$5f$	$7f$	$9f$	$11f$	$13f$	$15f$	$17f$
	No. of poles	p	$(3p)$	p	p	$(3p)$	p	p	$(3p)$	p
Space harmonics	Frequency	f	f	f	f	f	f	f	f	f
	No. of poles	p	0	$5p$	$7p$	0	$11p$	$13p$	0	$17p$

SPACE HARMONICS OF MOTOR WINDINGS

Sine.	q any.	p any.	n	3	5	7	9	11	13	15	17	19	21
			$a_n =$ $b_n =$	zero									
$Q-0$	4	0	$a_n =$	+ 0·333	− 0·200	− 0·1429	+ 0·1111	+ 0·0909	− 0·0769	− 0·6667	+ 0·0588	+ 0·0526	− 0·0476
			$b_n =$	+ 0·1111	− 0·0400	− 0·0204	+ 0·0123	+ 0·0082	− 0·0059	− 0·0044	+ 0·0035	+ 0·0028	− 0·0023
$S-0$	6	0	$a_n =$	+ 0·6667	+ 0·2000	− 0·1429	− 0·2222	− 0·0909	+ 0·0769	+ 0·1333	+ 0·0588	− 0·0526	− 0·0952
			$b_n =$	+ 0·2222	+ 0·0400	− 0·0204	− 0·0247	− 0·0082	+ 0·0059	+ 0·0089	+ 0·0035	− 0·0028	− 0·0045
$T-0$	3	0	$a_n =$	0	− 0·2000	+ 0·1429	0	− 0·0909	+ 0·0769	0	− 0·0588	+ 0·0526	0
			$b_n =$	0	− 0·0400	+ 0·0204	0	− 0·0082	+ 0·0059	0	− 0·0035	+ 0·0028	0
$S-\frac{1}{3}$	6	$\frac{1}{3}$	$a_n =$	0	− 0·2000	+ 0·1429	0	− 0·0909	+ 0·0769	0	− 0·0588	+ 0·0526	0
			$b_n =$	0	− 0·0400	+ 0·0204	0	− 0·0082	+ 0·0059	0	− 0·0035	+ 0·0028	0
$T-\frac{1}{3}$	3	$\frac{1}{3}$	$a_n =$	0	+ 0·2000	− 0·1429	0	− 0·0909	+ 0·0769	0	+ 0·0588	− 0·0526	0
			$b_n =$	0	+ 0·0400	− 0·0204	0	− 0·0082	+ 0·0059	0	+ 0·0035	− 0·0028	0

Full-pitch quarter-phase $Q-0$
Full-pitch six-phase $S-0$
Full-pitch three-phase $T-0$
Two-thirds pitch six-phase $S-\frac{1}{3}$
Two-thirds pitch three-phase $T-\frac{1}{3}$

CHAPTER IX

SPEED CONTROL OF INDUCTION MOTORS

THE normal induction motor is practically a constant-speed machine, the slip at full load on large machines being about 2 per cent. Many methods have been developed by which it is possible to obtain large speed variations to meet commercial requirements. Some of these methods are inefficient; others are efficient.

We will now discuss the methods for obtaining speed variation. They are as follows—

(*a*) Rheostatic control by the introduction of resistance in the rotor circuit.

(*b*) Pole changing.

(*c*) Cascade connection.

(*d*) Change of supply frequency.

(*e*) By the use of a synchronous converter in circuit with the rotor. (Kraemer's system.)

(*f*) By the use of three-phase series and shunt alternating-current commutator motors in conjunction with the induction motor.

(*g*) By the use of resistance and reactance in parallel in the rotor circuits. (Boucherot motor.)

(*h*) By the revolution of the stator.

The simplest method of varying the speed of an induction motor is to insert resistance in the rotor circuit. By this means any speed required, below synchronous speed, may be obtained for a certain load, with a corresponding loss in efficiency depending on the load and the reduction in speed.

With simple rheostatic control, we are limited by the speed variation which occurs with varying load, since at light loads the motor speed will rise to approximately the maximum value, no matter what the full-load speed may be.

It is chiefly used in the following cases—

1. During starting a motor, where the energy wasted in the resistance bears a small proportion to the energy of the total run.

2. For the speed control of haulage motors of medium size, where the wind is a long one.

3. In rolling-mill motors and winding-gear equalizers.

For rolling-mill work, where a mill has to roll a great variety of

sections, it is obvious that some speed regulation is necessary. Smaller sections must be finished at a higher speed than the larger ones, as otherwise the metal would cool too rapidly and could only be formed by the expenditure of a great deal of power, which increases the liability of breakage of the mill, and the accuracy of sections and quality of product may also be affected. To obtain a reasonable production from the mill, the smaller sections must be rolled at as high a speed as possible. Within certain limits, where the speed regulation does not exceed about 10 to 15 per cent, the rheostatic method of control is the simplest and most satisfactory, and under mill-operating conditions probably the most economical.

The behaviour of the motor working in conjunction with flywheels and slip regulators is discussed in the following article by the author, which is reprinted from *The Electrician.*

Perhaps no development in electrical engineering has been so rapid as the application of the alternating-current motor to the driving of rolling mills and to electric winding. The first great impetus given to the electrification of steam mills seems to have been made when the U.S. Steel Corporation decided to adopt the electric drive about eleven years ago.

The Westinghouse companies alone have been responsible for half a million brake-horse-power of such rolling-mill plant, and since the war began the figures for this country alone are an ample testimony to the increased output, decreased cost of product, and much more satisfactory service given by the adoption of the electric drive. In such applications it is of great importance to equalize the load on the generating plant as much as possible to ensure that maximum efficiency shall be obtained in operation. To that end a flywheel is coupled to the induction motor, which is caused to drop its speed by the introduction of resistance in the rotor circuits, and thus enables the flywheel to give up some of its energy when peak loads have to be met. The energy of a flywheel $= W.v^2/2g$, where $W =$ weight in lb., $v =$ velocity in feet per minute, and $g =$ acceleration due to gravity in feet per second per second. The energy given up will therefore be proportional to the difference in the squares of velocity. With a 20 per cent slip of the induction motor, the percentage of energy given up by the flywheel will be 36 per cent.

The resistance is introduced into the rotor circuits in two ways. In one, the resistance is inserted permanently in circuit, and constitutes what is commonly known as the "Continuous Slip Regulator." In the other, the resistance is introduced at a pre-determined value of the load, and its function is to keep the current to

"Intermittent Slip Regulator." It is a matter of considerable interest to analyse the behaviour of the motor in the two cases; to investigate the advantages and disadvantages of both systems; and to show how the speed, torque, output of motor, and input from the line vary under load conditions.

We will consider, in the first place, a rolling mill of the continuous type, and deduce the torque and speed equations when permanent resistance is adopted.

Let $I =$ moment of inertia of the flywheel in lb. (ft.)^2

$\omega =$ angular velocity in radians per second at any time t sec. from the commencement of the pass

$T_3 =$ total torque in the pass, assumed constant in lb.-ft.

$T =$ torque exerted by the induction motor at any time t sec. from commencement of pass in lb.-ft.

$T_2 =$ maximum torque exerted by the induction motor in the pass in lb.-ft.

$\omega_0 =$ synchronous speed in radians per second

$\omega_f =$ speed of motor in radians per second at full load with permanent resistance in

The torque equation is

$$T - I\frac{d\omega}{dt} = T_3 \quad . \quad . \quad . \quad . \quad . \quad . \quad (287)$$

During the pass $\frac{dw}{dt}$ is negative, since the speed is falling.

In the normal range of operation of the induction motor the torque exerted is in proportion to the slip.

That this is so can be shown as follows—

The torque in synchronous watts is equal to the input to the rotor; i.e.

$$T = EI \cos\phi \times m$$

where $E =$ voltage in the rotor per phase at standstill

$I =$ rotor current per phase in amperes

$\cos\phi =$ power factor in rotor circuit

$m =$ number of rotor phases

$R =$ resistance of rotor per phase in ohms

$L\omega =$ reactance of the rotor per phase at standstill in ohms

$$s = \text{slip} = \frac{\text{synchronous speed} - \text{speed}}{\text{synchronous speed}}$$

Now $$I = \frac{sE}{\sqrt{R^2 + s^2L^2\omega^2}}$$

since the E.M.F. generated per phase $= sE$ at slip s

$$\therefore T = \frac{E \times sE}{\sqrt{R^2 + s^2L^2\omega^2}} \times \frac{R \times m}{\sqrt{R^2 + s^2L^2\omega^2}} = \frac{sE^2R \times m}{R^2 + s^2L^2\omega^2} \quad . \quad (288)$$

in synchronous watts.

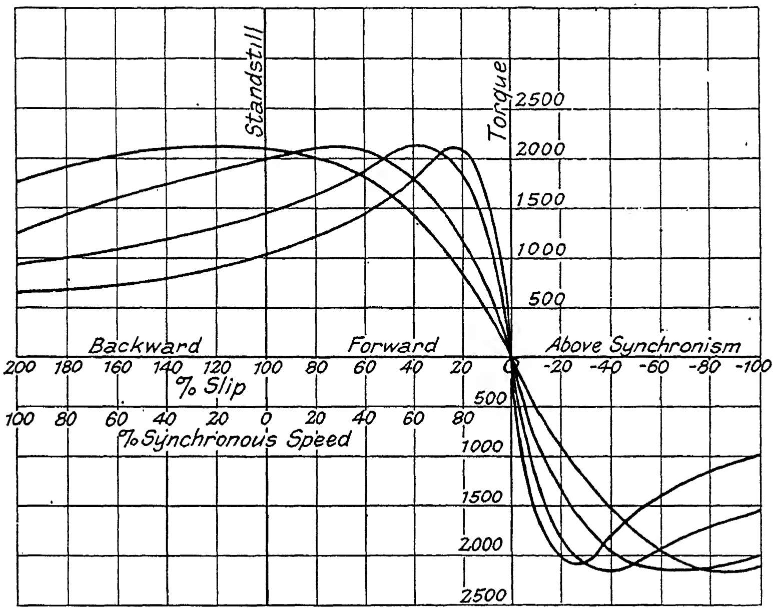


FIG. 38.—TYPICAL TORQUE-SPEED CURVES FOR INDUCTION MOTORS
With different values of resistance in the rotor circuits

This, then, is the equation connecting slip and torque.

When s is small, $s^2L^2\omega^2$ is negligible, and the torque-slip relation is given by

$$T = \frac{sE^2 \times m}{R}$$

i.e. the torque $\propto$ slip.

The torque-slip curves of an induction motor for both positive and negative slip are given in Fig. 38.

The slip usually taken is about 20 per cent in the passes, and for such values our assumption will be sufficiently accurate for practical purposes, as will be seen from Fig. 38.

$$\therefore \quad \frac{T}{T_F} = \frac{\omega_0 - \omega}{\omega_0} \div \frac{\omega_0 - \omega_F}{\omega_0} = \frac{\omega_0 - \omega}{\omega_0 - \omega_F}$$

where $T_F =$ torque at full load.

$\therefore$ Equation (287) may be written

$$T_F \frac{\omega_0}{\omega_0 - \omega_F} - \frac{T_F\,\omega}{\omega_0 - \omega_F} - \frac{Id\omega}{dt} = T_3$$

Let
$$\frac{T_F}{\omega_0 - \omega_F} = a$$

then
$$a\omega_0 - a\omega - \frac{Id\omega}{dt} = T_3$$

i.e.
$$\frac{d\omega}{dt} + \frac{a\omega}{I} = \frac{a\omega_0 - T_3}{I} = \frac{c}{I}$$

where $c = a\omega_0 - T_3$

The integrating factor of this equation is $e^{\int \frac{a}{I} dt} = e^{\frac{a}{I} t}$

where e is the base of the Naperian logs.

$$\therefore \quad \frac{d}{dt}\left(\omega e^{\frac{a}{I} t}\right) = e^{\frac{a}{I} t} \times \frac{c}{I}$$

i.e.
$$\omega e^{\frac{a}{I} t} = \frac{c}{I}\int e^{\frac{a}{I} t} + E$$

where E is the constant of integration.

$$\therefore \quad \omega = \frac{c}{a} + Ee^{-\frac{a}{I} t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (289)$$

when $\qquad t = 0,\ \omega = \omega_1 \qquad \therefore\ \omega_1 = \dfrac{c}{a} + E \quad . \quad . \quad (290)$

and $\qquad E = \omega_1 - \dfrac{c}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad (291)$

$$\therefore \quad \omega = \frac{c}{a} + \left(\omega_1 - \frac{c}{a}\right) e^{-\frac{a}{I} t} \quad . \quad . \quad . \quad . \quad (292)$$

$$\therefore\ \omega = \omega_0 - \frac{T_3(\omega_0 - \omega_F)}{T_F} + \left\{\omega_1 - \omega_0 + \frac{T_3}{T_F}(\omega_0 - \omega_F)\right\} e^{-\frac{a}{I} t} \quad . \quad (293)$$

This is our speed equation, and from it the speed at any time of the pass can be determined.

At the commencement of the first pass, ω_1 will be the speed corresponding to friction load. At the beginning of the second

pass, it may happen that the speed has not fallen to the frictional load speed. In that case the speed at the end of the first interval between passes can be found, and this speed must be substituted in the equation for the speed at the beginning of the second pass.

The *average* speed during the pass

$$= \frac{1}{t}\int_0^t \omega dt, \text{ where } t = \text{time of pass}$$

$$= \frac{1}{t}\int^t \left\{\frac{c}{a} + \left(\omega_1 - \frac{c}{a}\right)e^{-\frac{a}{I}t}\right\}dt$$

$$= \frac{c}{a} + \left[-\frac{I}{a}\left(\omega_1 - \frac{c}{a}\right)e^{-\frac{a}{I}t}\right]_0^t \times \frac{1}{t}$$

$$= \frac{c}{a} - \frac{I}{a}\frac{\left(\omega_1 - \frac{c}{a}\right)e^{-\frac{a}{I}t}}{t} + \frac{I}{at}\left(\omega_1 - \frac{c}{a}\right)$$

$$= \frac{c}{a} + I\frac{\left(\omega_1 - \frac{c}{a}\right)}{at}\left\{1 - e^{-\frac{a}{I}t}\right\} \quad . \quad . \quad . \quad . \quad (294)$$

$\therefore$ Average speed during the pass

$$= \omega_o - \frac{T_3}{T_F}(\omega_o - \omega_F) + \frac{I}{tT_F}\left[\{\omega_o - \omega_F\}\left\{\omega_1 - \omega_o + \frac{T_3}{T_F}(\omega_o - \omega_F)\right\}\right]\left[1 - e^{-\frac{a}{I}t}\right] \quad . \quad . \quad (295)$$

During the intervals between passes, we have

$$T = T_o + I\frac{d\omega}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (296)$$

Here $d\omega/dt$ is positive and $T_o =$ frictional torque

$$T = T_F\frac{\omega_o}{\omega_o - \omega_F} - T_F\frac{\omega}{\omega_o - \omega_F} \quad . \quad . \quad . \quad . \quad (297)$$

$$\therefore \quad I\frac{d\omega}{dt} + a\omega = a\omega_o - T_o \quad . \quad . \quad . \quad . \quad (298)$$

i.e. $$\frac{d\omega}{dt} + \frac{a\omega}{I} = \frac{a\omega_o - T_o}{I} = \frac{f}{I}, \text{ where } f = a\omega_o - T_o \quad (299)$$

i.e. $$\frac{d}{dt}\left(\omega e^{\frac{a}{I}t}\right) = \frac{f}{I}e^{\frac{a}{I}t} \quad . \quad . \quad . \quad . \quad (300)$$

$$\therefore \qquad \omega e^{\frac{a}{I}t} = \frac{f}{I}\int e^{\frac{a}{I}t}\,dt + F \quad . \quad . \quad . \quad . \quad (301)$$

When $t = t_2$ = time to end of first pass ; $\omega = \omega_2$ = angular velocity at the beginning of the first interval, then

$$\omega_2 = f/_a + Fe^{\frac{a}{I}t_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (302)$$

$$\therefore \quad F = (\omega_2 - f/_a)e^{\frac{a}{I}t_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (303)$$

$$\therefore \quad \omega = f/_a + (\omega_2 - f/_a)e^{\frac{a}{I}t_2}e^{-\frac{a}{I}t} \quad . \quad . \quad . \quad . \quad (304)$$

$$= f/_a + (\omega_2 - f/_a)e^{\frac{a}{I}(t_2-t)} \quad . \quad . \quad . \quad . \quad (305)$$

$$\therefore \quad \omega \text{ in interval} = \omega_o - \frac{T_o}{T_F}(\omega_o - \omega_F)$$

$$+ \left\{\omega_2 - \omega_o + \frac{T_o}{T_F}(\omega_o - \omega_F)\right\}e^{\frac{a}{I}(t_2-t)} \quad . \quad (306)$$

The average speed during the interval

$$= \frac{1}{t_3 - t_2}\int_{t_2}^{t_3}\omega\,dt = \frac{1}{t_3 - t_2}\int_{t_2}^{t_3}\left\{f/_a + (\omega_2 - f/_a)\,e^{\frac{a}{I}(t_2-t)}\right\}dt$$

$$= \frac{1}{t_3 - t_2}\left[f/_a\,t - \frac{I}{a}(\omega_2 - f/_a)\,e^{\frac{a}{I}(t_2-t)}\right]_{t_2}^{t_3}$$

$$= f/_a - \frac{I}{a(t_3 - t_2)}(\omega_2 - f/_a)\,e^{\frac{a}{I}(t_2-t_3)} + \frac{I}{a(t_3 - t_2)}(\omega_2 - f/_a)$$

$$= f/_a + \frac{I}{a(t_3 - t_2)}(\omega_2 - f/_a)\left\{1 - e^{\frac{a}{I}(t_2-t_3)}\right\} \quad . \quad . \quad (307)$$

Coming now to the question of torque, we find during the pass

$$T = T_3 + I\frac{d\omega}{dt}$$

$$T = T_3 + I\left\{-\frac{a}{I}\left(\omega_1 - \frac{c}{a}\right)\right\}e^{-\frac{a}{I}t}$$

$$= T_3 - a\left(\omega_1 - \frac{c}{a}\right)e^{-\frac{a}{I}t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (308)$$

$$= T_3 - \frac{T_F}{\omega_o - \omega_F}\left\{\omega_1 - \omega_o + \frac{T_3}{T_F}(\omega_o - \omega_F)\right\}e^{-\frac{a}{I}t} \quad . \quad (309)$$

From this the torque at any instant during the pass can be determined.

Let $T_1 =$ torque exerted by the motor at the beginning of the pass, i.e. $t = o$, then from (308) we have

$$T_1 = T_3 - a(\omega_1 - c/a) \qquad (310)$$

and T_2 the torque at the end of the pass, $t = t_2$

$$T_2 = T_3 - a(\omega_1 - c/a)\, e^{-\frac{a}{I}t_2}$$

$$= T_3 - (T_3 - T_1)\, e^{-\frac{a}{I}t_2}$$

$$= T_3 + \frac{T_1 - T_3}{e^{\frac{a}{I}t_2}} \qquad (311)$$

$$\text{Now } \frac{at_2}{I} = \frac{T_F}{\omega_o - \omega_F} \times \frac{t_2}{I} = \frac{T_F t_2}{\omega_o\left(\dfrac{\omega_o - \omega_F}{\omega_o}\right) \times I}$$

$$= \frac{T_F t_2}{\omega_o s I}$$

$$\therefore\ T_2 = T_3 + \frac{T_1 - T_3}{e^{\frac{T_F t_2}{\omega_o s I}}} = T_3 + \frac{T_1 - T_3}{A^{t_2}} \qquad (312)$$

where $s = \dfrac{\omega_o - \omega_F}{\omega_o} =$ slip at full load and $A = e^{\frac{T_F}{\omega_o s I}}$

This is a useful form in which to have the maximum torque exerted by the motor during the pass. During the decelerating period the minimum value that T_2 may reach is $T_2 = T_1$, and during the accelerating period the maximum value that T_2 may reach is $T_2 = T_1$, and this condition is reached when $A^t = 1$.

The maximum value that T_2 may reach during the decelerating period is $T_2 = T_3$ approx., and the minimum value that T_2 may reach during the accelerating period is also $T_2 = T_3$.

This is reached when $(T_1 - T_3)/A^t$ is very small as compared with T_3, and as a limit $\dfrac{T_1 - T_3}{A^t} = \dfrac{T_3}{100}$ may be set.

T_2 is about 20 per cent of T_3 during the decelerating period. It will be found that all cases in practice are included between the limits of $A^t = 1$ and 1000. The calculation of this quantity is very readily obtained by the aid of a slide rule possessing a log.

The torque exerted by the motor during the interval

$$T = T_o + I\frac{d\omega}{dt}$$

$$= T_o + I\frac{d}{dt}\left\{f/a + (\omega_2 - f/a)\, e^{\frac{a}{I}(t_2 - t)}\right\}$$

$$= T_o - I \times \frac{a}{I}(\omega_2 - f/a)\, e^{\frac{a}{I}(t_2 - t)}$$

$$= T_o - a\,(\omega_2 - f/a)\, e^{\frac{a}{I}(t_2 - t)} \quad . \quad . \quad . \quad . \quad (313)$$

When $t = t_2$, $T = T_2$

$$\therefore\ T_2 = T_o - a(\omega_2 - f/a)$$

and when $t = t_3$ the time of the beginning of the second pass, $T = T'_3$

$$\therefore\ T'_3 = T_o - a(\omega_2 - f/a)\, e^{\frac{a}{I}(t_2 - t_3)}$$

$$\therefore\ T'_3 = T_o + (T_2 - T_o)\, e^{\frac{a}{I}(t_2 - t_3)} \quad . \quad . \quad . \quad . \quad (314)$$

It is not our purpose to go into the considerations which determine the relative proportions of flywheel and motor. It may be remarked, however, that, where the power is generated at the works, it is important to use relatively heavy flywheels to keep the load on the generating plant as constant as possible. In these cases, where power is supplied from an outside source, the method of charging affects the proportions, a heavy flywheel being necessary when the maximum demand system is in vogue, and a small flywheel and relatively large motor when power is charged for on the flat-rate system, it being very important in the latter case to reduce the friction losses as much as possible.

We will illustrate the application of our formulae to a 20″ – 3′ high cogging mill for rolling ingots from 10″ × 10″ to 6″ × 6″.

The number of passes is 6, and speed of the mill 70/59·5 r.p.m.

The torque time diagram is given in Fig. 39.

The flywheel employed has a moment of inertia of $0{\cdot}54 \times 10^6$ lb. (ft.)^2, and has a diameter of 20 ft.

The motor has a full-load output of 630 h.p. at 70 r.p.m. normal slip, and the speed drop arranged for is from 70 to 59·5 r.p.m.

The maximum torque exerted by the motor = 83,700 ft.-lb. = 1·765 full load, and the speed is then 59·5 r.p.m.

The slip is therefore $\dfrac{71{\cdot}5 - 59{\cdot}5}{71{\cdot}5} = \dfrac{12}{71{\cdot}5} = 16{\cdot}8\%$

At full load the torque is 83,700/1·0765 lb.-ft. = 47,500 lb.-ft.
Let S = slip, corresponding to full-load torque, then

$$\frac{47{,}500}{S} = \frac{83{,}700}{0{\cdot}168}$$

$$\therefore\ S = \frac{47{,}500 \times 0{\cdot}168}{83{,}700} = 0{\cdot}095 = 9{\cdot}5\%$$

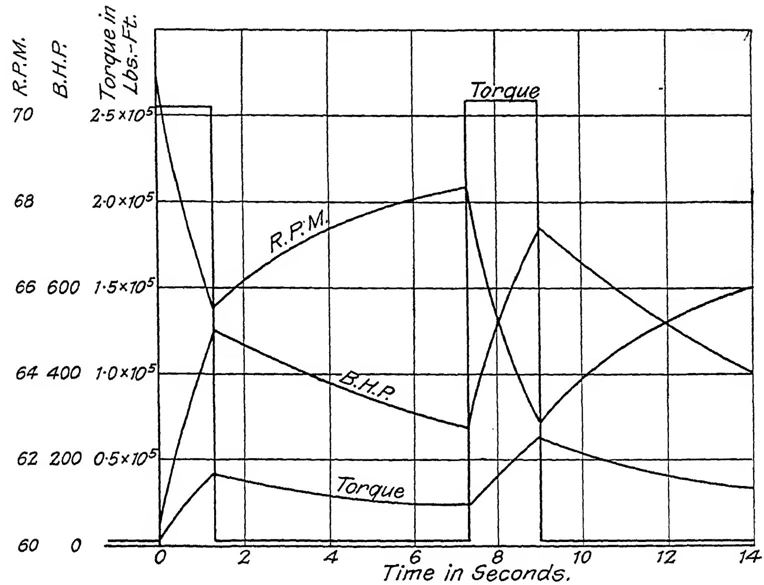


FIG. 39.—CURVES SHOWING PERFORMANCE OF THE INDUCTION MOTOR FOR PERMANENT RESISTANCE

The full-load speed is, therefore, 64·7 r.p.m., and the full-load output is

$$\frac{47{,}500 \times 2\pi \times 64{\cdot}7}{33{,}000}\ \text{h.p.} = 585\ \text{h.p.}$$

We will calculate the torque and speed in the first pass and interval.

1. In the pass, ω, the angular speed in radians per second,

$$= \frac{c}{a} + \left(\omega_1 - \frac{c}{a}\right) e^{-\frac{a}{I}t}$$

$$\therefore \frac{c}{a} = \frac{a\omega_o - T_3}{a} = \omega_o - \frac{T_3}{T_F}(\omega_o - \omega_F)$$

$$\omega_o = 2\pi \times \frac{71\cdot5}{60} = 7\cdot48 \text{ radians per second}$$

$$\omega_F = 2\pi \times \frac{64\cdot7}{60} = 6\cdot78 \text{ radians per second}$$

$T_3 = 2\cdot505 \times 10^5$ lb.-ft. ; friction torque $= 4000$ lb.-ft.

$$\frac{T_3}{T_F} = \frac{2\cdot5 \times 10^5}{4\cdot75 \times 10^4} = 5\cdot28 \text{ ; } \omega_o - \omega_F = 0\cdot7$$

$\frac{c}{a} = 3\cdot78$; no-load slip $= 0\cdot8\%$; revs. per min. at no load, $70\cdot9$ r.p.m.

$$\frac{c}{a} = 3\cdot62 \text{ ; } \frac{a}{I} = \frac{47{,}500}{0\cdot095 \times 7\cdot48 \times 0\cdot54 \times 10^6} = 0\cdot1236$$

Using Equation (292) for the speed and (308) for the torque, we will tabulate the various quantities for the first pass—

Time : Seconds.	ω	R.P.M.	Torque (lb.-ft.).	$e^{-\frac{a}{I}t}$	H.P. Output of Motor.
0 (beginning)	7·4	70·9	4,000	1·000	54
0·3	7·27	69·3	13,500	0·965	179
0·6	7·14	68·0	22,500	0·928	292
0·9	7·0	66·8	31,500	0·39	409
1·27 (end)	6·88	65·5	41,900	0·856	524

For the first interval, we use Equation (305) for the speed and Equation (313) for the torque; $f/a = 7\cdot421$; $\omega_2 - f/a = 6\cdot88 - 7\cdot421 = -0\cdot541$.

Tabulating the results for the interval, we have as above—

Time : Sec.	ω	R.P.M.	Torque (lb.-ft.).	$e^{-\frac{a}{I}t}$	H.P. Output of Motor.
1·5	6·894	65·7	39,600	$e^{-\frac{a}{I}(t_2 - t)} = 0\cdot973$	495
2·5	6·955	66·5	35,600	,, $= 0\cdot862$	450
3·5	7·014	67·0	31,500	,, $= 0\cdot753$	402
4·5	7·057	67·5	28,600	,, $= 0\cdot673$	366
7·27	7·164	68·4	21,400	,, $= 0\cdot475$	279

For the second pass, we use Equation (292) for the speed, but for ω_1 we must put the value 7·164, the value of the speed at the end of the first pass. The tabulated results for the second pass are as follows—

Time : Seconds.	ω	R.P.M.	Torque (lb.-ft.).	$e^{-\frac{a}{I}t}$	H.P. Output of Motor.
0	7·164	68·4	21,400	1·0	279
0·3	7·05	67·5	30,000	0·965	384
0·6	6·91	66·0	39,000	0·928	490
0·9	6·79	64·7	48,000	0·89	592
1·2	6·69	63·8	54,000	0·865	656
1·525	6·57	62·8	62,000	0·827	740

$$\frac{c}{a} = a\omega_o - T_3\,;\ \text{here } T_3 = 2{\cdot}58 \times 10^5$$

$$\therefore\ \frac{c}{a} = \omega_o - \frac{T_3}{a} = \omega_o - \frac{T_3}{T_F}(\omega_o - \omega_F) = 7{\cdot}48 - \frac{2{\cdot}58 \times 10^5}{4{\cdot}75 \times 10^4} \times 0{\cdot}7$$

$$= 7{\cdot}48 - 3{\cdot}8 = 3{\cdot}68$$

$$\omega_1 - \frac{c}{a} = 7{\cdot}164 - 3{\cdot}68 = 3{\cdot}484$$

$$\text{Also } T = T_3 - a\left(\omega_1 - \frac{c}{a}\right)e^{-\frac{a}{I}t}\,;\quad a = \frac{T_F}{\omega_o - \omega_F} = \frac{47{,}500}{0{\cdot}7} = 67{,}855$$

In the interval of the second pass, we have

$$\omega = f/a + (\omega_2 - f/a)\,e^{\frac{a}{I}(t_2 - t)}\,;\ f = a\omega_o - T_o$$

$$\text{and } \frac{f}{a} = \omega_o - \frac{T_o}{a} = \omega_o - \frac{T_o}{T_F}(\omega_o - \omega_F)$$

$$= 7{\cdot}48 - \frac{4000}{47{,}500} \times 0{\cdot}7 = 7{\cdot}48 - 0{\cdot}059 = 7{\cdot}421$$

$$\omega_2 = 6{\cdot}57$$

$$\therefore\ \omega = 7{\cdot}421 - 0{\cdot}85\,e^{\frac{a}{I}(t_2 - t)}$$

Time from commencement of Second Pass in Seconds.	ω	R.P.M.	Torque (lb.-ft.).	$e^{-\frac{a}{I}(t_2 - t)}$	H.P. Output.
2	6·619	63·3	58,300	$e^{-\frac{a}{I}0\cdot475} = 0\cdot943$	700
3	6·718	64·4	51,700	$e^{-\frac{a}{I}1\cdot475} = 0\cdot827$	632
4	6·799	64·8	46,300	$e^{-\frac{a}{I}2\cdot475} = 0\cdot733$	570
5	6·871	65·5	41,200	$e^{-\frac{a}{I}3\cdot475} = 0\cdot646$	515
6·525	7·017	67·0	31,350	$e^{-\frac{a}{I}6} = 0\cdot475$	399

The average speed during the first pass is given by Equation (295), viz.,

$$\omega_{av} = \omega_o - \frac{T_3}{T_F}(\omega_o - \omega_F) + \frac{I}{tT_F}$$

$$\left[\left\{(\omega_o - \omega_F)\right\}\left\{\omega_1 - \omega_o + \frac{T_3}{T_F}(\omega_o - \omega_F)\right\}\right]\left[1 - e^{-\frac{a}{I}t}\right]$$

$$\omega_{av} = 7\cdot48 - 3\cdot7 - \frac{0\cdot54 \times 10^6}{1\cdot27 \times 4\cdot75 \times 10^4} \times 0\cdot7 \times (-0\cdot08 - 3\cdot7)$$

$$\left(1 - e^{-0\cdot1236 \times 1\cdot27}\right)$$

$$= 3\cdot78 + 6\cdot26 \times 3\cdot62 \times 0\cdot145$$

$$= 3\cdot78 + 3\cdot29$$

$$= 7\cdot07$$

$\therefore$ Average speed in revs. per min. in the first pass $= 67\cdot5$

The motor torque at any instant of the pass

$$T = T_3 - a\left(\omega_1 - \frac{c}{a}\right)e^{-\frac{a}{I}t}$$

the average torque during the pass

$$= \frac{1}{t}\int_0^t T\,dt$$

$$= \frac{1}{t}\int_0^t \left\{T_3 - a\left(\omega_1 - \frac{c}{a}\right)e^{-\frac{a}{I}t}\right\}dt$$

$$= \frac{1}{t}\left[T_3 t + I\left(\omega_1 - \frac{c}{a}\right)e^{-\frac{a}{I}t}\right]_o^t$$

$$= T_3 + \frac{I}{t}\left(\omega_1 - \frac{c}{a}\right)e^{-\frac{a}{I}t} - \frac{I}{t}\left(\omega_1 - \frac{c}{a}\right)$$

For the first pass

$$T = 2{\cdot}505 \times 10^5 + (0{\cdot}54 \times 10^6 \times 3{\cdot}62 \times 0{\cdot}855 - 0{\cdot}54 \times 10^6 \times 3{\cdot}62) \times \frac{1}{1{\cdot}27}$$

$$= 2{\cdot}505 \times 10^5 + 0{\cdot}54 \times 10^6 \times 3{\cdot}62\,(0{\cdot}855 - 1) \times \frac{1}{1{\cdot}27}$$

$$= 2{\cdot}505 \times 10^5 - \frac{0{\cdot}283}{1{\cdot}27} \times 10^6$$

$$= 2{\cdot}505 \times 10^5 - 2{\cdot}23 \times 10^5$$

$= 27{,}500$ lb.-ft.

The results for the two passes are shown in Fig. 39. In a similar manner the other passes can be figured out.

We will now proceed to investigate the case of "Constant Output," as distinct from "Constant Torque" in the passes.

The power equation for the pass is

$$T\omega + I\frac{d\omega}{dt}\omega = W \quad . \quad . \quad . \quad (315)$$

where W = constant power in pass.

As before, $T = a\omega_o - a\omega$ . . . . . . (316)

$$\therefore\ a\omega\,\omega_o - a\omega^2 - I\frac{d\omega}{dt}\omega = W \quad . \quad . \quad . \quad . \quad . \quad (317)$$

$$\therefore\ a\omega\,\omega_o - a\omega^2 - W = I\frac{d\omega}{dt}\omega \quad . \quad . \quad . \quad . \quad . \quad (318)$$

$$\therefore\ dt = \frac{I\omega\,d\omega}{a\omega\,\omega_o - a\omega^2 - W} \quad . \quad . \quad . \quad . \quad . \quad . \quad (319)$$

$$\therefore\ dt = -\frac{1}{2a} \times I\frac{(a\omega_o - 2a\omega)\,d\omega}{a\omega\,\omega_o - a\omega^2 - W} + \frac{I}{2}\frac{\omega_o\,d\omega}{a\omega\,\omega_o - a\omega^2 - W} \quad (320)$$

$$\therefore\quad t = -\frac{I}{2a}\log(a\omega\,\omega_o - a\omega^2 - W) + \frac{I\omega_o}{2}\int\frac{d\omega}{a\omega\,\omega_o - a\omega^2 - W} + C \quad (321)$$

Where C is the constant of integration,

$$\frac{I\omega_o}{2a}\int\frac{d\omega}{\omega\,\omega_o-\omega^2-\dfrac{W}{a}}=-\frac{I\omega_o}{2a}\int\frac{d\omega}{\omega^2-\omega\,\omega_o+\dfrac{W}{a}} \quad . \quad (322)$$

$$=-\frac{I\omega_o}{2a}\int\frac{d\omega}{(\omega-\frac{1}{2}\,\omega_o)^2+\left(\dfrac{W}{a}-\dfrac{\omega_o^2}{4}\right)} \quad . \quad . \quad . \quad (323)$$

$$=-\frac{I\omega_o}{2a\sqrt{\dfrac{W}{a}-\dfrac{\omega_o^2}{4}}}\tan^{-1}\frac{\omega-\frac{1}{2}\,\omega_o}{\sqrt{\dfrac{W}{a}-\dfrac{\omega_o^2}{4}}} \quad . \quad . \quad . \quad (324)$$

$$\therefore \quad t=-\frac{I}{2a}\log\,(a\omega\,\omega_o-a\omega^2-W)$$

$$-\frac{I\omega_o}{2a\sqrt{\dfrac{W}{a}-\dfrac{\omega_o^2}{4}}}\tan^{-1}\frac{\omega-\frac{1}{2}\,\omega_o}{\sqrt{\dfrac{W}{a}-\dfrac{\omega_o^2}{4}}}+C \quad . \quad . \quad . \quad (325)$$

To determine C, we have, when $t=o$, $\omega=\omega_1$

$$\therefore \quad o=-\frac{I}{2a}\log\,(a\omega_o\,\omega_1-a\omega_1^2-W)$$

$$-\frac{I\omega_o}{2a\sqrt{\dfrac{W}{a}-\dfrac{\omega_o^2}{4}}}\tan^{-1}\frac{\omega_1-\frac{1}{2}\,\omega_o}{\sqrt{\dfrac{W}{a}-\dfrac{\omega_o^2}{4}}}+C \quad . \quad . \quad . \quad (326)$$

$$\therefore \; C=\frac{I}{2a}\log\,(a\omega_o\,\omega_1-a\omega_1^2-W)$$

$$+\frac{I\omega_o}{2a\sqrt{\dfrac{W}{a}-\dfrac{\omega_o^2}{4}}}\tan^{-1}\frac{\omega_1-\frac{1}{2}\,\omega_o}{\sqrt{\dfrac{W}{a}-\dfrac{\omega_o^2}{4}}} \quad . \quad . \quad . \quad (327)$$

$$\therefore \quad t=\frac{I}{2a}\log\left(\frac{a\omega_o\,\omega_1-a\omega_1^2-W}{a\omega_o\,\omega-a\omega^2-W}\right)$$

$$+\frac{I\omega_o}{2a\sqrt{\dfrac{W}{a}-\dfrac{\omega_o^2}{4}}}\left[\tan^{-1}\frac{\omega_1-\frac{1}{2}\,\omega_o}{\sqrt{\dfrac{W}{a}-\dfrac{\omega_o^2}{4}}}-\tan^{-1}\frac{\omega-\frac{1}{2}\,\omega_o}{\sqrt{\dfrac{W}{a}-\dfrac{\omega_o^2}{4}}}\right] \quad (328)$$

In passes, the last term in brackets is usually negligibly small,

and where it is not negligible, the inverse tangent can be expanded, and the first term taken. An approximation can thus be obtained.

$$\therefore\ t = \frac{I}{2a} \log \frac{a\omega_o \omega_1 - a\omega_1^2 - W}{a\omega_o \omega - a\omega^2 - W} \quad . \quad . \quad . \quad . \quad (329)$$

$$\frac{2at}{I} = \log \frac{a\omega_o \omega_1 - a\omega_1^2 - W}{a\omega_o \omega - a\omega^2 - W} \quad . \quad . \quad . \quad . \quad (330)$$

Put $a\omega_o \omega_1 - a\omega_1^2 - W = d$

$$\text{then } \frac{d}{a\omega_o \omega - a\omega^2 - W} = e^{-\frac{2at}{I}} \quad . \quad . \quad (331)$$

$$de^{-\frac{2at}{I}} = a\omega_o \omega - a\omega^2 - W \quad . \quad . \quad . \quad . \quad (332)$$

$$\text{i.e. } a\omega^2 - a\omega_o \omega + W + de^{-\frac{2at}{I}} = o \quad . \quad . \quad (333)$$

$$\therefore\ \omega = \frac{a\omega_o \pm \sqrt{a^2 \omega_o^2 - 4\left(W + de^{-\frac{2at}{I}}\right)a}}{2a} \quad . \quad (334)$$

The speed is thus determined at any time during the pass.

The output of the induction motor during the pass

$$= W + I\omega \frac{d\omega}{dt}$$

$$\frac{d\omega}{dt} = \frac{1}{4a} I \frac{8da^2e^{-\frac{2at}{I}}}{\sqrt{a^2 \omega_o^2 - 4\left(W + de^{-\frac{2at}{I}}\right)a}}$$

$$= I \frac{2dae^{-\frac{2at}{I}}}{\sqrt{a^2 \omega_o^2 - 4\left(W + de^{-\frac{2at}{I}}\right)a}} \quad . \quad (335)$$

The motor output can thus be determined for constant total output in the passes. In the interval between the passes, the power equation is

$$I\frac{d\omega}{dt}\omega + W_o = T\omega \quad . \quad . \quad . \quad (336)$$

where W_o = frictional power, which is assumed to remain constant.

The solution of this type of equation has already been indicated.

The case of the automatic slip regulator. We will now consider the case of a rolling-mill cycle, in which an automatic slip regulator

It may be remarked that ω_1 is easily determined, for it is simply the angular speed in radians per second when the regulator comes in. In other words, it will be the full-load speed of the motor, without external resistance, if the regulator is set to operate at the full-load of the motor.

It will be seen, in this case, that the speed-time curve in the pass is a straight line. This can otherwise be seen to be true, since the torque of the motor remains constant in value, the torque exerted by the flywheel is constant, since the total torque in the pass is constant.

The torque exerted by the flywheel

$$= I\frac{d\omega}{dt} \qquad (343)$$

$$\text{i.e.} \quad I\frac{d\omega}{dt} = \text{a constant} \qquad (344)$$

$$\therefore \quad \frac{d\omega}{dt} = \text{constant} \qquad (345)$$

i.e. the angular speed varies uniformly with time.

In the interval between passes, our torque equation is

$$T - I\frac{d\omega}{dt} = T_o \qquad (346)$$

Where T_o = friction torque, here $d\omega/dt$ is positive.

The solution is

$$\omega = \frac{T - T_o}{I}t + F \qquad (347)$$

When $t = t_2$, the time to the end of the first pass, $\omega = \omega_2$

$$\therefore \omega_2 = \left(\frac{T - T_o}{I}\right)t_2 + F \qquad (348)$$

$$\therefore F = \omega_2 - \left(\frac{T - T_o}{I}\right)t_2 \qquad (349)$$

$\therefore$ ω in the interval

$$= \omega_2 + \left(\frac{T - T_o}{I}\right)(t - t_2) \qquad (350)$$

The speed, therefore, rises according to a linear law in the intervals.

The power of the motor at any time during the pass with the regulator in

$$= T\omega = T\omega_1 + T\left\{\frac{T - T_3}{I}\right\}(t - t_1) \qquad . \qquad . \quad (351)$$

The average speed in the pass during the time the regulator is in action $= \frac{1}{2}(\omega_1 + \omega_2)$, where $\omega_1 =$ angular speed at entrance of regulator, $\omega_2 =$ angular speed at end of pass

$$= \tfrac{1}{2}(\omega_1 + \omega_1 + (T - T_3)(t_2 - t_1) \div I)$$

$$= \omega_1 + \frac{T - T_3}{I}\frac{(t_2 - t_1)}{2} \qquad . \qquad . \qquad . \quad (352)$$

In the interval, the torque equation is

$$T - I\frac{d\omega}{dt} = T_o \quad . \quad . \quad . \quad . \quad . \quad (353)$$

where $T_o =$ frictional torque; $d\omega/dt$ is positive in the interval between passes.

It is interesting to inquire what values of rotor resistance are required in order that the torque of the motor shall remain constant.

Referring to Equation (288), we have the torque in synchronous watts—input to rotor—i.e. torque $\times$ synchronous speed (expressed in watts)

$$= \frac{mE^2Rs}{R^2 + s^2L^2\omega^2}$$

Let $T =$ torque exerted by the motor in kilogrammetres, then $T\omega_o\, 9{\cdot}81 =$ torque in synchronous watts.

$$\therefore \quad T\omega_o\, 9{\cdot}81 = \frac{mE^2Rs}{R^2 + s^2L^2\omega_e^2} \qquad . \qquad . \qquad . \quad (354)$$

$m =$ number of rotor phases; $s =$ slip $= \dfrac{\omega_o - \omega}{\omega_o}$; $L\omega_e = L$ $\times\ 2\pi \times$ frequency $=$ reactance of rotor per phase at standstill; $R =$ rotor resistance per phase $+$ external resistance per phase.

$$\therefore\ T\omega_o\, 9{\cdot}81R^2 - mE^2Rs + T\omega_o\, 9{\cdot}81s^2L^2\omega_e^2 = 0 \quad . \qquad . \quad (355)$$

From this

$$R = \frac{mE^2s \pm \sqrt{(m^2E^4s^2 - 386T^2\omega_o^2\,\omega_e^2s^2L^2)}}{19{\cdot}62T\,\omega_o} \qquad . \qquad . \quad (356)$$

$$\therefore\ R = \frac{mE^2s \pm s\sqrt{(m^2E^4 - 386T^2 \times \omega_o^2 \times \omega_e^2 \times L^2)}}{19{\cdot}62T\,\omega_o} \quad . \quad (357)$$

$$\therefore \frac{R}{s} = \frac{mE^2 \pm \sqrt{(m^2E^4 - 386T^2\omega_o^2\omega_e^2L^2)}}{19{\cdot}62T\omega_o} \quad . \quad . \quad . \quad (358)$$

$$= \frac{mE^2 \pm \sqrt{(m^2E^4 - 15{,}200T^2\omega_o^2 \times f^2 \times L^2)}}{19{\cdot}62T\,\omega_o} \quad . \quad . \quad (359)$$

$\omega_e = 2\pi \times$ frequency $= 2\pi \times f$ must not be confused with ω, the angular velocity of the machine.

We have shown the relation between resistance and slip. We shall go now a step further, and get the relation between resistance and time.

$$R = s\left\{\frac{mE^2 \pm \sqrt{(m^2E^4 - 15{,}200T^2\omega_o^2 \times f^2 \times L^2)}}{19{\cdot}62T\,\omega_o}\right\} \quad . \quad (360)$$

and $s = \dfrac{\omega_o - \omega}{\omega_o}$

$$\therefore \quad R = \frac{\omega_o - \omega}{\omega_o}\left\{\frac{mE^2 \pm \sqrt{(m^2E^4 - 15{,}200T^2\omega_o^2 \times f^2 \times L^2)}}{19{\cdot}62T\,\omega_o}\right\} (361)$$

in the pass.

$$\therefore \quad R =$$

$$\frac{\omega_1 - \left(\dfrac{T - T_3}{\mathrm{I}}\right)(t - t_1) + \omega_o}{\omega_o}\left\{\frac{mE^2 \pm \sqrt{(m^2E^4 - 15{,}200T^2\omega_o^2f^2L^2)}}{19{\cdot}62\,T\omega_o}\right\}$$

(362)

It will be noticed that in Equation (359) the right-hand side of the equation is constant—assuming a constant coefficient of self-induction for the motor.

$$\therefore R = Ks$$

i.e. the resistance should be directly proportioned to the slip for constant torque.

We shall next take the case of the continuous mill with an automatic slip regulator and constant output during the pass. Our power equation is

$$T\omega - I\frac{d\omega}{dt}\omega = W \quad . \quad . \quad . \quad . \quad (363)$$

Where $W =$ constant output during the pass,

$$\frac{I\omega d\omega}{T\omega - W} = dt \quad . \quad . \quad . \quad . \quad (364)$$

$$\frac{I\omega d\omega}{W\left(\frac{T}{W}\omega - 1\right)} = dt \quad . \quad . \quad . \quad (365)$$

$$\therefore \quad \frac{I}{W}\left\{\frac{W}{T}\left(\frac{1}{\frac{T}{W}\omega - 1} + 1\right)\right\}d\omega = dt \quad . \quad (366)$$

Integrating, we have

$$C + \frac{IW}{T^2}\log\left(\frac{T}{W}\omega - 1\right) + \frac{I}{T}\omega = t \quad (367)$$

To determine C, put $t = t_1$, then $\omega = \omega_1$, the speed at time t_1, when the regulator comes in ;

then $$C + \frac{IW}{T^2}\log\left(\frac{T}{W}\omega_1 - 1\right) - \frac{I}{T}\omega_1 = t_1 \quad . \quad (368)$$

$$\therefore \quad C = -\frac{IW}{T^2}\log\left(\frac{T}{W}\omega_1 - 1\right) + \frac{I}{T}\omega_1 + t_1 \quad . \quad (369)$$

$$\therefore \quad t = \frac{IW}{T^2}\log\frac{\left(\frac{T}{W}\omega - 1\right)}{\left(\frac{T}{W}(\omega_1 - 1)\right)} + \frac{I}{T}(\omega - \omega_1) + t_1 \quad (370)$$

$$\therefore \ t = \frac{IW}{T^2}\log\left\{1 + \frac{\frac{T}{W}(\omega - \omega_1)}{\frac{T}{W}(\omega_1 - 1)}\right\} + \frac{I}{T}(\omega - \omega_1) + t_1 \, . \quad . \quad (371)$$

Expanding the logarithm, we have

$$\log\left\{1 + \frac{\frac{T}{W}(\omega - \omega_1)}{\frac{T}{W}\ \omega_1 - 1}\right\} = \frac{\frac{T}{W}(\omega - \omega_1)}{\frac{T}{W}\ \omega_1 - 1} - \frac{1}{2}\left\{\frac{\frac{T}{W}(\omega - \omega_1)}{\frac{T}{W}\ \omega_1 - 1}\right\}^2 \text{etc.}$$

(372)

The second and higher powers are negligibly small.

$$\therefore \quad t = \frac{IW}{T^2}\left\{\frac{\frac{T}{W}(\omega - \omega_1)}{\frac{T}{W}\ \omega_1 - 1}\right\} + \frac{I}{T}(\omega - \omega_1) + t_1 \quad . \quad (373)$$

$$\therefore \quad t - t_1 = \frac{I}{T}\left\{\frac{\omega - \omega_1}{\frac{T}{W}\omega_1 - 1} + \omega - \omega_1\right\} \quad . \quad . \quad . \quad . \quad (374)$$

$$\therefore \quad t - t_1 = \frac{I}{T}\left\{\frac{\omega - \omega_1 + \frac{T}{W}\omega_1\,\omega - \frac{T}{W}\omega_1^2 - \omega + \omega_1}{\frac{T}{W}\omega_1 - 1}\right\} \quad . \quad (375)$$

$$= \frac{I}{T}\left\{\frac{\frac{T}{W}\omega_1(\omega - \omega_1)}{\frac{T}{W}\omega_1 - 1}\right\} \quad . \quad . \quad . \quad . \quad (376)$$

$$\therefore \quad (t - t_1)\left(\frac{T}{W}\omega_1 - 1\right) = \frac{I}{W}\omega_1\,\omega - \frac{I}{W}\omega_1^2 \quad . \quad . \quad (377)$$

$$\therefore \quad \omega = \frac{\left(\frac{T}{W}\omega_1 - 1\right)(t - t_1) + \frac{I}{W}\omega_1^2}{\frac{I}{W}\omega_1} \quad . \quad . \quad . \quad (378)$$

and

$$\omega = \omega_1 + \left(\frac{T}{I} - \frac{W}{I\omega_1}\right)(t - t_1)\,. \quad . \quad . \quad . \quad (379)$$

If, as is sometimes the case, $\frac{IW}{T^2}$ is fairly large, it will be necessary to take the second or even the third power of the expansion of the logarithm, and these will give a quadratic and a cubic equation in ω which can be solved by the usual methods.

We will take as an example for the automatic slip regulator a steel tube mill. This is an admirable case for such an application for the passes, and intervals are relatively long.

The cycle consists of 1 pass of 45 sec. and 1 interval of 75 sec.

The total torque required in the pass = 26,000 lb.-ft.

The friction torque = 2100 lb.-ft.

The speed of the mill is 150 r.p.m.

The motor is geared to the mill shaft, and the reduction ratio = 3·23.

The total torque-time in lb.-ft. sec. = $26{,}000 \times 45 + 2100 \times 75 = 1{\cdot}17 \times 10^6 + 1{\cdot}575 \times 10^5 = 1{\cdot}327 \times 10^6$

$$\text{Average torque in the cycle} = \frac{1\cdot327 \times 10^6}{120} = 11\cdot05 \times 10^3$$
$$= 11{,}050 \text{ lb.-ft.}$$

To find the size of the flywheel, we proceed as follows—

Let I = moment of inertia of the flywheel in lb. (ft.)^2

ω_1 = angular velocity in radians per second at the beginning of the pass

ω_2 = angular velocity in radians per second at the end of the pass

The energy given out by the wheel in the pass $= \frac{1}{2}I(\omega_1^2 - \omega_2^2)$

$\omega_1 = 51\cdot8$ radians per second $= 495$ r.p.m.

$$\omega_2 = 2\pi \times \frac{450}{60} = 47\cdot1 \text{ radians per second}$$

The torque given out by the flywheel in the pass

$$= \frac{(26{,}000 - 11{,}050)}{3\cdot23} = \frac{14{,}950}{3\cdot23} = 4640 \text{ lb.-ft.}$$

The torque of the flywheel in lb.-ft. $\times$ average angular velocity in radians per second in the pass $\times$ time of pass in seconds = energy given out by flywheel in the pass.

$$\therefore \tfrac{1}{2}I(51\cdot8^2 - 47\cdot1^2) = 4640 \times 2\pi \times \frac{474}{60} \times 45$$

$$\therefore \quad I \text{ in lb. (ft.)}^2 = \frac{2 \times 4640 \times 2\pi}{460} \times \frac{474}{60} \times 45$$
$$= 45{,}000 \text{ lb. (ft.)}^2$$

Average horse-power of the motor

$$= \frac{11{,}050 \times 2\pi \times 474}{3\cdot23 \times 550 \times 60 \times 0\cdot95} = 326 \text{ h.p.}$$

This, of course, will not be the rating of the motor. The motor will be required to be put in for the R.M.S. brake-horse-power over the cycle. We have seen that the speed line is a straight line after the regulator comes in. Before its introduction, however, the motor is working with permanent resistance in its circuit—viz., its own rotor resistance. Our speed and torque Equations (292) and (308).

We will apply Equation (292),

$$\omega = \frac{c}{a} + \left(\omega_1 - \frac{c}{a}\right)e^{-\frac{a}{I}t}$$

$$\frac{c}{a} = \omega_o - \frac{T_3}{a}; \ \omega_o = 500 \times \frac{2\pi}{60} = 52{\cdot}4 \text{ radians per second}$$

$$T_3 = \frac{26{,}000}{3{\cdot}23} = 8050$$

$$a = \frac{T_f}{\omega_o - \omega_F} = \frac{11{,}050 \times 60}{3{\cdot}23 \times 2\pi \times 15} = 2180$$

Natural slip at full load = 3%

$$\frac{c}{a} = 52{\cdot}4 - \frac{8050}{2180} = 52{\cdot}4 - 3{\cdot}69 = 48{\cdot}71$$

$$\frac{a}{I} = \frac{2180}{45{,}000} = 0{\cdot}6485$$

The time to reach full-load torque is given by

$$50{\cdot}85 = 48{\cdot}71 - 3{\cdot}09e^{-\frac{a}{I}t}$$

$$\therefore \quad \frac{2{\cdot}14}{3{\cdot}09} = 0{\cdot}692$$

$$0{\cdot}0485t = 0{\cdot}368$$

$$\therefore \quad t = 7{\cdot}6 \text{ sec.}$$

The torque equation for the beginning of the pass up to the point when the regulator comes in is

$$T = T_3 - a\left(\omega_1 - \frac{c}{a}\right)e^{-\frac{a}{I}t}$$

$$= 8050 - 2180(51{\cdot}8 - 48{\cdot}71)\,e^{-\frac{a}{I}t}$$

$$= 8050 - 6745\,e^{-\frac{a}{I}t}$$

We will tabulate the results for the pass.

Time in Seconds.	ω	R.P.M.	$e^{-0{\cdot}0485\,t}$	Torque (lb.-ft.)	H.P. of Motor.
0	51·8	495·0	1	1305	122·7
2	51·51	492·0	0·905	1950	182·5
4	51·25	490·0	0·822	2520	235·1
6	51·01	487·5	0·746	3025	281·1
7·5 Reg. comes in	50·84	485·0	0·692	3390	311·0
45 End of Pass	47·84	457·0	—	3390	295·0

From the points K to H, i.e. when the regulator is in action, the speed falls in a *st* line, as shown in Equation (342).

$$\therefore \quad \omega = \omega_1 + \frac{T - T_3}{I}(t - t_1)$$

$\therefore$ at the end of the pass

$$51{\cdot}8 + \frac{(3390 - 8050)}{45{,}000} \times 37{\cdot}5$$
$$= 51{\cdot}8 - 3{\cdot}89 = 47{\cdot}91$$

In the interval, the speed rises in a *st* line, the torque remaining constant, till the natural torque slip curve of the motor is reached. This is seen from Fig. 40. Any further increase in speed beyond that corresponding to the natural slip of the motor results in a reduction in torque, and our speed and torque equations are, from this point, simply those for permanent resistance (viz., the resistance of the rotor windings)—namely, (305) and (313).

We must find the time taken to reach the natural torque slip curve, i.e. the torque-slip curve corresponding to no external resistance in the rotor circuits.

In the interval the speed rises in a *st* line, while the regulator is in, according to Equation (350), viz.,

$$\omega = \omega_2 + \frac{T - T_o}{I}\,(t - t_2)$$

$$\therefore \quad \frac{d\omega}{dt} = \frac{T - T_o}{I}$$

$$= \frac{3390 - 650}{45{,}000} = \frac{2740}{45{,}000} = 0{\cdot}061$$

The speed of the motor at full-load torque $= 2\pi \times \dfrac{485}{60} = 50{\cdot}84$

increase in speed from end of pass $= 50{\cdot}84 - 47{\cdot}91 = 2{\cdot}93$

$\therefore$ time to reach the natural slip-curve $= \dfrac{2{\cdot}93}{0{\cdot}061}$ sec. $= 48$ sec.

The remaining 27 sec. is taken up on the permanent resistance of the rotor.

Equation (305) applies to this period, viz.,

$$\omega = f/_a + (\omega_2 - f/_a)\, e^{\frac{a}{I}(t_2 - t)}$$

$$f/a = \omega_o - \frac{T_o}{a} = 52{\cdot}25 - \frac{650}{2180} = 52{\cdot}25 - 0{\cdot}296 = 51{\cdot}954$$

$$\omega_2 = 50{\cdot}84\,;\ \omega_2 - f/a = -1{\cdot}114$$

Tabulating the results for the 27 sec. in the interval, we have—

Time from 48 sec. in Interval.	ω	R.P.M.	$e^{-0{\cdot}0485\,t}$	Torque.	H.P.
2	50·95	486	0·905	2850	263·5
4	51·024	488	0·822	2650	264
6	51·117	489	0·746	2460	229
10	51·27	490	0·61	2130	198·5
14	51·38	490·3	0·51	1890	176·5
17	51·465	491	0·433	1700	158·5
27	51·648	493	0·275	1318	124

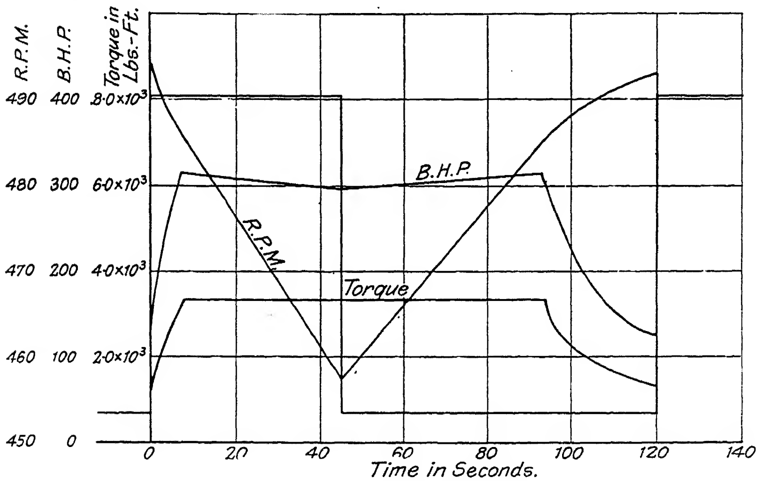


FIG. 40.—DIAGRAM SHOWING PERFORMANCE OF INDUCTION MOTOR OPERATING WITH FLYWHEEL AND AUTOMATIC SLIP REGULATOR

The corresponding torque

$$T = T_o - a(\omega_2 - f/_a)\, e^{\frac{a}{I}(t_3 - t)}$$

$$T = 650 - 2180(-1{\cdot}114)\, e^{\frac{a}{I}(t_2 - t)}$$

$$= 650 + 2430\, e^{0{\cdot}0485\,(t_2 - t)}$$

These results are plotted in Fig. 40.

We will now derive an expression for the R.M.S. horse-power or R.M.S. power.

Let t_1 = time in the pass, till regulator comes in

t_2 = time till end of pass from beginning of pass

t_3 = time in interval that the motor is on the regulator

t_4 = time till end of the cycle

The power in time t_1 at any instant

$$= \left\{\frac{c}{a} + \left(\omega_1 - \frac{c}{a}\right)e^{-\frac{a}{I}t}\right\}\left\{T_3 - a\left(\omega_1 - \frac{c}{a}\right)e^{-\frac{a}{I}t}\right\} \quad . \quad . \quad (380)$$

$$= T_3\frac{c}{a} + T_3\left(\omega_1 - \frac{c}{a}\right)e^{-\frac{a}{I}t} - c\left(\omega_1 - \frac{c}{a}\right)e^{-\frac{a}{I}t} - a\left(\omega_1 - \frac{c}{a}\right)^2 e^{-\frac{2a}{I}} \quad (381)$$

(Inst. power)^2

$$= W^2 = T_3^2\frac{c^2}{a^2} + 2T_3^2\frac{c}{a}\left(\omega_1 - \frac{c}{a}\right)e^{-\frac{a}{I}t} - 2T_3\frac{c^2}{a}\left(\omega_1 - \frac{c}{a}\right)e^{-\frac{a}{I}t}$$

$$- 2T_3c\left(\omega_1 - \frac{c}{a}\right)^2 e^{-\frac{2a}{I}} + T_3^2\left(\omega_1 - \frac{c}{a}\right)^2 e^{-\frac{2a}{I}t} - 2T_3a\left(\omega_1 - \frac{c}{a}\right)^3 e^{-\frac{3a}{I}t}$$

$$- T_3c\left(\omega_1 - \frac{c}{a}\right)^2 e^{-\frac{2a}{I}t} + 2ac\left(\omega_1 - \frac{c}{a}\right)^3 e^{-\frac{3a}{I}t}$$

$$+ c^2\left(\omega_1 - \frac{c}{a}\right)^2 e^{-\frac{2a}{I}t} + a^2\left(\omega_1 - \frac{c}{a}\right)^4 e^{-\frac{4a}{I}t} \quad . \quad . \quad . \quad (382)$$

$$\therefore \int_0^{t_1} W^2\,dt = T_3^2\frac{c^2}{a^2}t_1 - 2T_3^2\frac{c}{a^2}I\left(\omega_1 - \frac{c}{a}\right)e^{-\frac{a}{I}t_1} + \frac{2T_3^2cI}{a^2}\left(\omega_1 - \frac{c}{a}\right)$$

$$+ 2T_3\frac{c^2}{a^2}I\left(\omega_1 - \frac{c}{a}\right)e^{-\frac{a}{I}t_1} - 2T_3\frac{c^2}{a^2}I\left(\omega_1 - \frac{c}{a}\right)$$

$$+ \frac{T_3cI}{a}\left(\omega_1 - \frac{c}{a}\right)^2 e^{-\frac{2a}{I}t_1} - \frac{T_3cI}{a}\left(\omega_1 - \frac{c}{a}\right)^2$$

$$- \frac{T_3^2I}{2a}\left(\omega_1 - \frac{c}{a}\right)^2 e^{-\frac{2a}{I}t_1} + \frac{T_3^2I}{2a}\left(\omega_1 - \frac{c}{a}\right)^2$$

$$+ \frac{2}{3}T_3I\left(\omega_1 - \frac{c}{a}\right)^3 e^{-\frac{3a}{I}t_1} - \frac{2}{3}T_3I\left(\omega_1 - \frac{c}{a}\right)^3$$

$$+ \frac{T_3cI}{2a}\left(\omega_1 - \frac{c}{a}\right)^2 e^{-\frac{2at_1}{I}} - \frac{T_3cI}{2a}\left(\omega_1 - \frac{c}{a}\right)^2$$

$$-\frac{2}{3}Ic\left(\omega_1-\frac{c}{a}\right)^3 e^{-\frac{3a}{I}t_1}+\frac{2}{3}Ic\left(\omega_1-\frac{c}{a}\right)^3$$
$$-\frac{Ic^2}{2a}\left(\omega_1-\frac{c}{a}\right)^2 e^{-\frac{2a}{I}t_1}+\frac{Ic^2}{2a}\left(\omega_1-\frac{c}{a}\right)^2$$
$$-\frac{aI}{4}\left(\omega_1-\frac{c}{a}\right)^4 e^{-\frac{4a}{I}t_1}+\frac{aI}{4}\left(\omega_1-\frac{c}{a}\right)^4 \quad . \quad . \quad . \quad (383)$$

The instantaneous power in time

$$t_2-t_1 = T\omega_1 + T\left(\frac{T-T_3}{I}\right)(t-t_1)$$
$$= T\omega_1 + \frac{T^2-TT_3}{I}(t-t_1) \quad . \quad . \quad . \quad (384)$$

(Here t = time from commencement of pass.)

$$W_2^2 = T^2\omega_1^2 + 2T\frac{(T^2-TT_3)}{I}\omega_1(t-t_1)$$
$$+\left(\frac{T^2-TT_3}{I}\right)^2\{t^2-2tt_1+t_1^2\} \quad . \quad . \quad . \quad (385)$$

$$\therefore \int_{t_1}^{t_2} W_2^2 dt = T^2\omega_1^2(t_2-t_1)+\omega_1 2T\frac{(T^2-TT_3)}{I}\left[\tfrac{1}{2}t^2-t_1t\right]_{t_1}^{t_2}$$
$$+\left(\frac{T^2-TT_3}{I}\right)^2\left\{\frac{t^2}{3}-t^2t_1+t_1^2t\right\}_{t_1}^{t_2}$$
$$= T^2\omega_1^2(t_2-t_1)+2T\omega_1\left(\frac{T_2^2-TT_3}{I}\right)\left[\tfrac{1}{2}t_2^2+\tfrac{1}{2}t_1^2-t_1t_2\right]$$
$$+\left(\frac{T^2-TT_3}{I}\right)^2\left[\frac{t_2^3}{3}-\frac{t_1^3}{3}-t_2^2t_1+t_1^2t_2\right] \quad . \quad . \quad . \quad (386)$$

A corresponding expression for the interval t_3-t_2 as this applies, and a similar expression to the first for the last part of the cycle. For example, the speed in the third part of the cycle

$$\omega = \omega_2 + \left(\frac{T-T_o}{I}\right)(t-t_2) \quad . \quad . \quad . \quad (387)$$

and inst. power $W_3 = T\omega = T\left[\omega_2+\left(\frac{T-T_o}{I}\right)(t-t_2)\right]$. (388)

$$\therefore \int_{t_2}^{t_3} W_3^2 dt = T^2\omega_2^2(t_3-t_2)+2T^2\left(\frac{T-T_o}{I}\right)\omega_2\left[\tfrac{1}{2}t_3^2+\tfrac{1}{2}t_2^2-t_3t_2\right]$$
$$+\left(\frac{T^2-TT_o}{I}\right)^2\left[\frac{t_3^3}{3}-\frac{t_2^3}{3}-t_3^2t_2+t_2^2t_3\right] \quad . \quad . \quad . \quad (389)$$

Again, for the last period,

$$\omega = f/a + (\omega_2 - f/a)\, e^{-\frac{a}{I}t} \quad . \quad . \quad . \quad (390)$$

and the torque $= T_o - a(\omega_2 - f/a)\, e^{-\frac{a}{I}t} \quad . \quad . \quad . \quad (391)$

$$W_4 = T\omega$$

$$\int_{t_3}^{t_4} W_4^2 dt = \int_{t_3}^{t_4} \left[\left\{ T_o - a(\omega_2 - f/a)\, e^{-\frac{a}{I}t} \right\} \left\{ f/a + (\omega_2 - f/a)\, e^{-\frac{a}{I}t} \right\} \right]^2 dt \quad . \quad . \quad . \quad (392)$$

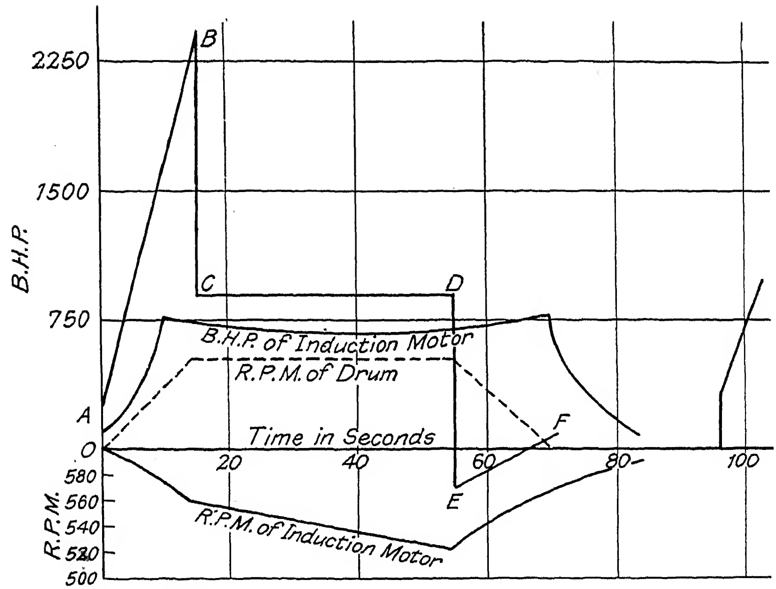


FIG. 41.—WINDING DIAGRAM FOR ILGNER SET

It will be noticed that this is the same product in form as for the first period, and the result will be obtained, if in that expression T_o is interchanged for T_3 and f/a for c/a; ω_2 for ω_1.

The R.M.S. power

$$= \frac{\sqrt{\left\{ \int_o^{t} W_1^2 dt + \int_{t_1}^{t_2} W_2^2 dt + \int_{t_2}^{t_3} W_3^2 dt + \int_{t_3}^{t_4} W_4^2 dt \right\}}}{t_4} \quad . \quad (393)$$

It is simpler in any given case to square the ordinates at each

point and find the area with the planimeter. Then find the mean ordinate and take the square root.

The Ward-Leonard system. We will consider next the case of the electric winder operating on the Ward-Leonard system with a flywheel provided on the induction motor shaft, and an automatic slip regulator in the rotor circuits.

Fig. 41 represents the b.h.p.-time diagram for an electric hoist. The speed-time curve is also shown.

It is hardly worth while to calculate the speed between A and the time when the regulator comes in, the time is so short; but to make the calculations complete we will attempt to calculate it.

Let W = output at B

t_1 = period of acceleration in seconds

The power equation is

$$T\omega - I\frac{d\omega}{dt}\omega = W\frac{t}{t_1} \quad \quad (394)$$

$$\frac{T}{T_F} = \frac{\omega_o - \omega}{\omega_o - \omega_F} \qquad \therefore T = \frac{T_F\omega_o}{\omega_o - \omega_F} - \frac{T_F\omega}{\omega_o - \omega_F} \quad \quad (395)$$

$$\therefore \frac{T_F\omega\,\omega_o}{\omega_o - \omega_F} - \frac{T_F\omega^2}{\omega_o - \omega_F} - I\frac{d\omega}{dt}\omega = W\frac{t}{t_1} \quad \quad (396)$$

$$\therefore a_1\omega - a_2\omega^2 - \omega\frac{d\omega}{dt} = c_1t; \text{ where } c_1 = \frac{W}{It_1} \quad \quad (397)$$

$$a_1 = \frac{T_F\omega_o}{(\omega_o - \omega_F)\,I}; \quad a_2 = \frac{T_F}{(\omega_o - \omega_F)\,I} \quad \quad (398)$$

The solution of this may be obtained approximately as follows—

Let $\omega = a$ when $t = o$, and let $\omega = ue^{-a_2t}$, so that $u = a$ when $t = o$

$$\frac{d\omega}{dt} = \frac{du}{dt}e^{-a_2t} - a_2ue^{-a_2t} \quad \quad (399)$$

$$\omega\frac{d\omega}{dt} = u\frac{du}{dt}e^{-2a_2t} - a_2u^2e^{-2a_2t} \quad \quad (400)$$

$$\therefore a_1ue^{-a_2t} - a_2u^2e^{-2a_2t} - u\frac{du}{dt}e^{-2a_2t} + a_2u^2e^{-2a_2t} = c_1t \quad \quad (401)$$

$$\therefore a_1ue^{-a_2t} - u\frac{du}{dt}e^{-2a_2t} = c_1t \quad \quad (402)$$

$$\therefore \; a_1 u e^{-a_2 t} - c_1 t = u \frac{du}{dt} e^{-2a_2 t} \quad . \quad . \quad . \quad (403)$$

$$\text{and } \frac{du}{dt} = a_1 e^{a_2 t} - \frac{c_1}{u} e^{2a_2 t} \times t \quad . \quad . \quad . \quad (404)$$

Expand u in a series of Maclaurin

$$u = f(t) = f(o) + t f'(o) + \frac{t^2}{\underline{|2}} f''(o) + \ldots \quad . \quad . \quad (405)$$

$$f(o) = a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (406)$$

$$f'(t) = \frac{du}{dt} = a_1 e^{a_2 t} - \frac{c_1}{u} e^{2a_2 t} \times t \quad \therefore \; f'(o) = a_1 \quad . \quad (407)$$

$$f''(t) = a_1 a_2 e^{a_2 t} - \frac{c_1}{u} e^{2a_2 t} - \frac{2a_2 c_1 e^{2a_2 t}}{u} \times t$$

$$+ \frac{c_1 t e^{2a_2 t}}{u^2} \left(a_1 e^{a_2 t} - \frac{c_1}{u} e^{2a_2 t} \right) \quad . \quad (408)$$

$$\therefore \; f''(o) = a_1 a_2 - \frac{c_1}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad (409)$$

$$\therefore \qquad \omega = u e^{-a_2 t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (410)$$

$$= e^{-a_2 t} \left\{ a + a_1 t - \tfrac{1}{2} \left(\frac{c_1}{a} - a_1 a^2 \right) t^2 + \ldots \right\} \quad . \quad (411)$$

This expression is not of much use, but it represents the manner in which the speed of the induction motor falls during the accelerating stage of the winding cycle of the direct-current motor.

We must next find the speed and output during the remainder of the cycle for the induction motor.

The power equation is

$$T\omega - I \frac{d\omega \, \omega}{dt} = Ct \quad . \quad . \quad . \quad . \quad (412)$$

C = rate of increase of power with time on the accelerating part of the curve.

In this part of the cycle, the torque of the motor is constant.

$$\therefore \quad \frac{\omega \, d\omega}{dt} - \frac{T\omega}{I} + \frac{C}{I} t = o \quad . \quad . \quad . \quad (413)$$

To solve this, let $\omega = vt$,

$$\text{then} \quad \frac{d\omega}{dt} = t \frac{dv}{dt} + v \quad . \quad . \quad . \quad (414)$$

The equation may then be written

$$vt^2 \frac{dv}{dt} + v^2 t - \frac{T}{I} vt + \frac{C}{I} t = o \qquad . \qquad . \qquad . \quad (415)$$

$$vt^2 \frac{dv}{dt} + t\left(v^2 - \frac{T}{I} v + \frac{C}{I}\right) = o \qquad . \qquad . \quad (416)$$

$$vt \frac{dv}{dt} + \left(v^2 - \frac{T}{I} v + \frac{C}{I}\right) = o \qquad . \qquad . \qquad . \quad (417)$$

$$\therefore \quad -\frac{vdv}{v^2 - \frac{T}{I} v + \frac{C}{I}} = \frac{dt}{t}$$

$$\text{Let} \quad \frac{T}{I} = b \text{ and } \frac{C}{I} = d \qquad . \qquad . \qquad . \qquad . \qquad . \quad (418)$$

$$\int \frac{dt}{t} = -\int \frac{vdv}{v^2 - \frac{T}{I} v + \frac{C}{I}} = -\int \frac{vdv}{v^2 - bv + d}$$

$$\int \frac{dt}{t} = -\frac{1}{2}\int \frac{(2v - b)\, dv}{v^2 - bv + d} - \frac{b}{2}\int \frac{dv}{v^2 - bv + d} \qquad . \quad (419)$$

$$\therefore \log_e t = -\tfrac{1}{2} \log (v^2 - bv + d) - \frac{b}{2}\int \frac{dv}{v^2 - bv + d} + K \quad (420)$$

$\int \frac{dv}{v^2 - bv + d}$ depends on whether $b^2 \gtreqless 4d$, i.e. $\frac{T^2}{I^2} \gtreqless \frac{4C}{I}$

obviously $b^2 > 4d$

$$\text{for} \quad C = \frac{T\omega}{t} - \frac{I}{t}\frac{d\omega}{dt}\,\omega \qquad . \qquad . \qquad . \qquad . \qquad . \quad (421)$$

$$\frac{C}{I} = \frac{T}{I}\frac{\omega}{t} - \frac{\omega}{t}\frac{d\omega}{dt} \qquad . \qquad . \qquad . \qquad . \qquad . \quad (422)$$

$$\frac{T^2}{I^2} \gtreqless \frac{4T}{I}\frac{\omega}{t} - \frac{4\omega\, d\omega}{t\, dt} \qquad . \qquad . \qquad . \qquad . \qquad . \quad (423)$$

but the torque given by the flywheel is usually greater than the torque of the motor, i.e. $I \frac{d\omega}{dt} > T$

$$\text{i.e.} \quad \frac{d\omega}{dt} > \frac{T}{I} = m \frac{T}{I} \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \quad (424)$$

$$\therefore \quad \frac{T^2}{I^2} + 4m\frac{T}{I}\frac{\omega}{t} \gtreqless \frac{4T\omega}{It} \quad \therefore \frac{T^2}{I^2} > \frac{4C}{I} \quad . \quad . \quad (425)$$

$$\int \frac{dv}{v_2 - bv + d} = \int \frac{dv}{(v - \frac{1}{2}b)^2 - (\frac{1}{4}b^2 - d)} \quad . \quad . \quad (426)$$

$$= -\frac{1}{\sqrt{b^2/_4 - d}} \coth^{-1} \frac{v - \frac{1}{2}b}{\sqrt{\frac{1}{4}b^2 - d}} \quad . \quad (427)$$

$$\therefore \frac{b}{2}\int \frac{dv}{v^2 - bv + d} = -\frac{b}{2\sqrt{\frac{1}{4}b^2 - d}} \coth^{-1} \frac{v - \frac{1}{2}b}{\sqrt{\frac{1}{4}b^2 - d}} \quad . \quad . \quad (428)$$

$$\therefore \log_e t = -\tfrac{1}{2}\log(v^2 - bv + d) + \frac{b}{2\sqrt{\frac{1}{4}b^2 - d}} \coth^{-1} \frac{v - \frac{1}{2}b}{\sqrt{\frac{1}{4}b^2 - d}} + K$$

where K = constant of integration. (429)

When $t = t_1$, the time when the regulator comes in, $\omega = \omega_1$

$$\therefore \log_e t_1$$

$$= -\tfrac{1}{2}\log\left(\frac{\omega_1^2}{t_1^2} - \frac{b\omega_1}{t_1} + d\right) + \frac{b}{2\sqrt{\frac{1}{4}b^2 - d}} \coth^{-1} \frac{\frac{\omega_1}{t_1} - \frac{b}{2}}{\sqrt{\frac{1}{4}b^2 - d}} + K$$

(430)

$$\therefore K$$

$$= \log_e t_1 + \tfrac{1}{2}\log\left(\frac{\omega_1^2}{t_1^2} - \frac{b\omega_1}{t_1} + d\right) - \frac{b}{2\sqrt{\frac{1}{4}b^2 - d}} \coth^{-1} \frac{\frac{\omega_1}{t_1} - \frac{b}{2}}{\sqrt{\frac{1}{4}b^2 - d}}$$

(431)

$$\therefore \log_e \frac{t}{t_1} = \tfrac{1}{2}\log \frac{\left(\frac{\omega_1^2}{t_1^2} - \frac{b\omega_1}{t_1} + d\right)}{\left(\frac{\omega^2}{t^2} - \frac{b\omega}{t} + d\right)}$$

$$+ \frac{b}{2\sqrt{b^2/_4 - d}}\left[\coth^{-1} \frac{\frac{\omega}{t} - \frac{b}{2}}{\sqrt{\frac{1}{4}b^2 - d}} - \coth^{-1} \frac{\frac{\omega_1}{t_1} - \frac{b}{2}}{\sqrt{\frac{1}{4}b^2 - d}}\right]$$

(432)

Neglecting the second term, which does not usually affect the result sensibly,

$$\text{we have } \frac{t}{t_1} = \left(\frac{\frac{\omega_1^2}{t_1^2} - \frac{b\omega_1}{t_1} + d}{\frac{\omega^2}{t^2} - \frac{b\omega}{t} + d}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad (433)$$

$$\therefore \frac{t^2}{t_1^2} = \frac{\dfrac{\omega_1^2}{t_1^2} - \dfrac{b\omega_1}{t_1} + d}{\dfrac{\omega^2}{t^2} - \dfrac{b\omega}{t} + d} \quad . \quad . \quad . \quad . \quad (434)$$

$$\therefore \omega^2 - b\omega t + t^2 d - \omega_1^2 + b\omega_1 t_1 + t_1^2 d = o$$

$$\therefore \omega = bt \pm \frac{\sqrt{b^2t^2 - 4\,(t^2d^2 - \omega_1^2 + b\omega_1 t_1 + t_1^2 d)}}{2}$$

$$\therefore \omega = \frac{T}{I}t \pm t\,\frac{\sqrt{\dfrac{T^2}{I^2} - 4\left\{\left(\dfrac{C}{I} - \dfrac{\omega_1^2}{t^2} + \dfrac{T}{I}\dfrac{\omega_1}{t^2}t_1 + \dfrac{t_1^2}{t^2}\dfrac{C}{I}\right)\right\}}}{2} \quad (435)$$

This equation holds up to the peak. The output is constant from C to D, and our equation becomes

$$T\omega - I\frac{d\omega}{dt}\omega = E \quad . \quad . \quad . \quad (436)$$

where E = constant output.

This equation is exactly similar to Equation (315), and is solved in the same way.

(*b*) **Speed control by pole-changing.** The speed may be changed by changing the number of poles. This may be effected by a single winding by a regrouping of the coils, or two or more independent windings may be used on the stator. The change of speed by changing the number of poles involves the use of fractional pitch windings. A primary turn of full pitch for a given number of poles is of fractional pitch for a smaller number of poles, and more than a full pitch for a larger number of poles. In such cases a squirrel-cage rotor is generally used, since it adapts itself to any number of poles. When the speed ratio is 2 : 1, this type of motor can be applied advantageously, and a great many have been built and have been in successful operation for many years. A motor with this speed ratio is practicable for either a wound rotor or squirrel cage, since it requires only six slip-rings in the wound rotor type. There are a number of different ways of connecting the windings of such motors having a speed ratio of 2 to 1, the selection of the connection in any case depending on the relative maximum outputs required at the two speeds. The connection which is most frequently used, and in which the material is worked to the best advantage, has a half-speed rating of from 60 to 70 per cent of the full-speed rating.

The following table gives several different connections which

may be used, with the approximate relative outputs for the different connections—

Speed.		Connection.	Approximate Maximum Output.
(1)	100	2-circuit delta	100
	50	γ-delta	11
(2)	100	2-circuit γ	100
	50	1-circuit γ	22
(3)	100	2-circuit γ	100
	50	1-circuit Δ	66
(4)	100	1-circuit Δ	100
	50	2-circuit γ	117
(5)	100	1-circuit γ	100
	50	2-circuit γ	350
(6)	100	γ-delta	100
	50	2-circuit delta	700

These values are approximate only, and will vary slightly with the ratio of reactance to resistance and also with the ratio of coil-end to slot part of the reactance. The usual method of changing from the larger number of poles to half the number is to reverse

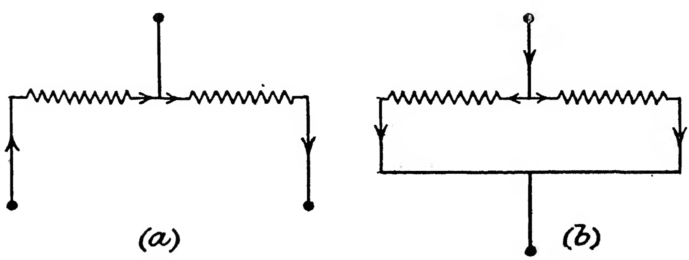


FIG. 42

the current in half the winding. This may be effected in the following ways. (Fig. 42.)

If we bring the current in at one end of a series of coils, as in Fig. 42A, we get the full number of poles; whereas by adopting the method shown in Fig. 42B, the current is reversed in half the winding for the smaller number.

If we wish to keep the same number of coils in series in each case, the winding may be arranged as in Fig. 43.

The two methods most commonly used for changing the number of poles in the ratio 2 : 1 are shown in Figs. 44A and 44B, and Figs. 46A and 46B.

In Figs. 44A and 44B, the winding is connected in star for

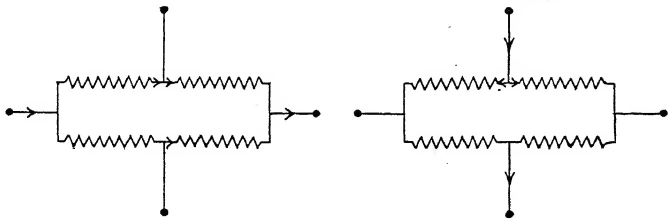
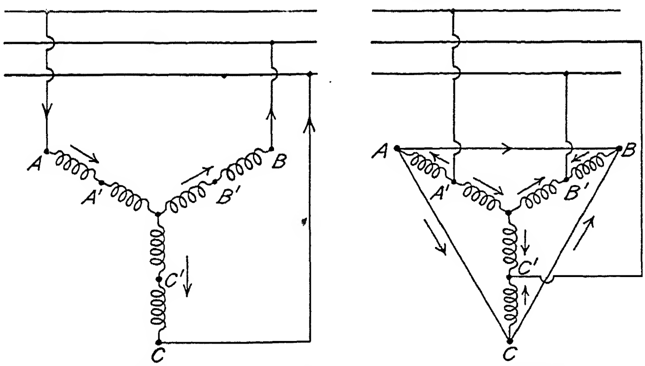


FIG. 43

the larger number of poles, and in two parallel star for the smaller number of poles. In the case of a single layer hemi-tropic winding for 8 poles, each phase will consist of four groups of coils, two groups straight and two groups bent, straight and bent coils alternating. Alternate coils are connected in series, i.e. two



(a) 8 poles FIG. 44 (b) 4 poles

straight coils are connected in series and two bent coils are connected in series. For the 8-pole connection, the circuit is completed in series through the two bent coils of the same phase, and a tapping is brought out at the junction of the straight and bent coils. A similar method is adopted in regard to the other phases. This method of connection is necessary in order that the direction of current in alternate groups of coils may be reversed when the two halves of each phase are connected in parallel. In order that

the direction of rotation shall remain unchanged when the poles are changed, two of the phases must be reversed in relation to the line wires with this connection. The larger number of poles is obtained by connecting the line wires to the ends of the winding; while the smaller number is obtained by connecting the lines to the tappings and short-circuiting the ends of the winding, as shown in Figs. 44A and 44B.

The development of a three-phase winding for pole-changing, according to Figs. 44A and 44B, is given in Fig. 45.

It will be seen from Fig. 45 that, with the smaller number of poles, certain conductors at the centre of each pole carry currents in the wrong direction. A portion of the winding is therefore ineffective. For this class of winding, one-third of the actual turns per pole are ineffective.

If T = number of turns per phase for the larger number of poles, then the number of turns per pole for the larger number of poles $= \dfrac{T}{p}$ (p = number of poles)

For the smaller number of poles, i.e. $\dfrac{p}{2}$, the effective turns per phase will be $\left[\dfrac{1}{2}\left(1\tfrac{2}{3}\dfrac{T}{p}\times\dfrac{p}{2}\right)\right] = \dfrac{5}{12}T$

If E = applied voltage per phase
and ϕ = flux per pole for larger number of poles
ϕ' = flux per pole for smaller number of poles
f = frequency
K = breadth factor

then (1) for the larger number of poles

$$E = \sqrt{2}\,\pi \times K_1 \times \phi \times T \times f \times 10^{-8} \quad . \quad . \quad . \quad (437)$$

(2) for smaller number of poles

$$E = \sqrt{2}\,\pi \times K_2 \times \phi' \times \frac{5}{12}T \times f \times 10^{-8} \times \sin\frac{\psi}{2} \quad . \quad (438)$$

where ψ = span of coil in electrical degrees for the smaller number of poles $\psi = 90$, corresponding to half pitch,

$$\therefore \sin\frac{\psi}{2} = \sin 45^\circ = 0{\cdot}707$$

$$\therefore \frac{\phi'}{\phi} = \frac{12}{5} \times 0{\cdot}707 = 3{\cdot}4 \quad . \quad . \quad . \quad . \quad . \quad (439)$$

since K_1 is practically $= K_2$

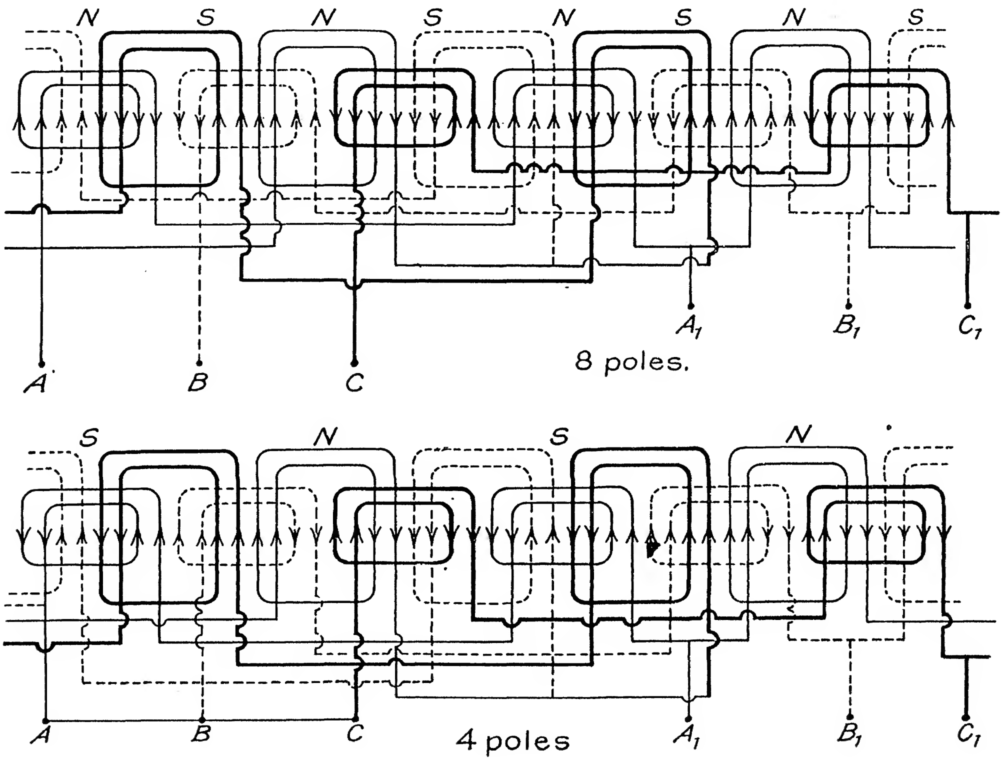


FIG. 45

Since the area at the pole face is doubled, the flux density in the gap with this connection

$= 1{\cdot}7$ times that for the larger number of poles.

If one neglects saturation, the magnetizing current is proportional to the gap density, and inversely proportional to the turns per pole per phase,

$$\therefore \frac{i_m}{i_m'} = \frac{B_m}{T/p} \div \frac{B_m'}{0{\cdot}707} \times \frac{5}{24}\frac{T}{p} \quad . \quad . \quad . \quad (440)$$

$$\frac{B_m'}{B_m} = 1{\cdot}7$$

$$\therefore \frac{i_m}{i_m'} = \frac{1}{1{\cdot}7} \times 2 \times 0{\cdot}707 \times \frac{5}{12} = 0{\cdot}347 \quad . \quad . \quad (441)$$

$$\therefore i_m' = 2{\cdot}88\, i_m \quad . \quad . \quad . \quad . \quad . \quad . \quad (442)$$

It is clear that this method of connection would require careful attention to the sectional areas of the teeth and core to avoid saturation on the smaller number of poles. It is not a good connection, and the second one is preferable and generally used. In this second method, shown in Figs. 46A and 46B, the winding is connected in delta for the larger number of poles, and in two parallel circuits star per phase for the smaller number of poles.

Let E = line voltage.

Then for the larger number of poles,

$$E = \sqrt{2}\,\pi\,\phi \times T \times f \times 10^{-8} \quad . \quad . \quad . \quad . \quad (443)$$

for the smaller number of poles.

$$\frac{E}{\sqrt{3}} = \sqrt{2}\,\pi\,\phi' \times \frac{5}{12} T \times 0{\cdot}707 \times f \times 10^{-8} \quad . \quad (444)$$

$$\phi = \frac{E \times 10^8}{\sqrt{2}\,\pi \times T \times f} \quad . \quad . \quad . \quad . \quad . \quad (445)$$

$$\phi' = \frac{E}{\sqrt{3}} \times \frac{12 \times 10^8}{5 \times 0{\cdot}707 \times T \times f \times \sqrt{2}\,\pi} \quad . \quad (446)$$

$$\therefore \frac{\phi'}{\phi} = \frac{12}{5 \times 0{\cdot}707 \times \sqrt{3}} = 1{\cdot}96 \quad . \quad . \quad . \quad (447)$$

$$\therefore \phi' = 1{\cdot}96\,\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (448)$$

and since the pole area in the gap is doubled with the smaller number of poles, the flux density $B_m' = 0{\cdot}98\,\phi$

$$\text{Also } \frac{i_m}{i_m'} = \frac{2 \times 0{\cdot}707 \times \frac{5}{12}}{0{\cdot}98} = 0{\cdot}6 \quad . \quad . \quad . \quad . \quad . \quad (449)$$

$$i'_m = 1{\cdot}66\, i_m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (450)$$

The second method of connection is preferable, since the performance is better, and the motor can be made smaller in external diameter and lighter than one connected, as in Figs. 44A and 44B.

It may also be mentioned that with the machine delta-connected

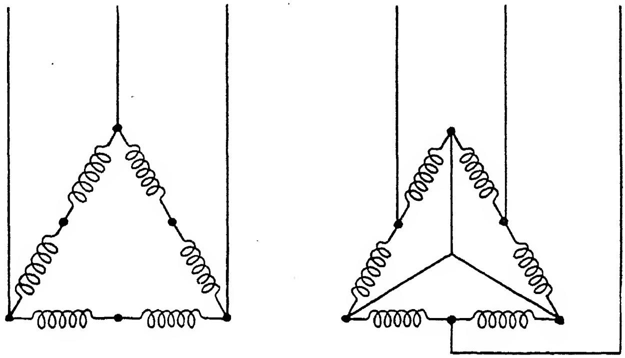


FIG. 46

for the larger number of poles and star-connected for the smaller number of poles, no reversal of line wires and phases is necessary.

Messrs. Brown, Boveri have adopted this method for the motors of the Simplon Tunnel locomotives; and the Oerlikon Co. have made several such pole-changing motors for driving ram pumps, etc. In one case the Oerlikon Co. supplied the Dolcoath Mines, Ltd., with four complete four-speed variable speed motors for driving ram pumps at speeds varying from 38 r.p.m. to 118 r.p.m. The machines were built to the following specification. (See *The Electrician*, 23rd August, 1918.)—

Speed step No.	1	2	3	4
Output	35	55	72·5	110 b.h.p.
Number of poles	72	48	36	24 b.h.p.
R.P.M.	38	60	82	118 r.p.m.
Voltage.	2700/3000			
Frequency	25 cycles			
Flywheel effect	8500 Kgm.^{2}			

The internal diameter of the stator core was 180 cm.

Gross length of core = 28 cm.

External diameter = 225 cm.

The motors were supplied at a lower voltage appropriate to the construction of the windings.

The motors had two distinct windings, which were embedded in 432 slots.

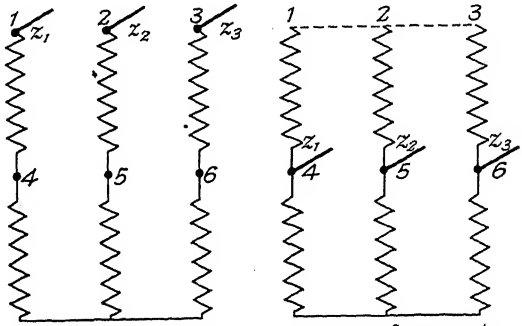


FIG. 47

The system for 24 and 48 poles consists of 48 coil-groups per phase, the connections for which are shown in Fig. 47.

Of the six connections of this winding system, those marked

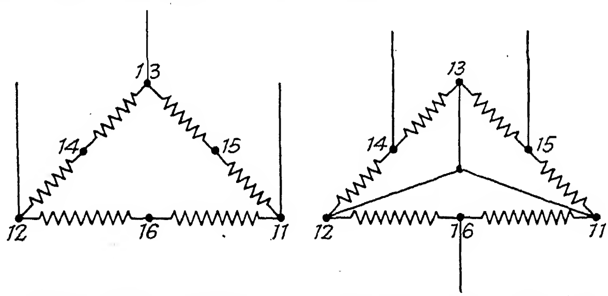


FIG. 48

with 4, 5, and 6 lead the corresponding sets of 24 coil-groups each. For 48 poles, the supply leads are connected by means of a pole-changing switch to 1, 2, and 3; while the tappings 4, 5, and 6 remain free. For 24 poles, the supply leads are connected to 4, 5, and 6, and the terminals 1, 2, and 3 are short-circuited. The winding system for 36 and 72 poles consists of 72 coils and is embedded in the same slots, but next the air-gap, since on the larger number of poles it is naturally desired to reduce the leakage reactance.

Fig. 48 shows the connections for 72 and 36 poles.

A section of the motor is shown in Fig. 49. The machine has bar windings.

The leakage factors for the various numbers of poles and the maximum power factors are shown below—

24 poles . . .	$\tau = 0{\cdot}035$	$\cos\phi$ max. = 93%
36 ,, . . .	$\tau = 0{\cdot}088$	,, = 86%
48 ,, . . .	$\tau = 0{\cdot}11$	,, = 82%
72 ,, . . .	$\tau = 0{\cdot}24$	,, = 68%

The oil-immersed pole-changing switch has three cast-iron cylinders interconnected by gearing. Each phase has its own contact drum.

The maximum torques obtainable on the various steps are—

On first speed step . . .	50%	above normal
,, second ,, . . .	100%	,,
,, third ,, . . .	300%	,,
,, fourth ,, . . .	300%	,,

A motor designed for 110 b.h.p. at 25 cycles for 24 poles, 118 r.p.m. would have the following dimensions—

Internal diameter of stator . . .	140 cm.
Gross length of core	22 cm.
External diameter of stator . . .	165 cm.

Its power factor at full load would be about 83%

It will be seen therefore that the additional winding and additional requirements increase the size of the motor to a great extent.

For larger speed ranges, additional windings may be used, but the control arrangements for changing the coil groups become more complicated.

The problem of multi-speed motor design is to arrange the windings that the change of connections in changing from one number of poles to another involves the least number of switching arrangements.

(*c*) **Speed control by cascade connection.** With this method of speed control, two motors are required at least, or the functions of the two machines can be performed in a single machine having the same magnetic structure, but the windings of the two machines merged into one, as in the Hunt motor.

Let us suppose we have two induction motors mechanically coupled together, and let the rotor winding of the first be connected to the stator of the second, and the rotor winding of the second short-circuited.

Such a combination enables us to get three speeds, viz., the speed of one alone, the speed of the second, and the cascade speed.

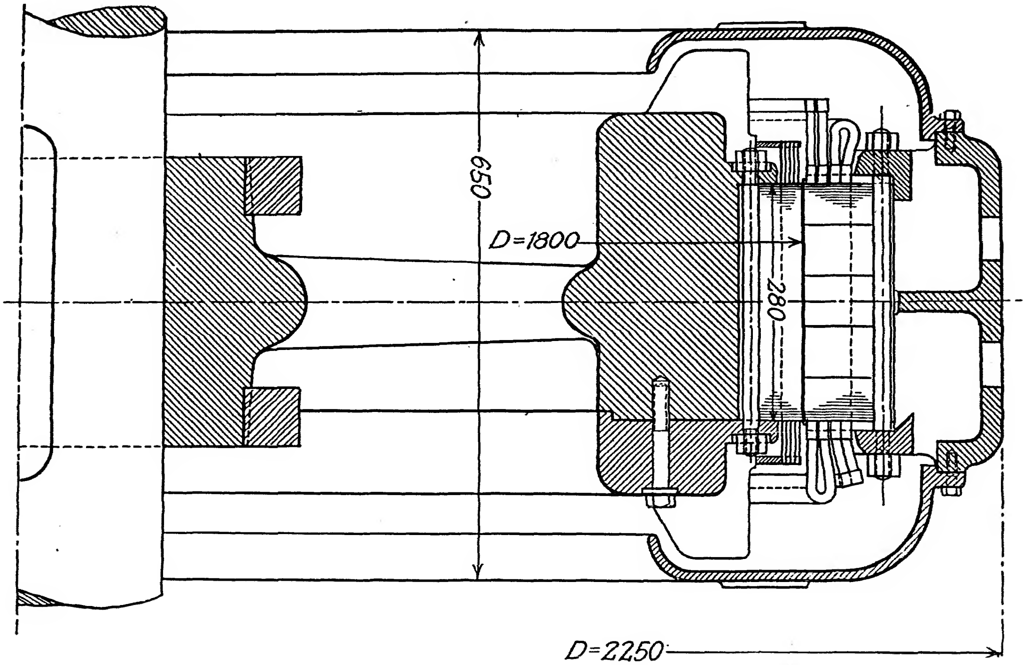


FIG. 49

It has the disadvantage that, at the two highest speeds, one machine is idle. With certain pole ratios, however, it is possible to wind the same structure to give at the same time two different numbers of poles, and in this case the material is used more efficiently.

Let p_1 = number of pairs of poles of the first machine
p_2 = number of pairs of poles of the second machine
f = supply frequency

The full synchronous speed of the first machine is $= \dfrac{f}{p_1}$ in revs. per sec.

The full synchronous speed of the second machine $= \dfrac{f}{p_2}$

Let us suppose the first machine is running with a slip s.

The speed of the first machine $= (1 - s)\dfrac{f}{p_1}$

The frequency of its rotor currents $= sf$, and this is the frequency of the currents supplied to the stator of the second machine.

The synchronous speed of the second motor at this frequency $= \dfrac{sf}{p_2}$

The speed of the first motor is the same as that of the second motor, and the slip of speed of the second motor below its synchronous speed

$$= \frac{sf}{p_2} - (1 - s)\frac{f}{p_1} = \left(\frac{s}{p_2} - \frac{1 - s}{p_1}\right) f \quad . \quad . \quad (451)$$

and the slip of frequency

$$s' = p_2\left(\frac{s}{p_2} - \frac{1 - s}{p_1}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (452)$$

$$= s - a\,(1 - s) = s\,(1 + a) - a$$

where $a = \dfrac{p_2}{p_1}$

The slip of the second motor becomes zero,

$$\text{i.e. } s' = o \text{ for } s_o = \frac{a}{1 + a} \quad . \quad . \quad . \quad . \quad . \quad . \quad (453)$$

The speed in this case is

$$s_o' = (1 - s_o)\frac{f}{p_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (454)$$

$$= \frac{f}{p_1 (1 + a)} = \frac{f}{p_1 + p_2} \quad . \quad . \quad . \quad . \quad (455)$$

Thus if $p_2 = p_1$, $s_o' = \dfrac{f}{2p_1}$ and the set runs at half the speed of either motor alone, or corresponding to a motor having $2p_1$ pairs of poles. If $a = 2$, $p_1 = 4$, $p_2 = 8$, $s_o = \dfrac{2}{3}$; $s_o' = \dfrac{f}{3p_1}$, i.e. two machines in cascade, in which the two fields revolve in the same direction, revolve at a speed corresponding to the sum of poles in each machine.

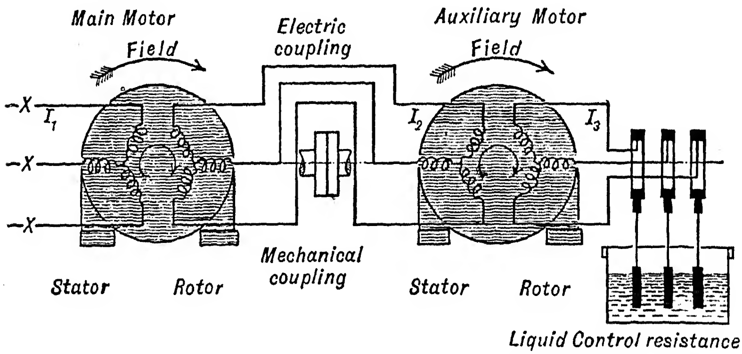


FIG. 50.—THE CASCADE SYSTEM OF INDUCTION MOTOR CONTROL

Fig. 50 shows the diagram of connections for such a set in cascade.

If the stator of the second motor is connected so that its magnetic field rotates in a direction opposed to that of the first motor stator field, the speed of the set corresponds to that of a motor having a number of poles equal to the difference of the numbers of poles, i.e. $\dfrac{f}{p_1 - p_2}$

The method of building up the circle diagram for two motors in cascade is shown in Fig. 51.

All the resistances and reactances are referred to the auxiliary motor rotor circuit in the usual manner.

R_1 = main motor resistance of stator per phase in ohms referred

R_2 = main motor rotor resistance per phase referred

R_3 = auxiliary motor stator resistance per phase referred

R_4 = auxiliary motor rotor resistance per phase

R_5 = external resistance added per phase in auxiliary motor rotor circuit to bring the set to rest

L_1 = coefficient of self-induction of main motor stator per phase referred to auxiliary rotor circuit

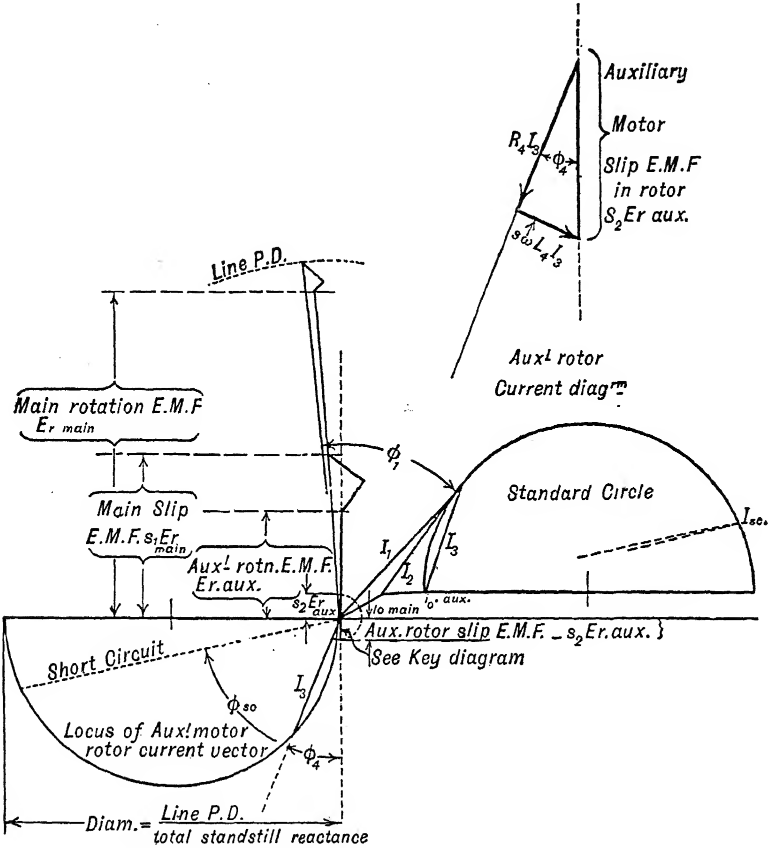


FIG. 51.—STANDARD CIRCLE DIAGRAM FOR TWO MOTORS IN CASCADE

L_2 = coefficient of self-induction of main motor rotor per phase referred

L_3 = coefficient of self-induction of auxiliary motor stator per phase referred

L_4 = coefficient of self-induction of auxiliary motor rotor per phase

Fig. 51 assumes the same number of turns in the stator and rotor of the two motors.

The auxiliary motor stator E.M.F. $= E_r$ (aux.) $= s_1 \times$ line P.D. (neglecting impedance drop)

Auxiliary motor rotor slip E.M.F. $= s_2 E_r$ (aux.)

$$= s_1 \times s_2 \times \text{line P.D.}$$

$$= s \times \text{line P.D.}$$

where $s = s_1 \times s_2$

$s_1 =$ slip of main motor

S_2 $\cancel{R_2} =$ slip of auxiliary motor

$$\text{The auxiliary motor rotor current} = \frac{\text{aux. motor slip E.M.F.}}{\text{aux. motor rotor impedance}}$$

$$= \frac{\text{line P.D.}}{\sqrt{R^2 + (L\omega)^2}} \quad . \quad . \quad (456)$$

where $R = R_1 + R_2 + R_3 + R_4 + R_5$

$L = L_1 + L_2 + L_3 + L_4$

The angle of lag ϕ_4 of the auxiliary rotor current

$$\phi_4 = \sin^{-1} \frac{L\omega}{\sqrt{R^2 + (L\omega)^2}}$$

$$\text{The ideal short-circuit current} = \frac{\text{line P.D.}}{L\omega} \quad . \quad . \quad . \quad (457)$$

$\therefore$ Aux. rotor current $I_3 =$ ideal short-circuit current $\times \sin \phi_4$ (458)

It is thus seen that the locus of I_3 is represented by a chord in the semicircle whose diameter $= \dfrac{\text{line P.D.}}{L\omega}$

The building up of the diagram is self-explanatory.

The calculation of the performance of a cascade set is probably performed most simply by the aid of complex quantities. One starts with the current in the rotor of the auxiliary motor and works back to the primary of the first machine.

Now, if $x_2 =$ reactance of the auxiliary rotor at full supply frequency and reduced to the same number of turns as the primary of the first machine

$e =$ back E.M.F. generated in the second motor by the flux linking both stator and rotor at full frequency

$I_2 =$ rotor current of auxiliary motor

$$\text{then } I_2 = \frac{s_2 e}{r_2 + j s_2 x_2} \quad . \quad . \quad . \quad . \quad . \quad (459)$$

where s_2 = slip of second motor

Now if s = slip of first machine,

the synchronous speed at slip s of auxiliary field $= \dfrac{sf}{p_2}$. (460)

$$\text{speed of set} = (1 - s)\frac{f}{p_1} \quad . \quad . \quad . \quad . \quad (461)$$

$\therefore$ relative speed of rotation with respect to the field in the auxiliary machine $= \dfrac{fs}{p_2} - (1 - s)\dfrac{f}{p_1}$ . . . (462)

and frequency of rotor currents of the second machine

$$= \left\{\frac{fs}{p_2} - (1 - s)\frac{f}{p_1}\right\} \times p_2$$

$$= f\left\{s - (1 - s)\frac{p_2}{p_1}\right\}$$

$$= f\{s - (1 - s)a\} \text{ where } a = \frac{p_2}{p_1}$$

$$= s_2 f \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (463)$$

$$\therefore \; s_2 = s(1 + a) - a \quad . \quad . \quad . \quad . \quad . \quad (464)$$

$$I_2 = \frac{s_2 e}{r_2 + j s_2 x_2} \quad . \quad . \quad . \quad . \quad . \quad (465)$$

where r_2 = rotor resistance per phase of auxiliary motor referred to primary of first machine

Multiplying numerator and denominator by

$$r_2 - j s_2 x_2$$

$$\text{we have } I_2 = \frac{s_2 r_2 e - j s_2^2 x_2 e}{r_2^2 + s_2^2 x_2^2} \quad . \quad . \quad . \quad . \quad (466)$$

$$= \frac{s_2 r_2 e}{r_2^2 + s_2^2 x_2^2} - \frac{j s_2^2 x_2 e}{r_2^2 + s_2^2 x_2^2} \quad . \quad . \quad (467)$$

$$= e(\alpha - j\beta) \quad . \quad . \quad . \quad . \quad . \quad (468)$$

$$\text{where } \alpha = \frac{s_2 r_2}{r_2^2 + s_2^2 x_2^2} \text{ and } \beta = \frac{x_2 s_2^2}{r_2^2 + s_2^2 x_2^2} \quad . \quad (469)$$

Now, if $g_2 - jb_2$ = exciting admittance of the second motor, the exciting current of this motor

$$= e(g_2 - jb_2) = I_{\mu_2} \quad . \quad . \quad . \quad (470)$$

$\therefore$ primary current of the second machine

$$= \dot{I}\mu_2 + \dot{I}_2 = \dot{I}_1 \quad . \quad . \quad . \quad (471)$$

$$= e(g_2 - jb_2) + e(\alpha - j\beta) \quad . \quad . \quad . \quad (472)$$

$$= e(g_2 + \alpha) - e(jb_2 + j\beta) \quad . \quad . \quad . \quad (473)$$

$$= e(x - jy) \quad . \quad . \quad . \quad (474)$$

where $x = g_2 + \alpha$ and $y = b_2 + \beta$ . . . . . (475)

The voltage induced in the secondary of the first machine is

$$s\dot{E}_1 = se + \dot{I}_1 Z \quad . \quad . \quad . \quad (476)$$

where $Z = (r_1 + r_2') + js(x_1 + x_2')$ . . . (477)

Here r_2' = resistance of primary of the second motor reduced to primary of the first machine

x_2' = reactance of stator of second machine reduced to full frequency and to the primary circuit

r_1 = resistance of rotor of first machine referred to primary of first machine

$\dot{E}_1$ = voltage induced in rotor of first machine at full frequency

It will be noticed that all circuits are referred to the primary circuit,

$$\therefore \dot{E}_1 = e + \frac{\dot{I}_1 Z}{s} \quad . \quad . \quad . \quad . \quad . \quad . \quad (478)$$

$$= e + \frac{e(x - jy)(r_1 + r_2' + js\ (x_1 + x_2')}{s} \quad . \quad (479)$$

$$= e\left\{1 + \frac{\{x - jy\}\ \{r_1 + r_2' + js\ (x_1 + x_2')\}}{s}\right\} \quad (480)$$

$$= e\{v + j\omega\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (481)$$

$$v = 1 + \frac{(r_1 + r_2')\ x + (x_1 + x_2')y}{s} \quad . \quad . \quad (482)$$

$$w = (x_1 + x_2')x - \frac{(r_1 + r_2')y}{s} \quad . \quad . \quad . \quad (483)$$

The exciting current of the first machine

$$= \dot{E}_1(g_1 - jb_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad (484)$$

$$= e(v + j\omega)(g_1 - jb_1) \quad . \quad . \quad . \quad . \quad (485)$$

$$= e(p - jq) = \dot{I}\mu_1 \quad . \quad . \quad . \quad . \quad (486)$$

where $p = g_1 v + wb_1$ . . . . . . (487)

$$q = vb_1 - wg_1 \quad . \quad . \quad . \quad . \quad . \quad (488)$$

The primary current of the first machine

$$= \underset{\cdot}{I} = \underset{\cdot}{I}_1 + \underset{\cdot}{I}\mu_1 \quad \ldots \quad (489)$$

$$= e(x - jy) + e(p - jq) \quad \ldots \quad (490)$$

$$= e\{(x + p) - j(y + q)\} \quad \ldots \quad (491)$$

$$= e(m - jn) \quad \ldots \quad (492)$$

$$m = x + p \quad \ldots \quad (493)$$

$$n = y + q \quad \ldots \quad (494)$$

The applied voltage to the primary of the first machine

$$= \underset{\cdot}{E}_1 + Z_1\underset{\cdot}{I} = \underset{\cdot}{E}' \quad \ldots \quad (495)$$

$$= e(v + j\omega) + (r_1' + jx_1')e(m - jn) \quad \ldots \quad (496)$$

$$= e(r + js) \quad \ldots \quad (497)$$

$$\text{where } r = v + r_1'm + x_1'n \quad \ldots \quad (498)$$

$$s = w + x_1'm - r_1'n \quad \ldots \quad (499)$$

$$\therefore \quad e = \frac{e'}{\sqrt{r^2 + s^2}} \quad \ldots \quad (500)$$

where e' = applied volts per phase

$$i = e\sqrt{m^2 + n^2} \quad \ldots \quad (501)$$

$$= e' \frac{\sqrt{m^2 + n^2}}{\sqrt{r^2 + s^2}} \quad \ldots \quad (502)$$

The power input to the set

$$= \text{real component of } \underset{\cdot}{E}'\underset{\cdot}{I} \quad \ldots \quad (503)$$

$$\text{Now } \underset{\cdot}{E}'\underset{\cdot}{I} = e(r + js)e(m - jn) \quad \ldots \quad (504)$$

$$= e^2\{rm - sn + j(sm - rn)\} \quad \ldots \quad (505)$$

$$\text{Real component} = e^2\{rm - sn\} \quad \ldots \quad (506)$$

$$= \frac{e'^2}{r^2 + s^2}(rm - sn). \quad \ldots \quad (507)$$

$$\text{Volt-amp. input} = e'i \quad \ldots \quad (508)$$

$$\text{The torque of auxiliary motor} = e^2 a \quad \ldots \quad (509)$$

The torque of main motor

$$= \text{real component of the product } \underset{\cdot}{E}_1\underset{\cdot}{I} \quad . \quad (510)$$

$$\text{Product of } \underset{\cdot}{E}_1\underset{\cdot}{I} = e(v + j\omega)e(m - jn)$$

$$\text{Real component} = e^2(vm - \omega n) \quad \ldots \quad (511)$$

= torque of main motor

With two similar motors in cascade, the torque becomes zero at cascade synchronous speed. Above this speed the torque of the second machine becomes a generator torque. Due to the reversal of current in the secondary of the first motor, its torque becomes negative also. In other words, above cascade synchronous speed the set acts as induction generator.

When approaching full synchronous speed, the generator torque of the auxiliary motor becomes very small, and the second motor acts merely as an impedance in the secondary circuit of the first machine, which again exerts a motor torque. Thus somewhere between half synchronous speed and full speed, the torque of the first motor becomes zero, while the second motor still gives a small generator torque. A little above this speed the torque of the set becomes zero, and above this speed the set gives a motor torque, although the auxiliary motor still gives a small negative torque.

Above full synchronous speed the set acts as generator, but practically only the first motor contributes to the generator torque above and the motor torque below full synchronous speed.

With high resistance in the rotor circuit of the auxiliary motor, the set gives a generator torque above half synchronous speed and remains a generator torque at all higher speeds. Instead of connecting the rotor winding of the first machine to the stator of the second, we may connect the two rotor windings in series and insert the resistance for starting or speed control in the stator of the second machine, and thus avoid the use of slip-rings.

The phases of the second rotor must be so connected as to give a direction of rotation to the auxiliary motor field, which is opposite to the direction of rotation of the set. Any number of machines may be connected in cascade, but it is hardly feasible to use more than two, for the performance becomes increasingly bad. With two machines in cascade, the first motor carries the magnetizing current of both machines.

The total self-inductive impedance of the couple is the sum of the separate impedances of each. It is clear therefore that the dispersion coefficient of two such similar machines in cascade is practically four times that of one machine, and therefore the power-factor of such a set is rather low even when each machine has a small number of poles. The output of the set when running in cascade is equal approximately to that of one machine at full speed.

Attempts were made to combine the two stator cores and two rotor cores, and Mr. F. Lydall patented such a motor in 1902 in which the two windings were arranged in the same slots in stator

and rotor. The two sets of windings were wound for dissimilar numbers of poles, so that the stator windings were mutually non-inductive and were only connected through the agency of the rotor.

The necessity of using deep slots to accommodate both windings caused a high leakage reactance and poor power-factor, efficiency, and overload capacity. To Louis J. Hunt, Esq., with whom the author had the privilege of serving his apprenticeship, belongs the credit of producing a really successful single cascade motor. Mr. Hunt conceived the idea of making one winding perform the function of two separate windings.

The stator winding had not only to be suitable for the circulation of two independent currents, viz., the main current of supply frequency and the induced currents of slip frequency, but it had to be so connected that the induced or low-frequency currents could only circulate when paths outside the winding were provided for them. This was necessary in order that the starting torque and speed could be controlled by means of starting resistances. Two sets of terminals were required: one set for connecting to the mains and the other set to the resistances. The main currents must not be allowed to flow through the external secondary paths, and consequently it was essential to connect the secondary terminals to two points in the winding between which no "main" potential difference existed. A parallel connected-winding was necessary.

Fig. 52 shows such a stator winding suitable for 12, 4, or 8 poles. The figures and letters on the key diagram correspond to those on the winding diagram.

Fig. 53 shows one phase of a change pole winding suitable for 4 or 8 poles.

The cascade motor is essentially a low-speed machine and, as pointed out by Mr. Hunt, it shows to best advantage when designed for speeds which are abnormal for single-field motors.

The numbers of poles for which the machine can be constructed are determined by the following conditions—

(*a*) The numbers of poles in the two fields must be so chosen that their windings are mutually non-inductive.

(*b*) The two fields when superimposed must not produce an unbalanced radial pull on the rotor.

Conditions (*a*) and (*b*) are satisfied when the two numbers of poles are such that, when divided by their greatest common factor, the quotient is in one case even and in the other odd. Further, the greatest common factor must be greater than two.

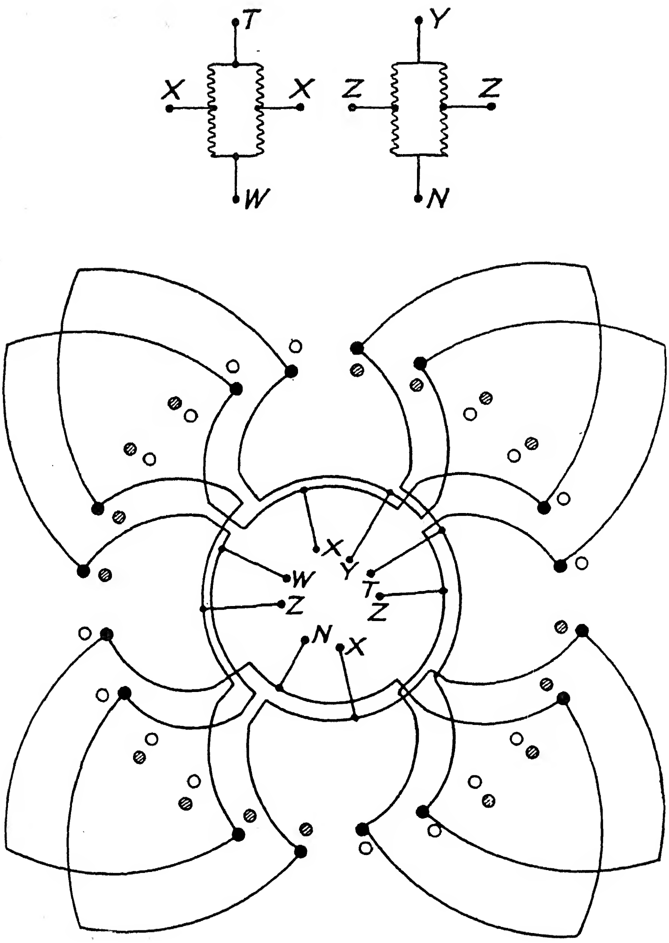


FIG. 53

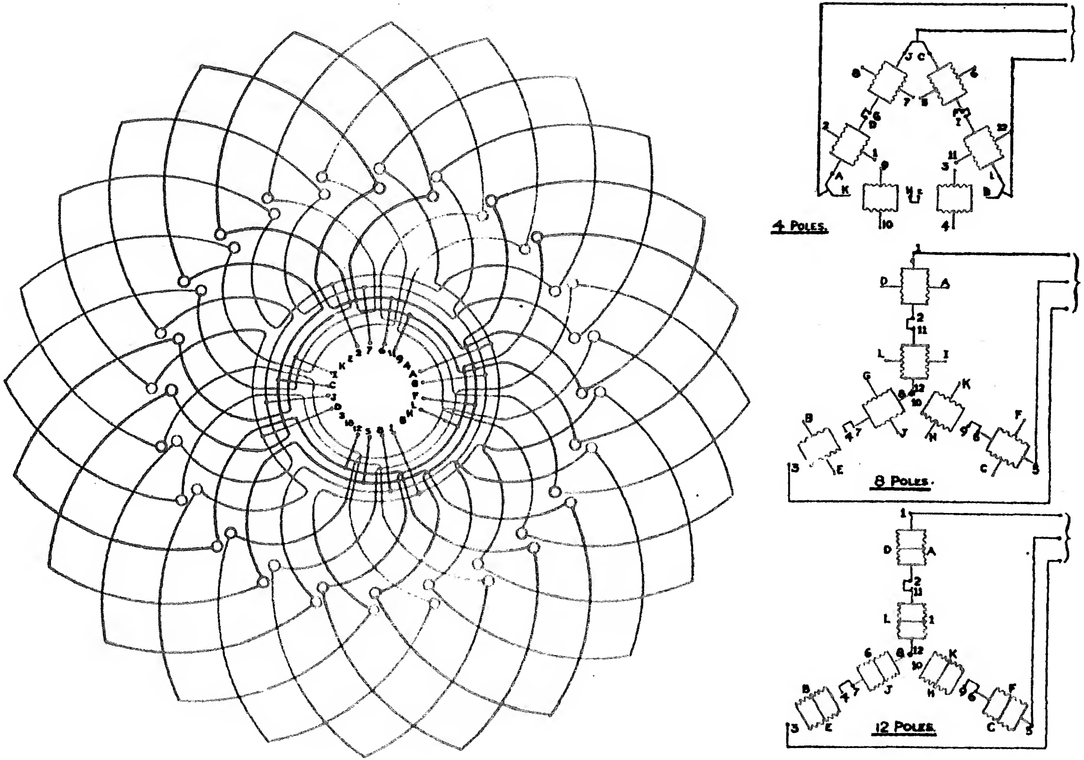


Fig. 52.—Stator Winding suitable for 4, 8, or 12 Poles (12-Pole Cascade)

Fig. 54 shows the windings of one phase of an 8-pole stator. One slot per pole per phase is shown to simplify the diagram. Each radial pair of small circles represents the two coil sections occupying one slot, and the crosses and dots indicate the directions of the main currents. The main currents produce 8 poles, and the coils must be connected so that currents induced by a 4-pole field can circulate in them. This 4-pole field is shown in Fig. 54. Four of the slots are opposite the centres of the poles

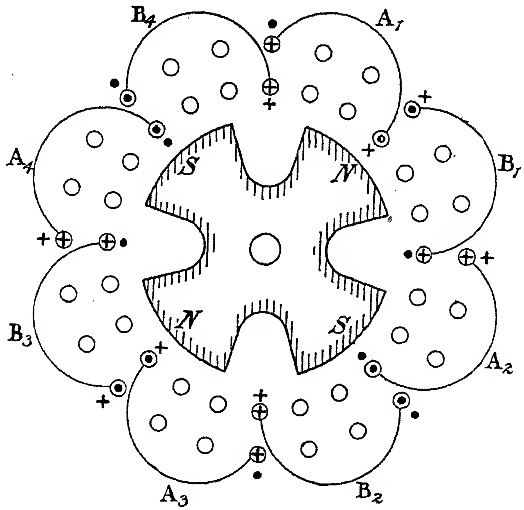


FIG. 54

and four are situated midway between. The E.M.F.'s induced in the parts of the winding in the slots at the centres of the poles will be in quadrature with those induced in the slots midway between the poles. The coils are marked *A* and *B* alternately, and all the induced currents in the *A* coils are in phase with one another and in quadrature with those in the *B* coils.

Fig. 55 shows the key diagram, and Fig. 56 shows one phase connected up to comply with this diagram.

The main currents in the key diagram (Fig. 56) are shown by arrows inside the windings; the induced currents are shown by the arrows outside. All the *A* coils are included between *T* and *C*, and the *B* coils between *C* and the star-point. The rotor winding is exceedingly ingenious.

The ratio of the numbers of turns in the main and auxiliary windings of the rotor is 1·73 to 1·0. This gives an auxiliary field

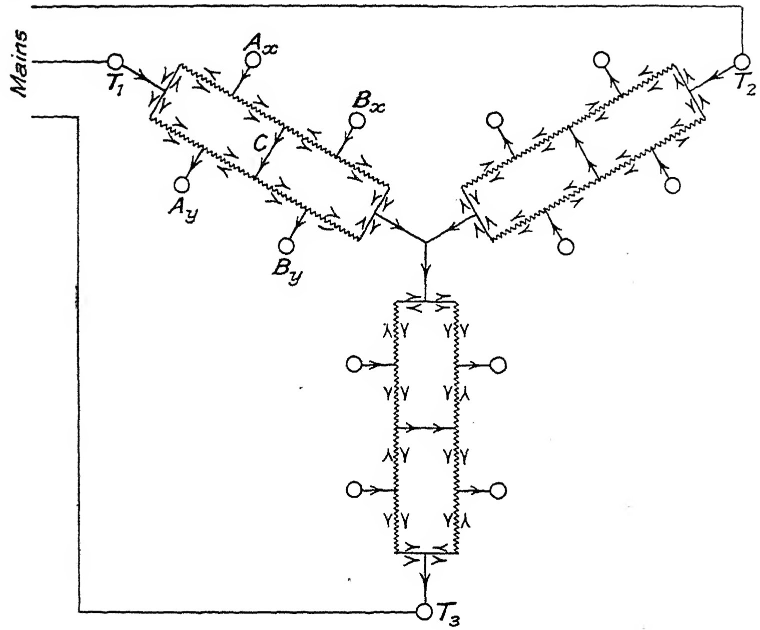


Fig. 55

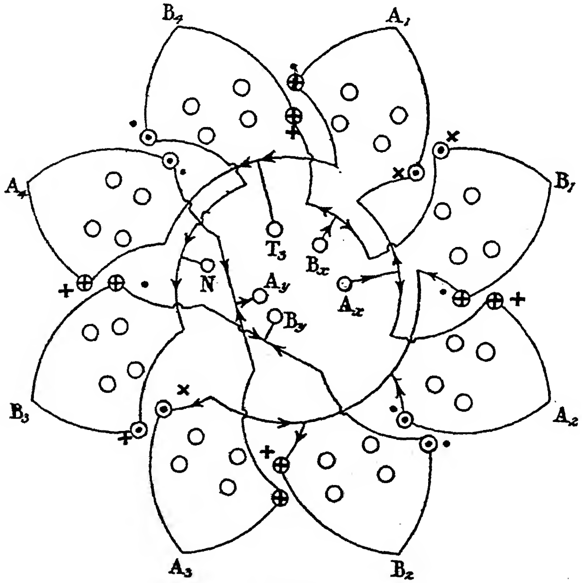


Fig. 56

with a flux per pole 73 per cent greater than the main flux, and the gap density is 86·5 per cent of that of the main field. The auxiliary magnetizing current is 75 per cent of the main field magnetizing current. With a given stator winding, the magnetizing current of the auxiliary field varies inversely as the square of the number of turns in the auxiliary winding.

The rotor windings consist of a main rotor winding connected in star and an auxiliary rotor winding connected mesh.

Fig. 57 shows the elements of the two windings and the method of connection.

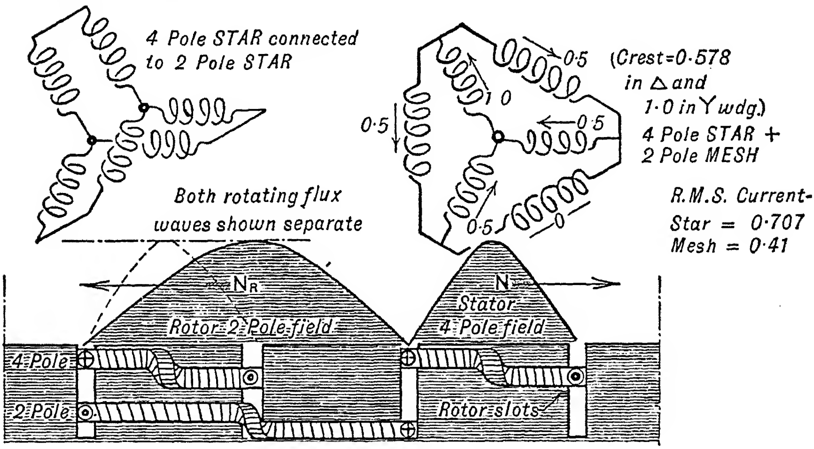


FIG. 57.—ELEMENTS OF "HUNT" ROTOR WINDING

The E.M.F.'s, due to the main field, in the mesh-connected winding are balanced by the E.M.F.'s generated by the auxiliary field, and current can circulate only by way of the star.

Fig. 58 shows a cascade rotor winding for 12 poles.

The main and auxiliary rotor windings are superimposed to form a single winding.

In order to show how this superposition and combination is effected, a 4-pole and 2-pole combination is shown in Fig. 59. This is merely for the purpose of illustration, for a 4-pole + 2-pole combination could not be used in practice for reasons previously given, but it serves as a unit from which to build up other pole combinations. The top diagram of Fig. 59 shows the component windings of a 4-pole + 2-pole rotor wound in 12 slots. The current in phase 1 is assumed at its crest value.

It will be noticed that in some slots the currents are equal and

flow in opposite directions, and hence neutralize each other. These are omitted as shown in the diagram below. (Fig. 59.)

The remaining conductors are then connected up to form a

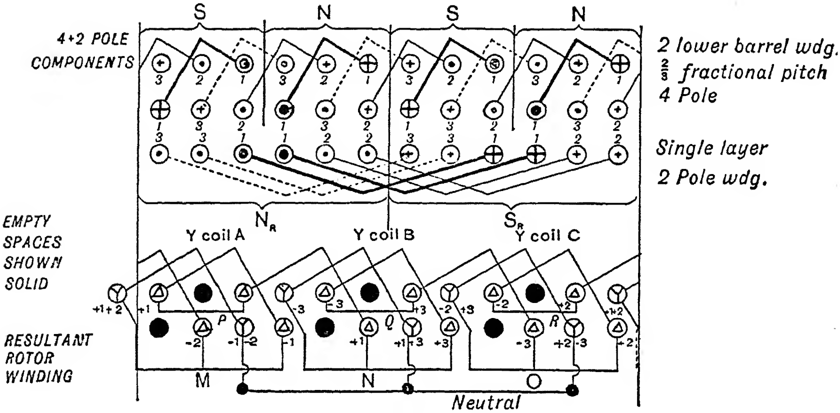


FIG. 59

resultant winding, which produces the 4-pole and 2-pole fields. Six slots have 3 bars, and six have 1 bar per slot. The three bars are combined into two, so that one carries a single current and the other a double current, i.e. two currents differing in phase by

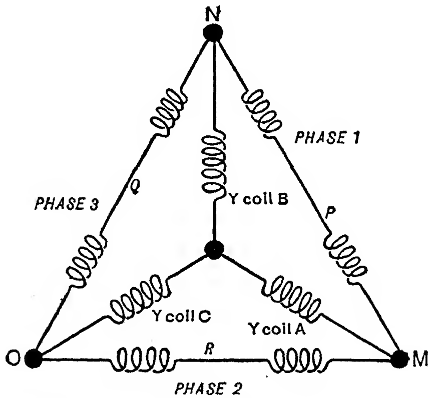
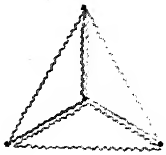


FIG. 60.—DIAGRAM OF Y-Δ ROTOR RESULTANT WINDING

60°. These bars carry $\sqrt{3}$ times the current in the other bars; they are then connected star and supply the single-current bars which are connected in delta. This arrangement saves 25 per cent of the bars and reduces the inductance.



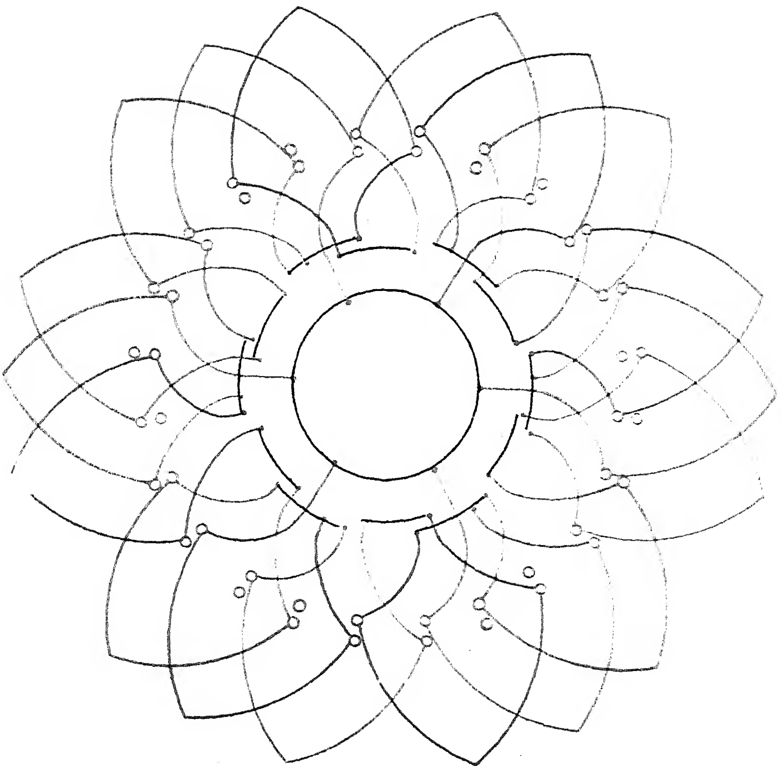


Fig. 60 shows the resultant diagram.

Speeds, other than cascade speed, are obtained by bringing the ends of the mesh to slip-rings, which can be short-circuited. Thus, if we short-circuit the mesh windings, the motor will run as an induction motor having the main number of poles. The two-speed motor can be operated at a third speed if the stator windings are so arranged that they can produce two different numbers of poles.

Fig. 61 shows an (8 + 4)-pole stator winding with two groups of coils in each phase divided and the neutral point opened.

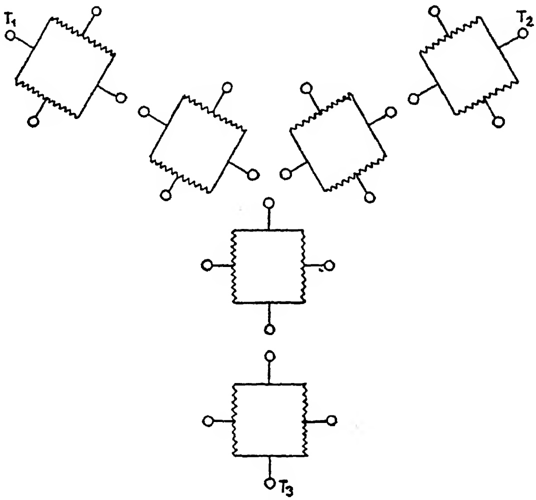


FIG. 61

To connect this winding for 8 poles, two groups of coils in adjacent sections are placed in series, and then, by means of a suitable switch, the whole winding is connected in star. This is shown in Fig. 62.

Fig. 63 shows the same winding reconnected to give 4 poles.

Fig. 64 shows a cascade rotor winding suitable for 4, 8, or 12 poles (12 poles cascade).

(*d*) **Change of speed by varying supply frequency.** By changing the supply frequency, the speed may be varied. The generators, driven by Diesel engines or steam turbines, may have their speeds adjusted over a fair range, and the induction motors connected to the generators will vary their speed as the supply frequency is varied. This is usually done on ships having an electric drive.

On war-ships, where cruising and fighting speeds are required, a low-frequency generator is provided for the low speed, and separate

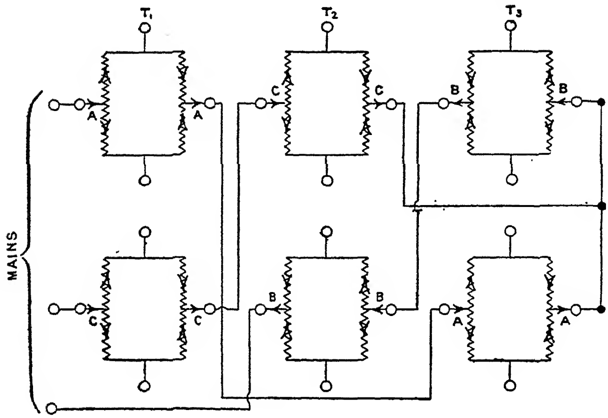


FIG. 62

and more powerful generators of higher frequency for the higher speed. If it is desired to use all the generators at the high speed,

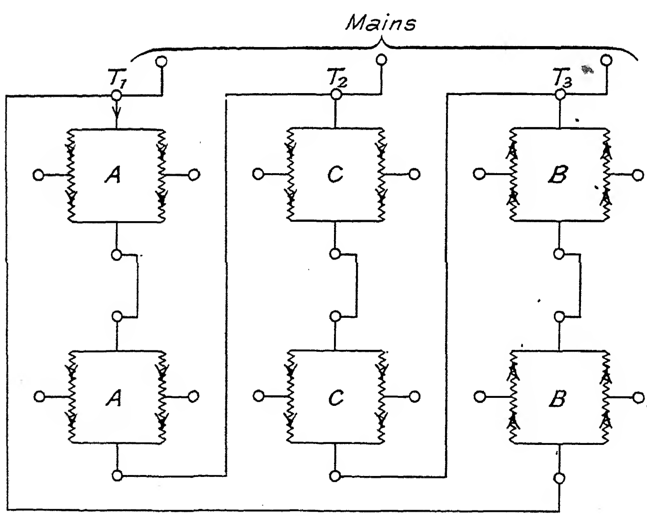
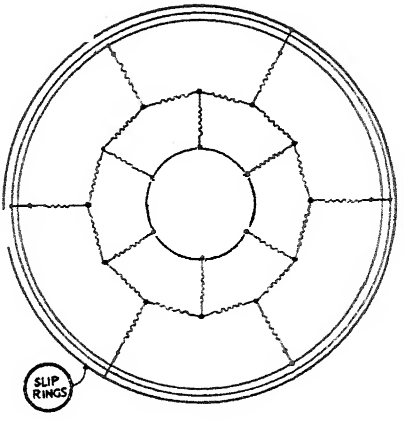


FIG. 63

then all that is necessary is to use motors giving the higher speed at the low frequency.



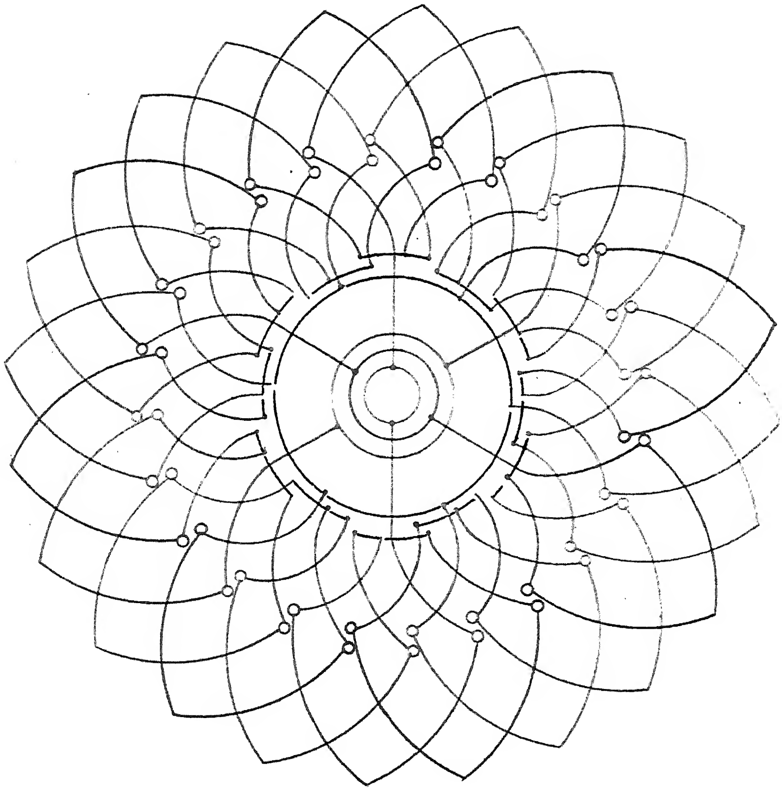


Fig. 64.—Cascade Rotor Winding, suitable for 4, 8, or 12 Poles (12-Pole Cascade)

(*e*) **By the use of a rotary converter in series with the slip-rings of the rotor.** (Krämer's system.) In this system the rotor winding of the induction motor is connected to the slip-rings of a rotary converter. The slip energy is converted into continuous-current energy, which can be given to a continuous-current network, or to a continuous-current auxiliary motor coupled to the shaft of the induction motor. In the latter case, as the slip increases, the power given out by the auxiliary motor rises, by the same amount as that given out by the main motor falls. Thus the sum of the outputs of the two motors remains constant over the whole speed range, i.e. the torque varies inversely as the speed. This is a property frequently required in rolling-mill work.

When working in conjunction with a flywheel, the requisite drop in speed can be obtained by a compound winding on the auxiliary D.C. motor.

The auxiliary motor determines the speed of the set. To lower the speed, the field of the D.C. motor must be increased. With the connections shown in Fig. 65, no special synchronizing gear is necessary. On switching on the excitation of the auxiliary motor and converter, the latter runs up to speed and falls into step.

The maximum speed of the set is determined by the lowest frequency necessary for stable running of the rotary converter. This is about 2 or 3 cycles per second, so that with a frequency of 50 cycles the highest speed will be about 4 to 6 per cent below synchronous speed of the main motor; or if artificially raised above synchronism, 4 to 6 per cent above this will be the lowest stable speed.

If the voltage on the slip-rings becomes small as compared with the ohmic drop, the converter becomes unstable. The power factor of the set can be made unity, over nearly the whole range, by adjusting the excitation of the converter.

If the speed of the induction motor is very low, the cost of a D.C. motor on the same shaft may be excessive. It is preferable in this case to return the slip-energy to the supply system by means of a high-speed motor-generator set. The generator may be of the asynchronous type. For work which requires constant torque, the latter method is suitable.

Fig. 65 shows the diagram for the Krämer system.

(*f*) **Variation of speed and power factor by the use of three-phase commutator motors used in conjunction with induction motors.** Instead of converting the slip-currents into continuous current, the rotor currents may be supplied to a three-phase commutator

motor. This three-phase auxiliary motor is then coupled direct or by means of a belt-drive to the main motor.

The diagram of connections is shown in Fig. 66.

According to the characteristic required, a series motor with

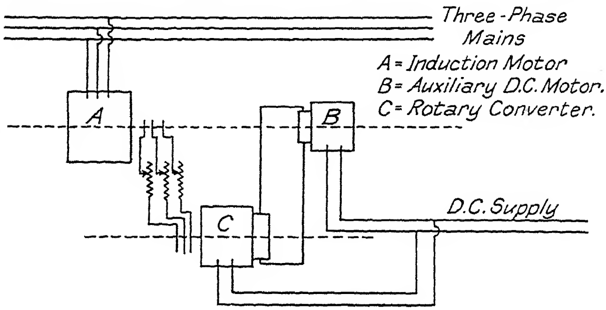


FIG. 65

brush-shifting device, or a shunt or compound motor with pressure regulation, is used as auxiliary motor. With this arrangement, the output is constant for a given input, so that the torque varies inversely as the speed.

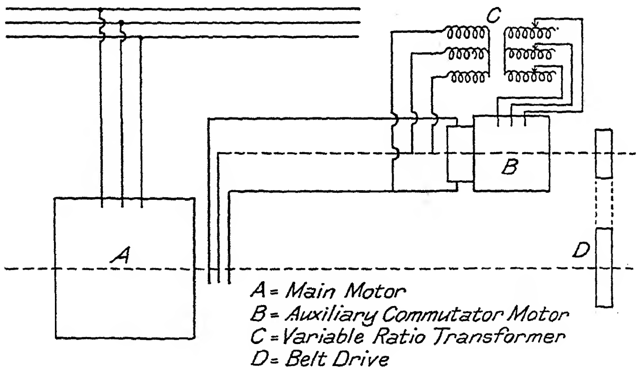


FIG. 66

In the Scherbius system used by Brown-Boveri, the rotor current is led to a three-phase commutator motor. The latter is not coupled to the main motor, but to a three-phase generator which returns the slip energy to the line. The generator is of the induction type, running somewhat above synchronous speed. The speed characteristic obtained depends on the type of commutator motor, i.e. whether series, shunt, or compound.

This system is used chiefly for drives where the power falls with the speed, as in centrifugal machines. It has the advantage that the main motor can be run independently of the auxiliary set.

The most common arrangements of regulating sets are: (1) A series commutator machine for single-range regulation, i.e. for speed ranges below synchronism; (2) a shunt-wound commutator machine for single-range regulation; and (3) a shunt-wound commutator machine with special exciter for double-range speed regulation above and below synchronism.

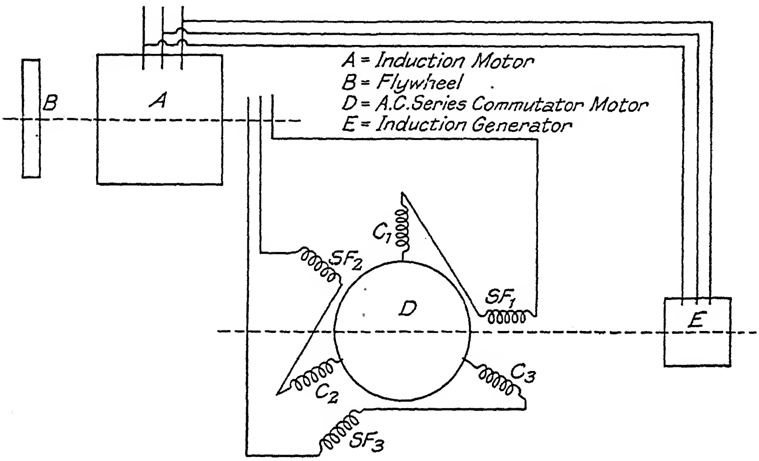


FIG. 67.—NEUTRALIZED SERIES-EXCITED THREE-PHASE A.C. COMMUTATOR MOTOR AND CONNECTIONS

For automatic single-range regulation of induction motor equiped with flywheel to reduce peaks on line. SF_1, SF_2, SF_3 are the series coils. C_1, C_2, C_3 are neutralizing windings which neutralize the armature amp.-turns

The commutator machine is mechanically coupled to an induction machine, and in the case of single-range regulation below synchronism, the induction machine acts as generator. In double-range sets, the induction machine will act as generator below synchronism and as motor above synchronism.

The speed of the regulating set will change but little with change of speed of the main motor, since a very small percentage change of speed is sufficient to change from full-load motor to full-load generator action. It may be regarded therefore as having an approximately constant value. In all these cases an E.M.F. is injected into the rotor circuit of each phase. If this injected E.M.F. has a component in phase opposition to the actual E.M.F. producing the rotor current, i.e. the E.M.F. overcoming the resistance drop

and tending to reduce the rotor current, it is apparent that the slip will increase to such a value that the E.M.F. generated in the rotor, due to that slip, will be sufficient to overcome the injected E.M.F. and the resistance drop due to the current required to produce the required load torque.

It is apparent, therefore, that by adjusting the phase of the injected E.M.F., we can readily control both speed and power factor.

Let us consider, first, a series commutator motor for single-range variation of speed below synchronism.

The diagram of connections is shown in Fig. 67.

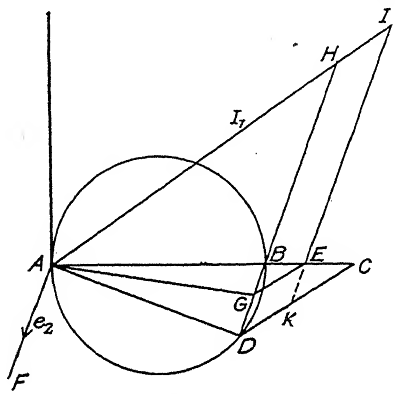


Fig. 68

By arranging the phases of the series coil, one can inject an E.M.F. having any constant phase angle with respect to the rotor current.

Mr. John I. Hull has adopted a form of circle diagram which shows very clearly the performance of an induction motor working in conjunction with commutator sets.

The following is taken from his article in the *Journal of the American Institute of Electrical Engineers* for June, 1920[1]—

$AH = I_1$ (proportional to primary current) and represents the flux-linking primary and secondary which would be produced by the primary current alone neglecting saturation

$HI = C_1 I_1$ represents primary leakage flux

$HG = IE = I_2$ (proportional to secondary current) represents the flux-linking secondary and primary which would be produced by the secondary current alone neglecting saturation

$GD = C_2 I_2$ represents secondary leakage flux

[1] Dr. Meyer-Delius presented similar treatment in the *ETZ*, 1913.

AE represents the total flux linking the primary. Neglecting resistance drop, E is a fixed point for constant line voltage and frequency, since AE is the flux which generates the counter E.M.F. to balance the applied voltage.

HG intersects AE at B, and it is clear that

$$\frac{AB}{AE} = \frac{I_1}{I_1\,(1 + C_1)} \quad . \quad . \quad . \quad . \quad . \quad (512)$$

since AE is constant, AB is constant, so B is a fixed point. AD is the resultant of I_1 and $I_2 + C_2I_2$, and generates all secondary E.M.F.'s, except the resistance drop, which is in phase opposition to e_2, set up in the secondary in quadrature to flux AD.

AF is therefore parallel to HD and to IE, and the angle ADH is therefore a right angle. Since A and B are fixed points, the locus of D is the arc of a circle.

A line parallel to AI from D intersects AE produced in C.

$$BD = HD - HB = I_2(1 + C_2) - \frac{I_2}{1 + C_1} \quad . \quad . \quad . \quad (513)$$

$$\text{since} \quad \frac{HB}{I_2} = \frac{I_1}{I_1\,(1 + C_1)} \quad . \quad . \quad . \quad . \quad . \quad (514)$$

$$\therefore \quad BD = \frac{I_2}{1 + C_1}\,\{C_1 + C_2(1 + C_1)\} \quad . \quad . \quad (515)$$

$$\frac{CD}{AI} = \frac{BD}{IE} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (516)$$

$$\therefore \quad CD = I_1(1 + C_1) \times \frac{I_2\{C_1 + C_2(1 + C_1)\}}{I_2(1 + C_1)} \quad (517)$$

$$= I_1[C_1 + C_2(1 + C_1)] \quad . \quad . \quad . \quad (518)$$

EA represents the flux whose counter E.M.F. balances the applied voltage. Let us represent EA by I_m. Then

$$BA = I_o = \frac{I_m}{1 + C_1} \quad . \quad . \quad . \quad . \quad . \quad (519)$$

$$\frac{CB}{CD} = \frac{AE}{AI} = \frac{I_m}{I_1(1 + C_1)} \quad . \quad . \quad . \quad (520)$$

$$\therefore \quad CB = CD \times \frac{I_m}{I_1(1 + C_1)} \quad . \quad . \quad . \quad (521)$$

$$= I_m\,\frac{[C_1 + C_2(1 + C_1)]}{1 + C_1} \quad . \quad . \quad . \quad (522)$$

$$= I_o[C_1 + C_2(1 + C_1)] \quad . \quad . \quad . \quad (523)$$

$$CA = EA + CE = I_m(1 + C_2) \quad . \quad . \quad (524)$$

$$\text{since } \frac{CE}{EK} = \frac{I_m}{I_2} \quad \therefore \quad CE = I_m \times \frac{C_2 I_2}{I_2} \quad . \quad . \quad (525)$$
$$= C_2 I_m$$

$$CA = I_m(1 + C_2) = \frac{I_m(1 + C_2)}{C_1 + C_2(1 + C_1)} \times [C_1 + C_2(1 + C_1)] \quad (526)$$

CA is $\therefore$ constant for I_m constant, and C is therefore a fixed point.

By selecting a suitable scale, I_m could be made to represent the magnetizing current for the whole flux I_m, which is the usually calculated quantity. I_o could be made to represent the true running light current, primary reactance considered. I_1 the primary current and I_2 the secondary current. By changing the scale of the diagram by the factor $C_1 + C_2(1 + C_1)$, we may say that

$$\frac{i_m}{1 + C_1} = CB \quad . \quad . \quad . \quad . \quad (527)$$

which equals the true running light current I_o; the primary current $I_1 = CD$ . . . . (528)

$$\text{and} \quad \frac{I_2}{1 + C_1} = DB \quad . \quad . \quad . \quad . \quad . \quad (529)$$

At standstill, with zero secondary resistance, AD, the resultant secondary flux, must be zero, which means that D coincides with A and $CD = CA$, i.e. the ideal short-circuit current at standstill $= CA$.

Since $C_1 I_1$ is defined as the primary leakage flux, the primary reactance drop with current i_m is $C_1 e_1$, since i_m produces the flux which generates e_1. If X_1 be the primary reactance, $C_1 e_1 = i_m X_1$

$$\therefore \quad C_1 = \frac{i_m X_1}{e_1} \text{ and } C_2 = \frac{i_m X_2}{e_1} \quad . \quad . \quad . \quad (530)$$

In order to draw the diagram, we need to know the primary and secondary reactances X_1 and X_2, and the nominal magnetizing current i_m.

The diagram is shown in Fig. 69.

$$CB = i_o \quad . \quad . \quad . \quad . \quad . \quad (531)$$

$$CA = \frac{i_m(1 + C_2)}{C_1 + C_2(1 + C_1)} \quad . \quad . \quad . \quad (532)$$

$$CM = \text{watt component of input current} \quad . \quad . \quad (533)$$

$$I_1 = CD \quad . \quad . \quad . \quad . \quad . \quad . \quad (534)$$

The secondary circuit is $(1 + C_1)DB$. To find the secondary voltage $e_2 = AF$, one can first find its value at full frequency. Since AE (Fig. 68) is proportional to e_1 at full frequency,

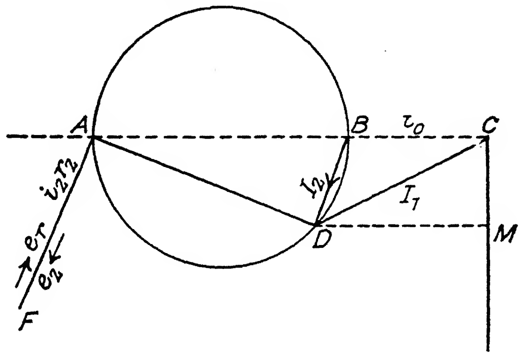


Fig. 69

$$AB = \frac{e_1}{1 + C_1} \qquad \text{Also } AD = \frac{AD}{AB} \cdot \frac{e_1}{(1 + C_1)} \text{ volts}$$

at full frequency. The actual value of the secondary volts

$$= i_2r_2, \text{ so the \% slip} = \frac{i_2r_2}{AD} \quad . \quad . \quad . \quad (535)$$

$$\text{The torque in synchronous watts} = BD \times AD \quad . \quad . \quad (536)$$

$$\text{The output} = (1 - s)BD \times AD \quad . \quad . \quad (537)$$

$$\text{The efficiency} = \frac{(1 - s)BD \times AD}{e_1MC} \quad . \quad . \quad . \quad (538)$$

$$\cos \phi = \frac{MC}{CD} \quad . \quad . \quad . \quad . \quad . \quad . \quad (539)$$

Now it is clear from this simple diagram that if an injected E.M.F. e_r, due to the commutator motor, exists in the rotor, and if this E.M.F. e_r is in phase opposition to the secondary current, the slip of the motor must increase to make the generated E.M.F. of the main rotor $e_2 = i_2r_2 + e_r$. In this case the slip is increased, for the effect of the injected E.M.F. is equivalent to an addition of

resistance in the rotor circuit. If e_r is in phase with e_2 and assisting e_2 the slip will decrease, for e_2 must equal in this case the algebraic sum of e_l and i_2r_2.

Now it is possible to inject, by means of the commutator motors, E.M.F.'s having any phase angle with the rotor current. If the injected E.M.F. has a component which is 90° ahead of BD, the effect will be to make BD lead. In other words, speed control and power factor correction are possible.

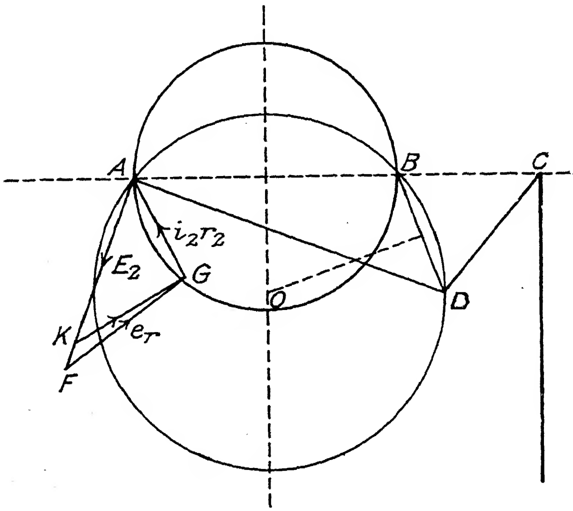


FIG. 70

Now with the neutralized series commutator motor, shown in Fig. 67, the E.M.F.'s at the terminals of the commutator motor are the leakage reactance drop, the resistance drop, and the rotational voltage of the armature. This rotational E.M.F. is proportional to the flux and speed of rotation, and the flux is proportional to the secondary currents. Since the speed of the commutator motor is practically constant, the injected E.M.F. is proportional to the rotor current of the main motor. Further, the phase angle between the rotor current and injected E.M.F., i.e. between resistance drop and rotational voltage, is constant, and can only be changed by altering the construction of the machine. The diagram for this case is shown in Fig. 70.

Since $FG \propto AG$ and $\angle FGA$ is constant,

$\therefore$ $\angle FAG$ is constant. Also AF is at right angles to AD

$\therefore\ \angle GAD = 90° - \angle FAG =$ constant; but AG is parallel to BD, i.e. i_2

$$\therefore\ \angle GAD = \angle BDA$$

$\therefore\ \angle BDA$ is constant; and since AB is constant in length, it follows that the locus of the point D is a circle. The centre of the circle is easily found, since the circle passes through A, B and D.

Bisect AB and BD, and draw perpendiculars to each through the points of bisection. Then the intersection of the perpendicular bisectors gives us the centre. Points A, B and C are determined as for Fig. 69, but now for X_2 we must substitute

$$X_2 + X_c + X_{cs},$$

where $X_c =$ leakage reactance of the regulating motor at primary frequency, and

$$X_{cs} = \frac{\text{volt. amp. to excite regulating motor}}{i_2^2 \times \sqrt{3}} \quad . \quad . \quad (540)$$

Now Mr. Hull shows that, for designs ordinarily encountered, it is permissible to neglect saturation, for saturation reduces X_{cs}, and thus gives us a new and larger circle; but the ratio of injected E.M.F. to rotor current or exciting current is reduced. The two effects partially offset one another. In any case, it is simple to draw the new circle, provided we can determine the excitation, at full frequency, from the magnetizing curve of the regulating motor. It is clear that the series regulating motor is most suitable for cases where large and rapid load fluctuations occur, and where it is desired to reduce the peak loads by means of the flywheel.

With a constant load, the speed may be regulated by hand as desired, if the field windings of the commutator motor are supplied through a suitable transformer with taps.

Speed regulation and power factor control by means of a three-phase shunt commutator motor. For some purposes, speed variation is required, but the speed should not vary greatly with the load. It is clear therefore that, if the flux of the commutator motor can be adjusted to constant value, the injected E.M.F. will be practically constant, since the speed of the commutator motor is practically constant. By adjusting the magnitude of the field, it is clear that the rotational E.M.F. can be given any suitable value, and that this E.M.F. will be independent of the load. The scheme of connections is shown in Fig. 71.

The field coils are fed from taps on the auto-transformer B. When the voltage across the rings increases, the slip frequency

increases at the same rate, and therefore the flux in the auto-transformer is independent of the slip. For the same reason, the

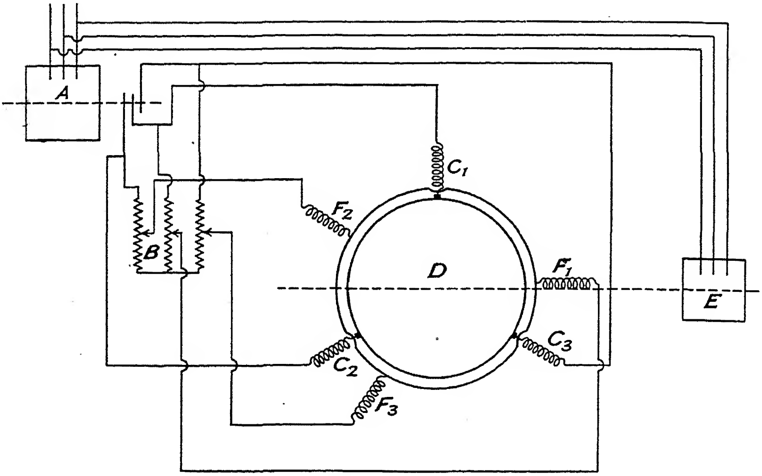


FIG. 71

flux in F_1, F_2 and F_3 is constant for a given tap position and independent of the slip. It can be changed, however, by altering the tapping point on B.

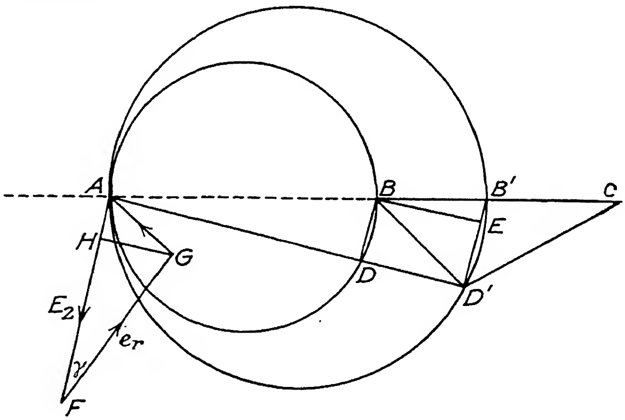


FIG. 72

The circle diagram for this case is shown in Fig. 72.

With the commutator motor stationary, we get the circle BDA. In constructing this we must include the leakage reactance of the

regulating motor, so that $X_{2+c} = X_2 + X_c$, and for C_2 we have C_{2+c}. With the commutator motor running, we get secondary current BD', and the total induced E.M.F. of the induction motor rotor AF is proportional to AD' and slip. FG, the injected E.M.F. of the commutator motor, is proportional to AD' and is at constant angle to AF. This angle is determined by the connections of the transformer and exciting windings F_1, F_2 and F_3.

Resolve the resistance drop GA into two components, HA in phase opposition to AF and GH in quadrature to AF, the corresponding components of secondary current being BD and DD'. $DD' \propto HG$. $HG = FG \sin\gamma$, so that $HG \propto FG$ for γ constant; but $FG \propto AD'$ $\therefore$ $DD' \propto AD'$

and $AD = AD' - DD'$ $\therefore$ AD is proportional to AD'

$$\therefore \frac{AB'}{AB} = \frac{AD'}{AD} = \text{constant, and } B \text{ is a fixed point.}$$

Therefore the locus of D' is a circle.

When running light, i.e. zero torque, BD and AH are zero. The torque of the motor is proportional to the product of mutual flux FG and the component of secondary current in quadrature with it, i.e. to $BD \times AD$.

It is zero therefore when BD is zero. BD is the torque-producing component of BD'.

The slip, when running light, $s_0 = \dfrac{FH}{AD'}$

and the additional slip, due to the load $= \dfrac{AH}{AD'}$

It will be seen that the running light slip is determined by the angle γ and the ratio of $\dfrac{FG}{AD'}$ which conditions are adjusted by the connections at the auto-transformer. The load slip s_1 is the same for all values of s_0, provided the angle γ is chosen so that $\dfrac{HG}{AD'}$ remains constant.

The power factor can obviously be improved and the maximum torque of the motor increased.

Characteristics intermediate between those obtained by means of a series and shunt A.C. motor can be obtained by means of a compound-excited neutralized commutator motor. It is necessary in this case to change the field voltage of the commutator motor, as the load changes in order to change the flux and hence the

injected E.M.F. This is effected by means of a series transformer which has an air-gap in the magnetic circuit, so that its flux is proportional to the resultant of the primary and secondary ampere-turns.

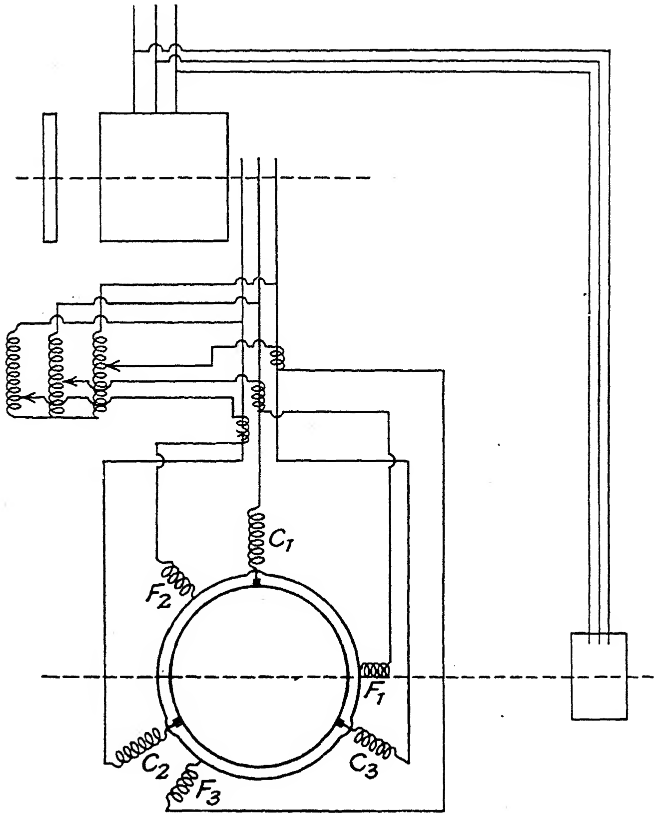


FIG. 73.—NEUTRALIZED COMPOUND-EXCITED, CONSTANT SPEED THREE-PHASE A.C. COMMUTATOR AND CONNECTIONS

For adjustable speed single-range control of induction motor, giving automatic drop in speed with load increase

Now, in cases where the speed regulation is required below and above synchronism, a special exciter or frequency changer is required. This case has the advantage that, for a given range of speed regulation, it requires a regulating set of half the output that would be required if the full-speed range were required below synchronism.

A second important advantage is that the synchronous speed of the main motor is in the middle of the speed range, so that many processes may be carried out running as a plain induction motor with the set shut down. This special exciter is simply a rotor mounted on the shaft of the main motor. It is provided with a closed circuit winding connected to slip-rings on one side and to a commutator on the other, just like the armature of a rotary converter. It is wound for the same number of poles as the induction motor, and the commutator has three brush sets per pole pair. The magnetic circuit of the armature is completed by a ring of laminated steel placed over the slots and rotating with the armature. The slip-rings are connected with the mains in such a manner that the rotating field rotates in a direction opposed to the direction of rotation of the rotor. Since the speed of the field relatively to the rotor is that of synchronism, the brush P.D. on the commutator (neglecting the drop in the rotor windings) will be practically constant. The frequency of the brush P.D. will be equal to the slip frequency of the main motor. It is obvious that this machine acts as a frequency changer, pure and simple, without altering the value of the P.D. In passing through synchronism, the phase sequence of the brush P.D.'s is automatically reversed. Now this exciter is used to excite the fields of the commutator machine. Imagine that from a suitable external source, whose frequency is always of slip frequency, we excite the commutator machine so as to reverse the phases of the E.M.F.'s which it generates before these are reduced to zero. Then these E.M.F.'s will be in phase with the rotor E.M.F.'s, and the immediate temporary effect is to increase the rotor currents and accelerate the rotor. As the speed increases, the rotor E.M.F. of the induction motor decreases, and the current decreases till it reaches the value required to produce the load torque. The rotor current is now only partially maintained by the rotor E.M.F., and is partially maintained by the commutator machine. The commutator machine is now acting as a generator, and is driven by the induction machine coupled to it. If the excitation of the commutator machine is now increased, the speed of the induction motor will rise and reach synchronism. At synchronism the induction motor rotor E.M.F. is zero, and the rotor current is entirely produced by the E.M.F. of the commutator machine. This current, being of zero frequency, is a continuous current. If this current is still further increased, the speed will rise above synchronism. The induction motor rotor will now generate E.M.F.'s of opposite phase sequence and of reversed phase. Let us suppose that as the main motor passes through

synchronous speed, the phase sequence of the source supplying the excitation of the commutator machine is reversed automatically, but without phase reversal of the E.M.F.'s. The commutator machine will have the same sequence of E.M.F.'s as the rotor of the induction motor E.M.F.'s, but the latter will oppose the former. The rotor currents flow in the direction of the commutator E.M.F.'s, and therefore this machine acts as generator supplying the rotor copper losses of the induction motor and also additional power, which is converted into mechanical power. The induction motor

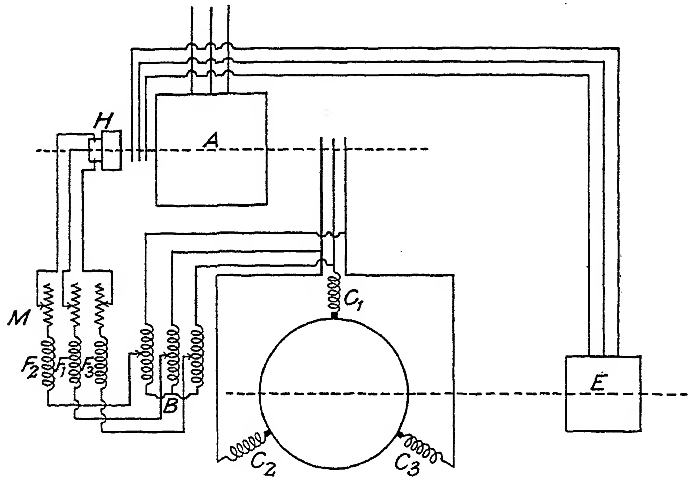


Fig. 74.—Neutralized Three-phase Shunt A.C. Commutator Machine and Connections

For adjustable speed control of induction motor below and above synchronous speed

is now doubly fed, both stator and rotor receiving power from external sources. With steadily increasing excitation of the commutator machine, the speed of the induction motor will increase.

It is clear therefore that what is required for excitation of the commutator machine is some source whose frequency is always that of slip of the induction motor, and whose phase sequence is reversed as the motor passes through synchronous speed. Such a source is the frequency changer already described.

Fig. 74 shows the diagram of connections for double range speed regulation.

The field coils of the commutator machine are shown at F_1, F_2 and F_3. They are fed from tappings on the auto-transformer B,

whose terminals are connected across the slip-rings of the induction motor. The inner ends of the field coils F are connected through the rheostats M to the commutator brushes of the frequency changer. When it is required to change the speed, the flux is increased by altering the tappings. The reactance drop component of the impedance drop of the field circuit, being proportional to the frequency as well as the flux, is proportional to the square of the slip, while the resistance drop is proportional to the field current and slip.

By connecting to taps of B whose distance from the star point is proportional to the slip, we get a voltage proportional to the square of the slip, since the total E.M.F. of B is proportional to the slip.

By changing the taps on resistance M so that the entire resistance of the circuit is proportional to $\frac{1}{\text{slip}}$ we just permit the constant voltage frequency changer to supply the resistance drop balancing E.M.F., while the auto-transformer B furnishes the reactance drop balancing E.M.F. One set of switches can be arranged to vary M and B at once.

A full account of the actions occurring can be found in Mr. Hull's article from which this is taken.

CHAPTER X

PHASE ADVANCING

THE power factor of lightly-loaded induction motors is very low, and likewise that of fully-loaded slow-speed motors. The deleterious effects of low power factor on the voltage regulation of the system are well known; and, further, the losses in generators and motors and transmission lines are increased due to low power factor. Further, in the design of induction motors, especially if of low speed, the output for a given frame is frequently reduced by the necessity of using shallow slots, for it is very important to reduce the leakage reactance. Several different types of machine have been devised for injecting a leading E.M.F. into the rotor circuits and thus to improve the power factor.

Leblanc's system. Leblanc proposed an arrangement of exciters for this purpose. Considering a two-phase induction motor, two exciters, one for each phase, are connected in series with the rotor winding, and the field of one exciter is excited by the rotor current of the second. These machines were ordinary commutating alternating-current generators. With this arrangement an E.M.F. is injected into one phase, which is 90° out of phase with the current carried by that phase; and if the polarity of the poles is properly arranged, this is a leading E.M.F. The objection to this scheme was the excessive cost. Leblanc has devised an exciter which embodies, in one machine, all the phases, and this is of a very simple nature.

The armature is made like an ordinary drum-wound continuous-current armature, and is surrounded by a simple ring of laminations having inwardly projecting poles, but without field windings. If such an armature is provided with four brushes, placed 90° apart on the commutator, and connected to the four slip-rings of a two-phase rotor of an induction motor, and is run at a speed which is high compared with the frequency of the rotor circuits, it will have the effect of producing leading currents in the rotor circuit. The beauty of the exciter is that the armature currents excite the field and produce a flux in the armature which is in such a phase as to generate an E.M.F. in each circuit which is in quadrature with the current carried by the circuit. By proper design and the use of carbon brushes, the commutation can be made sufficiently good. The rotors of induction motors of large power carry heavy

currents, and this necessitates a commutator of considerable dimensions.

The Scherbius system. In the Scherbius phase advancer, the external ring of laminations is omitted, and the armature winding is embedded in slots, which are some distance from the periphery. The path for the magnetic field is, therefore, through the iron above the slots. When used in conjunction with a three-phase rotor, three sets of brushes are provided on the commutator at 120 electrical degrees apart. These brushes receive the rotor currents of slip frequency. The currents of slip frequency will produce a revolving field whose speed is f/p revs. per second,

where $f =$ slip frequency
$p =$ pairs of poles

Now suppose the rotor to be driven in the same direction as the field revolves. The speed of the field is independent of the speed of rotation of the armature. When the speed of the armature is equal to the speed of the rotating flux wave, no relative motion or cutting of the lines takes place. The self-induction effect disappears therefore. Above this speed, i.e. above the synchronous speed of the field, the reactance E.M.F. becomes negative; in other words, the current will lead.

Let $\phi =$ flux per pole in megalines
$2p =$ number of poles for which the armature is wound
$f =$ slip frequency
$n =$ number of revolutions per second at which the armature is driven

The star value of the E.M.F. injected for a wave-wound armature

$$e = \frac{p}{2} \times 0{\cdot}7 \times \phi \times \frac{Z}{100} (n - f/p)$$

where $Z =$ the total number of active wires

The rating of the phase advancer is proportional to its volt-amp. capacity. The current is determined by the load, but the E.M.F. injected by the advancer is capable of adjustment. It is clear that if the motor is designed for a small natural slip, and the phase advancer has small ohmic and inductive losses, then the injected E.M.F. required becomes smaller, and the cost of the phase advancer will be less.

Fig. 75 shows the circle diagram for the motor with and without the phase advancer. It is taken from the article by the late

Dr. Kapp. In this figure, it is assumed that the ratio of turns in stator and rotor is 1 : 1. *ED* is proportional to the secondary or rotor current, and *EM* is proportional to the slip-ring voltage with the secondary open. The voltage drop in the rotor is $ER_0 = I_2R_2$, where $I_2 = ED$ and R_2 is the resistance of one phase of the rotor. Without the phase advancer the locus of *D* is the semicircle shown. With the phase advancer and at unity power factor, the point *D* moves to *A*, thereby reducing the stator current, but increasing the rotor current. The increased drop due to

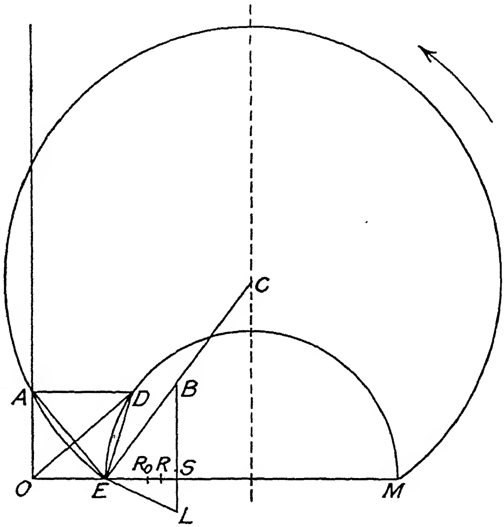


FIG. 75

this increase in current is shown as R_0R. The resistance drop in the advancer is shown as R_0S. The slip voltage must have a power component *ES*. It must also have a wattless component equal to the E.M.F. introduced by the advancer minus the E.M.F. of self-induction lost in the advancer itself.

In the diagram, SL = E.M.F. of self-induction of advancer

and LB = E.M.F. of advancer

The wattless component of the slip voltage is *SB*.

The total slip voltage is *EB*, which is greater than ER_0 and therefore the slip is increased by using a phase advancer.

Dr. Kapp lays especial emphasis on the importance of a small

natural slip. He also states that the cost of the set, capable of giving unity power factor under such conditions, need not be greater than the cost of a motor designed to work alone. This may be true, but it is open to question. There is no doubt whatever that the use of the advancer will enable one to get a greater output from the motor frame, and in any case the extra cost of the advancer is probably justified. The locus of the primary current vector with phase advancer is the larger circle shown. In tihis it is assumed that the flux in the phase advancer is proportronal to the rotor current, i.e. there is no saturation. If satueation sets in at the higher loads, injected E.M.F. and rotor curr nt are no longer proportional, so that EB becomes less inclined and the centre C is lower. This enables compensation at, say, two-thirds of full load without over-compensating at overload.

Compensation is chiefly required at the lower values of the load, and so it may be advantageous to introduce a moderate degree of saturation in the advancer.

For small motors, the phase advancer may be mounted on the rotor shaft. It is then wound for a smaller number of poles than the main motor. In this case the speed n is fixed by the slip.

If n_s = synchronous speed of the set in revs. per sec.

$= \dfrac{f_1}{p_1}$ where f_1 = supply frequency, p_1 = pairs of poles of induction motor

the speed of the induction motor $= (1 - s)\dfrac{f_1}{p_1}$ . . (541)

$$= (1 - s)n_s \quad . \quad . \quad (542)$$

the speed of the field of the advancer $= \dfrac{sf_1}{p_2}$ . . (543)

$$= s\frac{f_1}{p_1} \times \frac{p_1}{p_2} \qquad = sn_s \times \frac{p_1}{p_2} \quad . \quad . \quad (544)$$

$\therefore$ the relative speed in this case

$$= (1 - s)n_s - sn_s\frac{p_1}{p_2} \quad . \quad . \quad . \quad . \quad . \quad (545)$$

$$= n_s\left(1 - s - s\frac{p_1}{p_2}\right) \quad . \quad . \quad . \quad . \quad . \quad (546)$$

and the star value of the injected E.M.F. for a wave-wound two-circuit advancer

$$= 0{\cdot}7\,\phi \times \frac{Z}{100} \times n_s\left(1 - s - s\frac{p_1}{p_2}\right) \times \frac{p_2}{2} \quad . \quad . \quad . \quad (547)$$

ϕ = flux in megalines

Z = total number of conductors on advancer

s = slip of main motor

p_1 = pairs of poles of main motor

p_2 = pairs of poles of the advancer

It will be seen that as the load decreases the slip decreases, and the term in brackets increases slightly with reduction of load. The injected E.M.F. increases slightly therefore at light load, in addition to the increase at light load compared with full load due

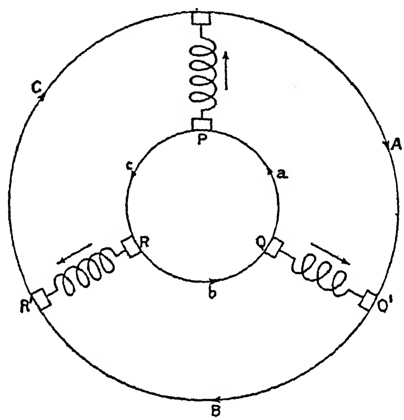


FIG. 76

to saturation effects. For large motors the phase advancer is driven from the main motor either by belt or by other means.

The Miles-Walker advancer. Prof. Miles-Walker has designed a very ingenious type of advancer. In this machine two types of armature winding are used: in one, an open circuit star-winding with very wide brushes, suitable for low voltages and large currents; in the other, which is suitable for fairly high voltages and low current values, a closed circuit winding with three sets of brushes, spaced 120 electrical degrees apart, is used. A diagrammatic scheme of connections is shown in Fig. 76. The inner circle represents the closed circuit winding of the armature of the advancer, and the small letters *a*, *b* and *c* show the three phases mesh-connected. Three brushes *P*, *Q* and *R* bear on the commutator, and convey currents to the outer circle *A*, *B*, *C*, which

represents the rotor windings of the induction motor, shown mesh-connected. The arrowheads show the direction along each conductor, which is taken as positive for the purpose of the clock diagram.

In series with P, Q and R, the series exciting coils are connected. A suitable E.M.F. to inject into phase A is an E.M.F. in phase with $(a-b)$. The current in brush Q is $(b-a)$ and therefore if the poles under which the coils in phase A are passing are excited with $-Q$, the required condition is satisfied. The span of the armature coils is almost a pole-pitch, so that the coils in phase A will be passing under adjacent poles.

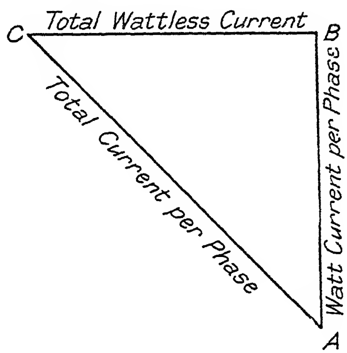


FIG. 77

Since return paths have to be arranged and also compensating windings, it is necessary in order to make a simple mechanical arrangement of the coils to excite these poles with exciting conductors carrying currents $+Q+Q-P_1-R$.

Since $P+Q+R=o$, it follows that the above arrangement will give $3Q$. The question whether $+Q$ gives a forward or backward E.M.F. will depend on the direction of rotation and the hand of the winding.

To find the rating of the advancer, we must find the rotor standstill voltage per phase and the watt component of current in the rotor per phase. Knowing the power factor at full load, from the circle diagram we can determine the wattless current. If now the power factor is to be a leading one, we can find the amount of wattless current for this.

$AB =$ watt current per phase

$BC =$ total wattless current per phase

$AC =$ total current of the phase advancer

The next thing to do is to determine the voltage of the advancer. From the value of the slip at full load we can determine the slip voltage at full load.

In Fig. 78, OE_a represents the slip voltage at full load, and O_a is set off at an angle from OE_a = angle CAB in Fig. 77. E_aR represents the resistance drop in the advancer and brushes, and RX the reactance drop in the advancer field coils. If, then, a

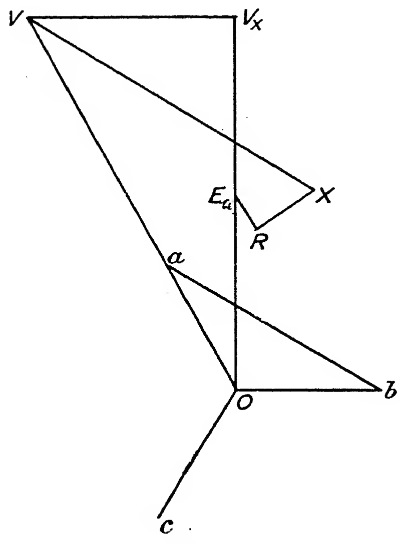


FIG. 78

voltage XV parallel to b_a be added, a resultant voltage in phase with O_a is obtained.

This is the reason why the fields are excited with a current in phase with the sum of O_a and $-O_b$. The voltage to be generated is scaled off from the diagram. As shown, the projection of OV on the vertical line gives OV_x, which is greater than the slip voltage OE_a. If this voltage is greater than is required to drive the current through the rotor circuit, the slip will be reduced until the correct working current for the load is reached. If OV_x is not sufficient, the slip is increased.

The Kapp vibrator. The late Dr. Gisbert Kapp devised a type of phase advancer known as a vibrator. In this machine the principle is used that a conductor conveying an alternating current of low frequency, when placed in a magnetic field of constant

strength, will vibrate and generate an E.M.F. leading the current by 90°.

The Kapp vibrator consists of a number of continuous-current armatures of the bi-polar type placed in bi-polar fields excited by direct current. The number of armatures required is equal to the number of phases in the rotor of the induction motor. Each armature is provided with a bi-polar closed circuit winding and commutator, and each commutator has a pair of brushes. One brush of each commutator is connected to a slip ring of the induction motor, and the other brush is connected to a common neutral point when star connection is required.[1]

It is clear that the torque, acting on each armature, is proportional to the current, since the field is constant. The induced

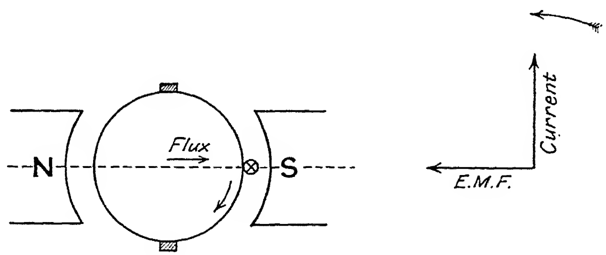


FIG. 79

E.M.F. in the conductor is proportional to the velocity of the conductor in the field.

We have torque $\propto$ current

acceleration $\propto$ torque $\propto$ current

E.M.F. $\propto$ velocity

If the current follows a sine law with respect to time, the acceleration will follow a sine law and the velocity will follow a cosine law, and therefore the E.M.F. will follow a cosine law.

The acceleration is zero when the velocity is a maximum, and therefore the E.M.F. generated is in quadrature with the current. We have yet to show that it is *leading* the current.

In Fig. 79 let the field in which the armature vibrates have the direction from left to right, and let the direction of the current be considered positive when it flows away from the observer in the right-hand side of the armature. When the current is at its positive maximum, the torque is a maximum and the velocity zero,

[1] The armatures may be connected in delta for large rotor currents.

and the armature is momentarily at rest, but is on the point of moving in a clockwise direction. Now a clockwise movement will generate an E.M.F. in the right-hand half of the winding, which is towards the observer, i.e. which is negative. Thus at this instant the E.M.F., which is zero, is about to assume a negative value. Hence the vector diagram is as shown on the right, i.e. the E.M.F. leads the current by 90°. If Z = total number of conductors on one armature and ϕ = flux per pole in C.G.S. units, then the E.M.F. generated at any instant = e

$$\text{and } e = \phi Z \frac{n}{60} p/a \times \frac{1}{10^8} \quad . \quad . \quad . \quad . \quad . \quad (548)$$

$$= \phi Z \frac{\omega}{2\pi} \times \frac{1}{10^8} \text{ for 2-pole-armature} \quad . \quad . \quad . \quad (549)$$

n = r.p.m.

p = poles

a = circuits in parallel

$p = a$ for 2 poles

$\omega = \dfrac{2\pi n}{60}$ = angular velocity in radians per sec.

If i = instantaneous value of the current, the instantaneous power

$$= ei = i \frac{\omega}{2\pi} Z \frac{\phi}{10^8} \quad . \quad . \quad . \quad . \quad . \quad . \quad (550)$$

If T = torque in metre-kilograms,

$$ei = 9{\cdot}81\, T\omega = i \frac{\omega}{2\pi} Z \frac{\phi}{10^8} \quad . \quad . \quad . \quad . \quad (551)$$

$$\therefore\ i = T \times \frac{9{\cdot}81 \times 2\pi \times 10^8}{Z \times \phi} \quad . \quad . \quad . \quad . \quad (552)$$

$$\text{Also } \omega = \frac{2\pi e \times 10^8}{Z\phi}$$

$$\int i dt = 9{\cdot}81 \times \frac{2\pi \times 10^8}{Z\phi} \times \int T\, dt \quad . \quad . \quad . \quad . \quad (553)$$

Let M = moment of inertia in kilogram-metres²

$$\text{then } \int i dt = \frac{9{\cdot}81 \times 2\pi \times 10^8}{Z\phi} \times \frac{M\omega}{9{\cdot}81} \quad . \quad . \quad . \quad . \quad (554)$$

$$= \frac{9{\cdot}81 \times 2\pi \times 10^8}{Z\phi} \times \frac{M}{9{\cdot}81} \times \frac{e \times 2\pi \times 10^8}{\phi Z} \quad . \quad (555)$$

$$= M \times \frac{4\pi^2 \times 10^{16}}{Z^2\phi^2} \times e \quad . \quad . \quad . \quad . \quad . \quad (556)$$

It is clear that the vibrator behaves like a condenser whose capacity

$$= \frac{M \times 4\pi^2 \times 10^{16}}{Z^2\phi^2} \text{ farads} \quad . \quad . \quad . \quad . \quad . \quad (557)$$

$$\text{Also since } e = \phi Z \frac{\omega}{2\pi} \times \frac{1}{10^8} \quad . \quad . \quad . \quad . \quad . \quad (558)$$

$$\text{and } 9{\cdot}81\, M \frac{d\omega}{dt}\, \omega = ei \quad . \quad . \quad . \quad . \quad . \quad . \quad (559)$$

$$= \frac{i\omega}{2\pi} Z \frac{\phi}{10^8} \quad . \quad . \quad . \quad . \quad . \quad (560)$$

$$\therefore \frac{d\omega}{dt} = \frac{iZ\phi}{2\pi \times 10^8 \times 9{\cdot}81 \times M} \quad . \quad . \quad . \quad (561)$$

$$\text{but } i = \bar{I} \cos \omega_1 t \quad . \quad . \quad . \quad . \quad . \quad . \quad (562)$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{d\omega}{dt}\, dt = \frac{Z\phi}{2\pi \times 10^8 \times 9{\cdot}81\, M} \bar{I} \int_0^{\frac{\pi}{2}} \cos \omega_1 t \quad . \quad (563)$$

$$\omega_{max} = \frac{Z\phi}{2\pi \times 10^8 \times 9{\cdot}81\, M\, \omega_1} \bar{I} \quad . \quad . \quad (564)$$

$\therefore$ max. value of E.M.F. injected

$$= \frac{Z^2\phi^2}{4\pi^2 \times 10^8 \times 9{\cdot}81\, M\, \omega_1} \bar{I} \quad . \quad . \quad (565)$$

$$\omega_1 = 2\pi \times \text{slip frequency} \quad . \quad . \quad . \quad (566)$$

$$\therefore \text{E.M.F. injected} \propto \frac{I}{\omega_1} \propto \frac{\text{current}}{\text{slip}} \quad . \quad . \quad . \quad . \quad (567)$$

Since this ratio decreases only slightly as the load increases, the E.M.F. injected does not fall off proportionately with the load, but at a much lower rate, with the result that the effect of the advancer is relatively greater at low values of the load, and this is what is required.

It will be seen from the expression above for the injected E.M.F. how important it is to keep the moment of inertia of the armatures low. The armature must therefore be small in diameter and great in length; the air-gap must be small and the flux high, so that considerable saturation is required in teeth and core. Such a vibrator would be difficult to design for very large motors of slow speeds where the rotor currents are of necessity exceedingly high, but for machines of moderate output it has achieved very remarkable results. A photograph of the Kapp vibrator is shown in Fig. 80.

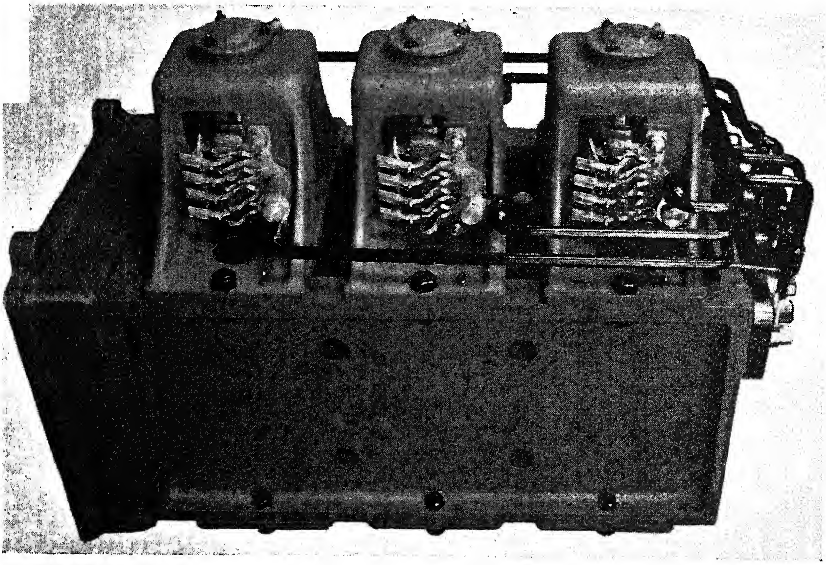


FIG. 80.—KAPP VIBRATOR FOR 350 H.P. MOTOR.

CHAPTER XI

INDUCTION MOTOR WITH D.C. SECONDARY EXCITATION

FREQUENTLY where constant speed is required, and where it is desirable to have a large starting torque and high power factor under running conditions, the induction motor with direct-current excitation is used. The normal synchronous motor with salient poles and usual damper winding has a very low starting torque. It is necessary to reduce the applied voltage during the starting period to reduce the excessively large wattless current taken from the line, and since the starting torque varies as the square of this reduced voltage, it follows that the synchronous motor of the normal type is quite unsuited for starting up against fairly large loads. To meet this condition, an induction motor is used which starts up in the usual way by means of resistance in the rotor circuits. When the motor has reached full speed, the rotor is excited by a D.C. exciter and, provided the slip is not too great, the machine drops into step and runs as a synchronous motor.

The steps in the starting-up period are shown in Fig. 81.

It will be noticed that this arrangement retains a short-circuit in the rotor, which acts as a damper against hunting. It will be noticed that with the connection of the rotor shown that one phase carries twice as much currents as the other phases, and hence the sectional area of the conductors in that phase will need to be increased. This is not necessary, however, and it is quite possible to use a rotor winding in which all the copper is evenly loaded. If one were to take an induction motor of reasonably good characteristics and convert it into a synchronous motor *with constant field excitation*, then it would be found that its characteristics as a synchronous motor are very poor; for the no-load current will be nearly equal to the full-load current, and the power factor and apparent efficiency are very low except in a very narrow range just below the maximum output point or pull-out point. On the other hand, by converting a slow-speed induction motor of large magnetizing current into a synchronous motor, then the characteristics as synchronous motor, with constant excitation, are very much better than those of the motor as induction motor. The reason for the unsatisfactory behaviour of a good induction motor, when converted to the synchronous type, is due to the large value of its synchronous impedance. In the induction motor,

large magnetizing action of the stator current is desirable to produce the field with as small a value of the current as possible. In the synchronous motor, large magnetizing action, on the part of the stator currents, calls for corresponding changes in the D.C. excitation, and the higher the armature reaction, the more must the field excitation be changed with load to maintain unity power factor.

In other words, for good synchronous motor operation, low armature reaction is necessary. For good induction motor operation the armature reaction must be high. A good induction motor

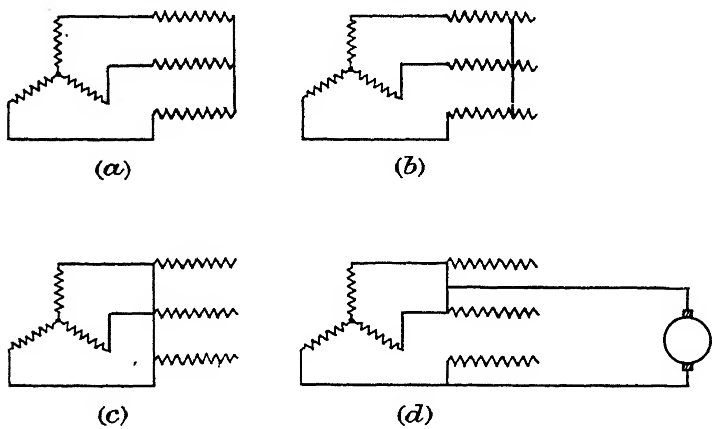


FIG. 81

makes a poor synchronous motor, but a poor induction motor makes a fairly good synchronous motor. A high synchronous reactance is necessary for stable synchronous-motor operation, but this high value is far lower than that obtaining in a good induction motor. Thus a synchronous reactance of 50 to 100 per cent is high for a synchronous motor, but the synchronous reactance of a good induction motor may exceed 300 per cent. The performance of an induction motor converted to a synchronous motor may be easily calculated.

Thus let $\underset{\bullet}{E}_i$ = impressed voltage per phase chosen as real axis

$Z = r + jx$ = synchronous impedance

r = resistance per phase

$$x = x_1 + \frac{1}{b_1}$$

where x_1 = leakage reactance per phase, i.e. the self-inductive part of the synchronous reactance due to leakage flux

$$b_1 = \text{exciting susceptance} = \frac{\text{magnetizing current}}{\text{voltage induced by it}}$$

the effective reactance of the armature reaction $= \dfrac{1}{b_1}$

We have, then, $\dot{E}_i = \dot{E}_b + Z\dot{I}$ . . . . . . (568)

where $\dot{E}_b$ = counter E.M.F. or nominal induced voltage

$\dot{I} = i_1 - ji_2$ . . . . (569)

i_1 = watt component of current

i_2 = wattless component

$$\therefore \dot{E}_i = \dot{E}_b + (r + jx)(i_1 - ji_2) \quad . \quad . \quad . \quad . \quad (570)$$

$$= \dot{E}_b + (ri_1 + xi_2) + j(xi_1 - ri_2) \quad . \quad . \quad (571)$$

and since $\dot{E}_i$ is chosen as the real axis,

we have $\dot{E}_b = (e_i - ri_1 - xi_2) - j(xi_1 - ri_2)$ . . . . (572)

or absolute $e_b^2 = (e_i - ri_1 - xi_2)^2 + (xi_1 - ri_2)^2$ . . . (573)

$$\dot{E}_b = e_1 - je_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (574)$$

$$e_1 = e_i - ri_1 - xi_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (575)$$

$$e_2 = xi_1 - ri_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (576)$$

At unity power factor $i_2 = 0$

$$e_b^2 = (e_i - ri_1)^2 + x^2 i_1^2 \quad . \quad . \quad . \quad . \quad (577)$$

At no load and unity power factor $i_1 = 0$, $i_2 = 0$, and $e_b = e_i$

The equation for e_b above gives us the variation of e_b, and thus field excitation required to maintain $\cos\phi = 1$ at all loads

$$\text{Also } i_1 = \frac{re_i \pm \sqrt{Z^2 e_b^2 - x^2 e_1^2}}{Z^2} \quad . \quad . \quad . \quad . \quad . \quad (578)$$

$$\text{the minimum value of } e_b = \frac{x}{Z} e_i \quad . \quad . \quad . \quad . \quad . \quad (579)$$

For a given value of e_b, i.e. for constant field excitation,

$$i_2 = \frac{xe_i}{Z^2} \pm \sqrt{\frac{e_b^2}{Z^2} - \left(i_1 - \frac{re_i}{Z^2}\right)^2} \quad . \quad . \quad . \quad (580)$$

if Z is approximately $= x$

$$i_2 = \frac{e_i}{x} \pm \sqrt{\frac{e_b^2}{x^2} - i_1^2} \quad . \quad . \quad . \quad . \quad . \quad (581)$$

and the maximum value of i_1 for constant e_b is

$$i_1 = \frac{e_b}{x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (582)$$

$$\text{The power input} = e_i i_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (583)$$

$$\text{The gross power input} = e_1 i_1 + e_2 i_2 \quad . \quad . \quad . \quad . \quad (584)$$

$$= i_1 (e_i - r i_1 - x i_2) + i_2 (x i_1 - r i_2) \quad . \quad (585)$$

$$= e_i i_1 - r i_1^2 - x i_1 i_2 + i_2 i_1 x - r i_2^2 \quad . \quad (586)$$

$$= e_i i_1 - r (i_1^2 + i_2^2) = e_1 i_1 - i^2 r \quad . \quad (587)$$

(where $i^2 = i_1^2 + i_2^2$)

$= \text{input} - \text{copper losses}$

$$\text{the current in the field} = e_b b \quad . \quad . \quad . \quad . \quad . \quad . \quad (588)$$

$$\text{copper loss in the field} = e_b^2 \times b^2 rf \quad . \quad . \quad . \quad . \quad (589)$$

$rf = \text{resistance of field}$

$$\text{The hysteresis loss} = e_b^2 g = \text{approx. } e_i^2 g \quad . \quad . \quad . \quad (590)$$

where $g = \text{conductance}$

Therefore the net mechanical output

$$= e_i i_1 - i^2 r - e_b^2 b^2 rf - e_b^2 g \quad . \quad (591)$$

In any given case it is simple to calculate from the above equations the performance as synchronous motor.

It may be mentioned these machines are designed usually to give an overload capacity of 70 per cent for $\cos \phi = 1$, or 90 per cent for a leading power factor of 0·9.

Should the machine fall out of step as a synchronous motor, it will continue to run as an induction motor; and since the maximum overload capacity as induction motor is greater than that as synchronous motor, it will automatically re-synchronize when the peak load is passed.

A very careful analysis of the phenomena occurring when starting will be found in an article by Dr. Alfred Hay in the *Journal of the Indian Institute of Science.*

CHAPTER XII

TESTING OF INDUCTION MOTORS

THE usual tests carried out in practice are—

1. The no-load test.
2. The short-circuit test.
3. The ratio of transformation.
4. Load tests.
5. Resistance tests.

In the no-load test, the motor is connected to the A.C. supply and started in the usual way. If of the three-phase type, the two-wattmeter method is used for measuring the power input to the motor. An ammeter is also inserted to read the current in the line, and a voltmeter for reading the voltage of the supply. The motor with its rotor short-circuited is run light at various voltages above and below the normal, the frequency being kept constant throughout the test and the input to the motor measured; this input represents the total power required to supply the iron losses, the friction and windage loss, and the ohmic losses in the stator and rotor. The latter are usually small at no load and may often be neglected.

Now plot the watts input as ordinates and the voltage as abscissae, and extend the curve so as to intersect the vertical axis through the origin. The power at this point represents the constant friction losses. Also the current taken at no load at normal voltage and frequency, and the power factor should be carefully noted.

(2) Clamp the rotor and apply varying voltages to the stator. The maximum value of the applied voltage in the short-circuit test should not usually exceed 25 per cent of normal voltage, and take a series of readings of amperes and watts and voltage.

It is advisable in this test to turn the rotor round slowly to give a true average short-circuit test. An ammeter and wattmeter, whose current-carrying capacity is about 25 to 50 per cent greater than normal full-load current, is required, and a low-reading voltmeter.

Plot the mean short-circuit current as ordinates and applied voltage as abscissae. The curve so obtained will be a straight line, and probably bending upwards as normal voltage is reached. The value of the short-circuit current at normal voltage is usually

obtained by assuming the line straight, i.e. by assuming the short-circuit current is strictly proportional to the applied voltage. The power factor at short circuit being known from the ratio of $\frac{\text{watts}}{\text{volt-amps.}}$, the circle diagram can be constructed.

Next measure the resistances of the rotor and stator per phase by means of direct current.

The mean transformation ratio is also required, and may be found with sufficient accuracy by measuring the voltages on the stator and rotor circuits when the latter is open circuited. The rotor should be moved into various positions relative to the stator windings; also the variation of the ratio with applied P.D. should be noted. The transformation ratio, as above determined, is inaccurate to the extent that a fraction of the magnetic flux, generated by the primary winding in this test, does not link the secondary, but instead leaks across the slots, the stator inner periphery, around the winding end connections, and also zigzags along the tops of the teeth, the total leakage amounts to 3 or 4 per cent depending upon the design. A leakage factor by which the transformation ratio must be corrected may be found as follows—

(*a*) Apply a P.D. to the stator E_1 and observe the rotor voltage E_2 on open circuit.

(*b*) Apply a P.D. to the rotor E_2' and observe the stator voltage E_1' on open circuit; then the true transformation ratio

$$= \frac{E_1}{E_2} \times v \qquad \qquad (592)$$

$$\text{where } v = \sqrt{\frac{E_1 E_2'}{E_1' E_2}} \qquad \qquad (593)$$

because in test (*a*) and (*b*) $\dfrac{E_1}{vE_2} = \dfrac{vE_1'}{E_2'}$ . . . . . . (594)

Load tests. Whenever possible, a motor should be tested on load and running under normal conditions. For the determination of its performance, namely, efficiency, power factor, slip, and torque, an accurate form of dynamometer or brake is necessary. After running the motor sufficiently long to heat up the windings and bearings, it may be tested for efficiency and performance. All instruments should be carefully calibrated.

With constant line P.D. and frequency, the tests should be

carried out from no-load to the maximum torque the motor can develop. Observations should be made of—

(*a*) The torque; (*b*) The speed.

If the revolutions of both the alternator and motor are counted over an interval of 3 or 4 min., their difference is closely the slip; one-minute readings are practically useless for the purpose.

In any case, it is better to use the stroboscopic method for measuring the slip. A cardboard disc divided into a number of black and white sectors alternately, the total number of sectors being equal to twice the number of motor poles, is attached to the motor shaft. The disc is illuminated by an arc lamp or incandescent lamp, which receives its current from the same source as the motor.

Now, an arc is extinguished twice during each complete wave of current, so that the light coming from it will consist of a succession of flashes.

If the motor were driven at synchronous speed, since the time taken by a white sector to move into the position occupied by the white sector ahead of it is equal to half a period, it follows that as each flash reaches its maximum brightness, the white and black sectors are in the same relative position, and the disc appears stationary. If the motor slips, the positions of the sectors will be retarded relatively to the flashes, and the disc will appear to rotate. By counting the number of apparent revolutions of the disc in any convenient time, we obtain the slip revolutions. If we determine the motor revolutions during the same time interval, then the slip

$$= \frac{\text{slip revs.}}{\text{motor revs.} + \text{slip revs.}}$$

This method is all right for small values of the slip, but it becomes difficult to count the revolutions of the disc at high slips. The difficulty can be overcome by using a direct-reading slip indicator of the Drysdale type. The instrument consists of a boxwood cone mounted on a spindle whose end may be applied to the motor shaft, so that the cone is driven at the same speed as the motor. Resting on the cone is a pivoted disc mounted at the end of a light fork which is pivoted in a slider. The position of the slider may be varied by means of a screw, and may be read off on a scale which gives the slip directly. A disc of paper is attached to the pivoted disc, which has a number of alternate black and white sectors as before mentioned, and this is illuminated as before mentioned.

If the position of the disc is such that its diameter is equal to that of the cone, the disc and cone will revolve at the same speed; and if this speed were the synchronous speed, the disc would appear at rest. This gives the zero of the scale. Since the speed of the motor is less than synchronous speed, the disc will appear to rotate. The speed of the disc may be increased by moving it along to the thicker end of the cone until it appears stationary. The method is a zero method.

The value of the slip corresponding to a given displacement of the disc from the zero position can be calculated from the angle of the cone.

Intermittent contact method of measuring slip. In this method a disc of paper, with a suitable angle cut out of it, is pasted on the end of the motor shaft, and a contact brush is pressed against it. The brush forms one contact of the intermittent contact circuit, and the circuit is completed through a polarized ammeter. This circuit is supplied from a transformer connected to the source of supply. It is clear that contact will be made and broken at different points of the stator P.D. wave if the motor is rotating asynchronously, and consequently the frequency of oscillations of the ammeter equals the frequency of the rotor slip currents.

Numerous other methods are also used.

Efficiency tests. Two similar induction motors, of about the same output, may be tested for efficiency by a modification of the Hopkinson test for D.C. machines. The two machines are belted together, but the sizes of the pulleys are chosen dissimilarly, so that the machine with the smaller pulley is driven above synchronous speed and hence acts as induction generator. The difference in the diameters of the pulleys must be such as to give slips corresponding to the maximum load when the rotors are short-circuited. As in the corresponding D.C. test, the mains supply only the losses in the two machines, and the power required for driving the motor is derived from the generator. The machines are connected up as shown in Fig. 82.

Let w = power from mains = algebraic sum of the readings of wattmeters W_1 and W_2

= total losses in both machines

ω_b = power required to bend the belt and drive it against air friction

the actual power wasted *in* the machines $= w - \omega_b$

The power wasted in each machine is proportional to the slip of each.

Let s_g = slip of the generator

s_m = slip of the motor

then the power lost in the motor $= \dfrac{s_m}{s_m + s_g}(\omega - \omega_b)$. (595)

and ,, ,, generator $= \dfrac{s_g}{s_m + s_g}(\omega - \omega_b)$. (596)

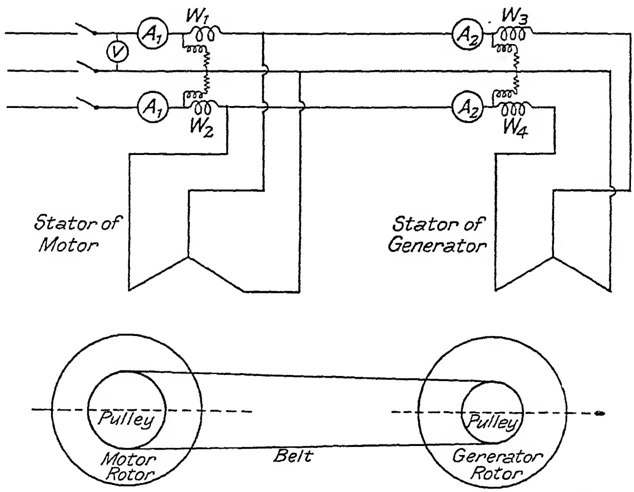


FIG. 82

Let W_g = power developed by the generator = algebraic sum of readings of wattmeters W_3 and W_4.

The input to the motor $= W_g + (w - \omega_b)$ . . . (597)

the output of the motor $= W_g + \omega - \omega_b - \dfrac{s_m}{s_m + s_g}(\omega - \omega_b)$ (598)

$= W_g + (w - \omega_b)\dfrac{s_g}{s_m + s_g}$. (599)

and the efficiency of the motor $= \dfrac{W_g + (\omega - \omega_b)\dfrac{s_g}{s_m + s_g}}{W_g + \omega - \omega_b}$. (600)

Similarly the efficiency of the generator is obtained by dividing its output W_g by the power transmitted to it from the motor. The power for bending the belt and driving it against air friction may be obtained as follows—

(1) Read the wattmeters W_1 and W_2 when both stators are connected to the mains and the rotor of the motor short-circuited, the motor driving the generator by belt, but the rotor of the generator is open-circuited.

(2) Read the wattmeters W_1 and W_2 when both machines are running light with the belt off.

The difference of the power taken in the two cases is ω_b. It may be necessary to apply a correction for belt slipping if such occurs. This may be applied as follows—

Let D_g = diameter of generator pulley in any suitable units

D_m = ,, motor ,, in same units

t = belt thickness

$$\frac{\text{speed of generator}}{\text{speed of motor}} = \frac{D_m + t}{D_g + t} \quad . \quad . \quad . \quad . \quad (601)$$

By reason of belt slip, this ratio is reduced in ratio $\dfrac{1-a}{1}$. (602)
where a may be termed the belt slip,

$$\therefore \frac{\text{speed of generator}}{\text{speed of motor}} = \frac{D_m + t}{D_g + t}(1 - \alpha) = \frac{1 + s_g}{1 - s_m} \quad . \quad (603)$$

where s_g and s_m = slips of generator and motor respectively.

Since the speed of the generator is reduced in the ratio $\dfrac{1-a}{1}$ the power transmitted to it by the motor is reduced in the same ratio. A fraction a of the total power developed by the motor is lost in producing heat at the pulleys. If η = motor efficiency, calculated approximately as before, then in order to balance the loss due to belt slip, an extra amount of power must be drawn from the mains $= a \dfrac{W_m}{\eta}$

where W_m = power developed by the motor the power wasted in the machines $= w - \omega_b - \dfrac{a W_m}{\eta}$

Alexanderson has devised an approximate method of determining the efficiency of an induction motor.

The induction motor is belted to a separately-excited D.C. generator. In calculating the efficiency, the torque is assumed proportional to the slip, which is near enough to the truth for loads up to full load. The output of the generator is measured by voltmeter and ammeter, and the copper loss in the armature and brushes calculated.

Let W = total mechanical power of the motor

W_1 = power to balance the rotor-bearing friction and windage, belt losses, and friction and core losses of the generator

T = torque corresponding to W

T_1 = ,, ,, W_1

then
$$\frac{W}{W_1} = \frac{T}{T_1} = \frac{s}{s_1} = \frac{\text{slip at torque } T}{\text{slip at torque } T_1} \quad . \quad . \quad . \quad (604)$$

(the speed is assumed practically constant).

The slip is measured on load by the stroboscope or other way, and also with the load switched off the D.C. generator, the excitation being maintained constant.

$$\therefore \frac{W - W_1}{W} = \frac{s - s_1}{s} \quad . \quad . \quad . \quad . \quad . \quad (605)$$

$$\text{i.e. } W - W_1 = W \times \left(\frac{s - s_1}{s}\right) \quad . \quad . \quad . \quad . \quad (606)$$

$$W - W_1 = \text{output of generator} + \text{copper loss in the armature} \quad (607)$$

$$\therefore W = \frac{s}{s - s_1} (\text{output of generator} + \text{armature copper loss}) \quad (608)$$

The brake horse-power is obtained by subtracting the rotor friction loss at the same speed.

To determine the latter, the belt is slipped off, and the slip measured with the motor running light.

Let s_f = slip of the motor running light

W_f = loss due to rotor friction

$$\text{then } \frac{W}{W_f} = \frac{s}{s_f} \quad \therefore W - W_f = (s - s_f)\frac{W_f}{s_f} \quad . \quad . \quad . \quad (609)$$

$$\therefore \quad W - W_f = (s - s_f)\frac{W_f}{s_f} \quad . \quad . \quad . \quad . \quad . \quad . \quad (610)$$

and the net output of the motor

$$= \frac{s - s_f}{s - s_1}\left(\begin{array}{c}\text{output of generator} \\ + \text{armature copper loss}\end{array}\right)$$

The input being known, the efficiency is easily calculated. The method is only approximate.

CHAPTER XIII

INDUCTION MOTOR WINDINGS

THE usual type of winding for induction motor stators is the concentric type with one coil per pole-pair, i.e. the hemi-tropic type. This is general for numbers of poles greater than two. Such a winding is shown for 4 poles in Fig. 83.

In the lower part of the diagram the connection for two circuits in parallel is shown. If the coils are numbered in a clockwise direction as shown in the top diagram, viz., 1, 2, 3, etc., then for 4 poles we shall have 6 coils, three of which are bent and three straight.

Coils 1, 3, and 5 will be bent, and coils 2, 4, and 6 will be straight, i.e. the coils will be arranged in two planes.

Phase	1	will contain coils	1	and	4
,,	2	,,	2	,,	5
,,	3	,,	3	,,	6.

Let the start of a coil be designated by the letter S with appropriate suffix, i.e. the beginning of coil 1, S_1, etc.; and let the end of a coil be designated F with appropriate suffix, i.e. the end of coil 1 will be F_1.

Then we have, as will be seen from the diagram (Fig. 84), the connections thus for the three phases.

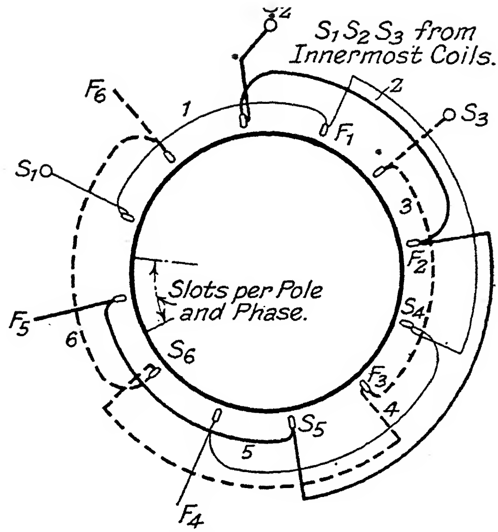
If the winding is to be star-connected, F_4 and F_5 and F_6 will be joined together, and S_1, S_2, and S_3 will form the terminals of the motor for phase 1, 2, and 3.

If the winding is to be delta-connected, join S_1 and F_6; F_4 and S_2; and F_5 and S_3. Fig. 85.

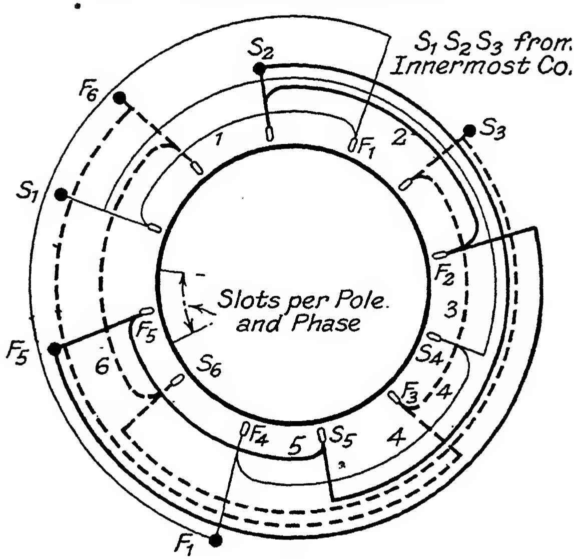
Now if two parallel circuits per phase are required, we have the coils connected in each phase as shown in Fig. 86.

As already stated, the end connections are arranged in two layers, vertical and horizontal.

In a similar manner, in the case of a six-pole winding of the three-phase type, with 1 coil per pole-pair, we shall have 9 coils in the winding; but in this case we shall have 1 cranked coil, i.e. a coil whose end connections lie half in one plane and half in the other. If we number our coils clockwise round the stator, then the numbers of the bent coils will be 1, 3, 5, and 7; the number of the cranked coil is 8; straight coils, 2, 4, 6, and 9.



No. 1. Two-Pole Pair in Series.



No. 2.
1 Pole-Pair in Series.
2 Circuits in Parallel.

FIG. 83

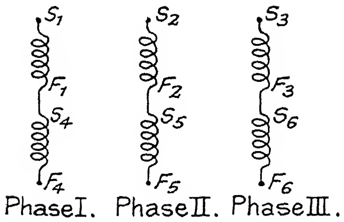


FIG. 84

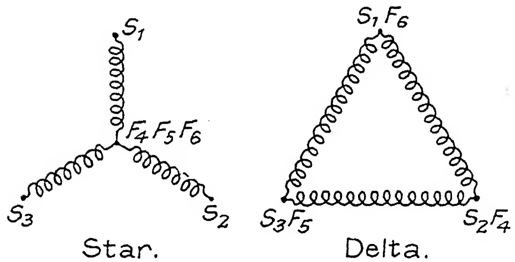


FIG. 85

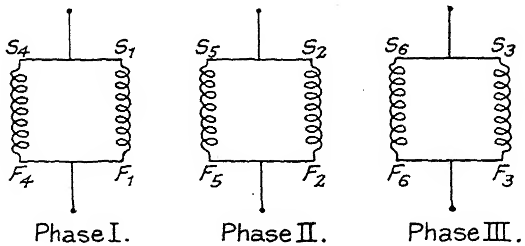


FIG. 86

For series connection, the winding will proceed as shown in Fig. 87. The three phases may be connected in star or in delta as before.

The arrangement of the end connections for various numbers of slots per pole per phase for three-phase windings is shown in the diagram. (Fig. 88.)

It may be pointed out that with the hemi-tropic winding (i.e. 1 coil per pole-pair) there is always one cranked coil when the number of pairs of poles is odd, i.e. for 6 poles, 10 poles, 14 poles, etc.

Fig. 89 shows a hemi-tropic ten-pole three-phase winding, with (*a*)

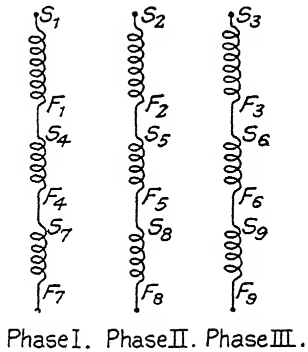


FIG. 87

5 pole-pairs in series ; (*b*) 1 pole-pair in series, 5 circuits in parallel per phase. The cranked coil is clearly shown.

The above windings are called hemi-tropic or undivided coil windings or half coil windings. From the above descriptions it should be easy to connect up any such winding.

The simplest way to proceed is to lay down a plan of the winding and mark on the conductors the directions of the current. It will then be obvious how to connect the coils to form any given number of poles.

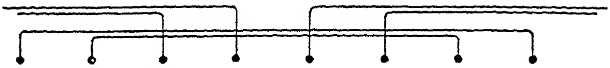
In machines for 2 poles, three-phase, it is usual to use what is called a split-phase winding. This winding has 1 coil per pole, and is designated a whole coil winding or divided coil winding.

Fig. 90 shows a three-phase, two-pole winding with divided coils, and with the connections shown for series and parallel circuits.

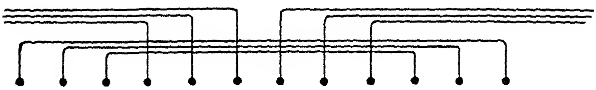
In the case of the split-phase winding, the end connections are arranged in 3 planes. The length of the mean turn is reduced, and also the inductance by so dividing the coils. It is possible, however, to arrange the overhang in 2 planes by the use of a

cranked coil as shown in Fig. 91, which relates to a hemi-tropic winding for 2 poles.

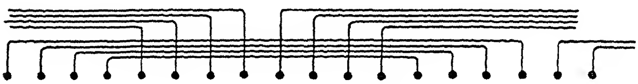
Another type of winding, which is quite common for small induction motors, for both stators and rotors, is called the "basket" type or "mush" type of winding. In this winding there is 1 coil side per slot, and it will be observed from the



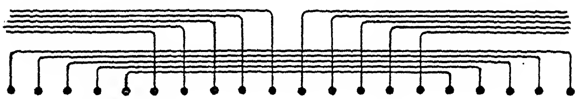
(*a*) 2 Slots per Pole and Phase.



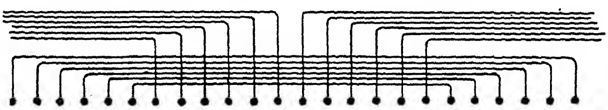
(*b*) 3 Slots per Pole and Phase.



(*c*) 4 Slots per Pole and Phase.



(*d*) 5 Slots per Pole and Phase



(*e*) 6 Slots per Pole and Phase.

FIG. 88
Diagrams of stator windings for three phases

diagram (Fig. 92) that coil ends from coils situated in consecutive slots cross one another, i.e. proceed to left and right alternately. The method of numbering the coils is shown in the diagram. In the top diagram is shown the mush winding for 4 poles and 36 slots. The pitch of the coils in this type of winding for the three-phase case must always be odd: thus coil sides 1 and 10 are joined to form one coil or a pitch of 9 teeth embraced. In the two-phase mush winding the pitch must be even.

It will be noticed that coils whose end connections leave the core in the same direction are numbered consecutively: thus coil in slot 1 is called 1, and coil in slot 3 is called coil 2, etc.

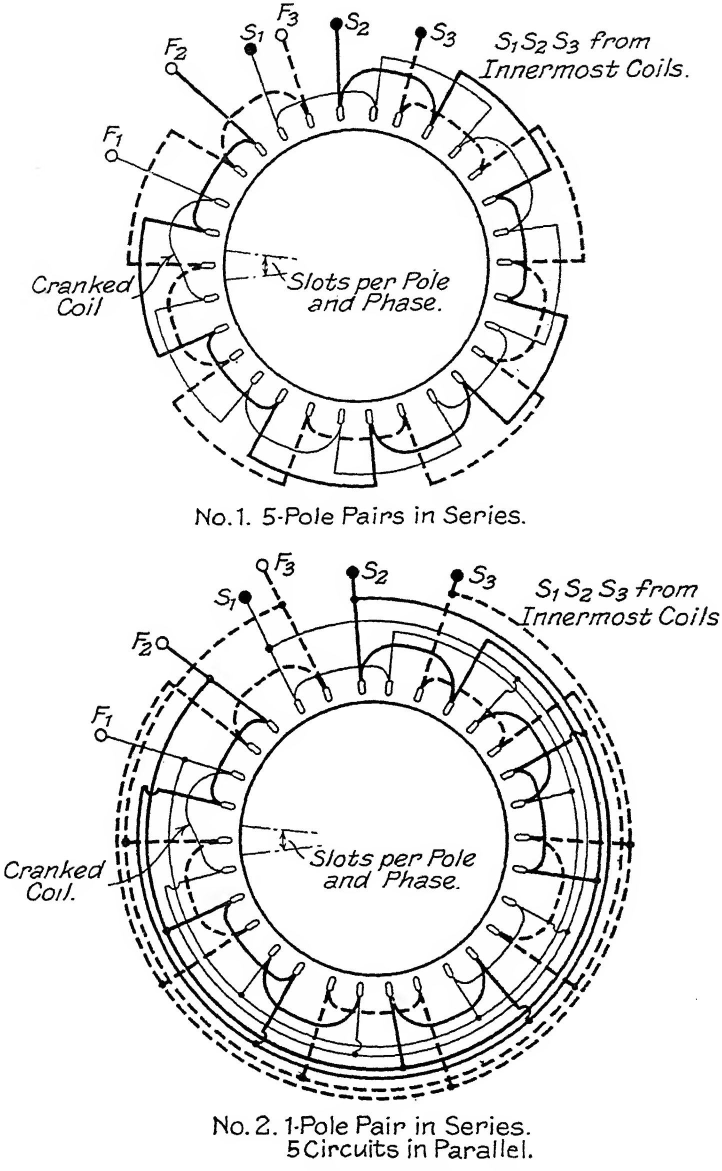
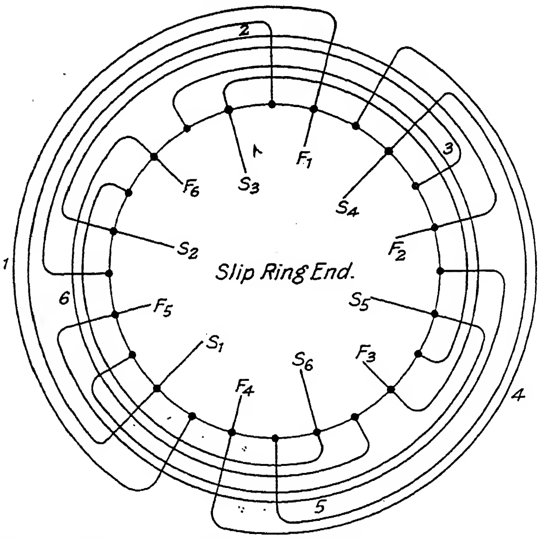


FIG. 89.—TEN-POLE STATOR WINDINGS. THREE PHASE

The connections of the coils to form a three-phase winding is as follows: For 4 poles, 36 slots, 18 coils—

Phase I	Phase II	Phase III
Start S_1 $F_1 - S^2$ $F_2 - F_{15}$ $S_{15} - S_{10}$ $F_{10} - S_{11}$ $F_{11} - F_6$ S_6 finish	Start S_4 $F_4 - S_5$ $F_5 - F_{18}$ $S_{18} - S_{13}$ $F_{13} - S_{14}$ $F_{14} - F_9$ S_9 finish	Start S_7 $F_7 - -S_8$ $F_8 - F_3$ $S_3 - S_{16}$ $F_{16} - S_{17}$ $F_{17} - F_{12}$ S_{12} finish

Note S_1 is the beginning of coil 1
F_1 is the end of coil 1



Plane I—Coils 1 and 4. Plane II—Coils 2 and 5. Plane III—Coils 3 and 6
Coils numbered round stator in clockwise direction
Start of coil to innermost slot coil of coil group
Finish of coil from outermost slot coil of coil group

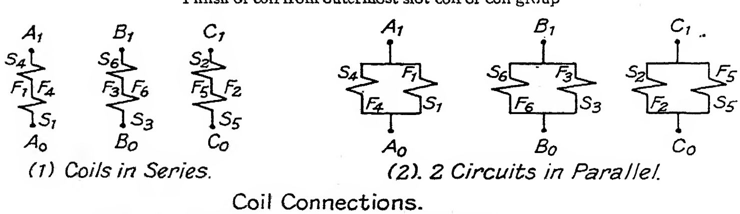


Fig. 90.—Three-phase Stator Winding for 2 Poles
Split coils in 3 planes

Again for 6 poles, 36 slots, 7 teeth embraced, we have—

PHASE I	PHASE II	PHASE III
Start S_1	Start S_3	Start S_5
$F_1 - F_4$	$F_3 - F_6$	$F_5 - F_8$
$S_4 - S_7$	$S_6 - S_9$	$S_8 - S_{11}$
$F_7 - F_{10}$	$F_9 - F_{12}$	$F_{11} - F_{14}$
$S_{10} - S_{13}$	$S_{12} - S_{15}$	$S_{14} - S_{17}$
$F_{13} - F_{16}$	$F_{15} - F_{18}$	$F_{17} - F_2$
S_{16} finish	S_{18} finish	S_2 finish

Again with open slots, frequently a 2 coil per slot or double-layer winding is used. Instances of this are shown in the Hunt windings, already discussed.

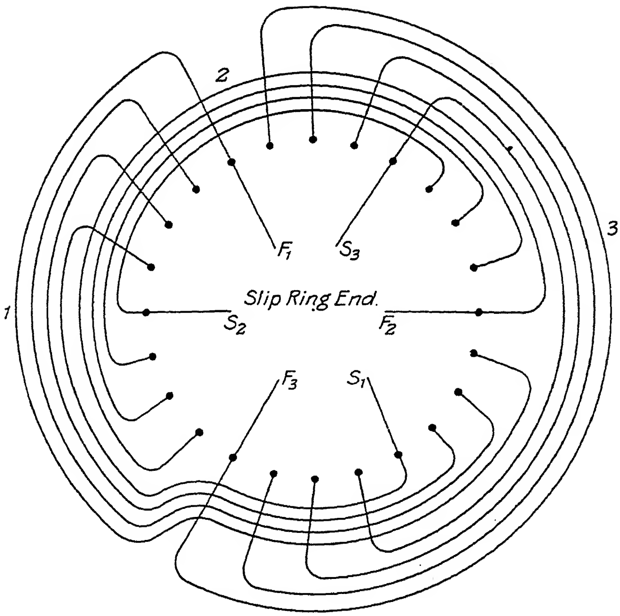


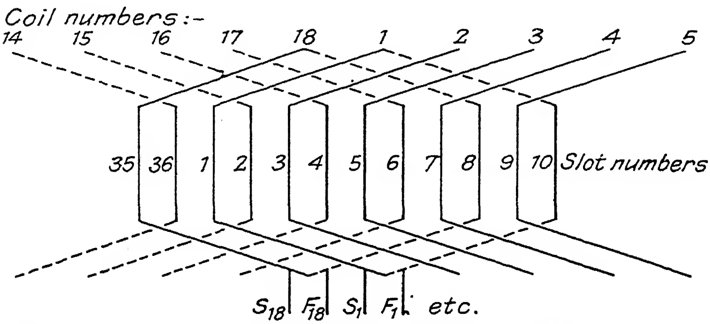
FIG. 91.—THREE-PHASE STATOR WINDING FOR 2 POLES
Coils in 2 planes. Phases not split
Coils numbered round stator in clockwise direction
Start of coil to innermost slot coil of group
Finish of coil from outermost slot coil group

Rotor windings. For small machines it is quite common practice to use mush windings for the rotor. These are similar to those already discussed.

For larger motors of the slip-ring type, a double-layer winding is used, generally of the two-bar per slot type.

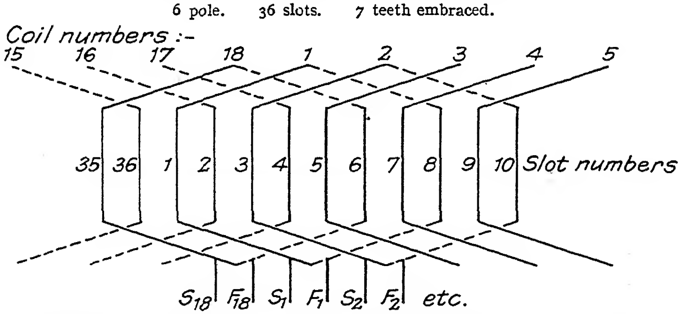
The winding is usually of the barrel type, and is of the wave type. A very common type of wave winding is that having an integral number of slots per pole per phase. In this type one proceeds with a normal step, but at every complete tour of the winding the step is either increased or decreased by unity to avoid closing the winding. This brings one to the next conductor to the right or

4-pole. 36 slots. 9 teeth embraced.



Position of slot 1 in each case to be on the left-hand side of stator and below the horizontal centre line by an amount equal to one *coil span*

6 pole. 36 slots. 7 teeth embraced.



Coils numbered round stator in clockwise direction looking from connection side

FIG. 92.—BASKET TYPE STATOR WINDING

Scheme for numbering of coils

left of the one from which we started. These steps are known as abnormal steps. After having completed half the winding, it is necessary to reverse the travel. It is best to number the bars in a clockwise direction, looking at the slip-ring end. The winding is usually left-handed, but this need not be so.

Let q = slots per pole per phase

p = number of poles

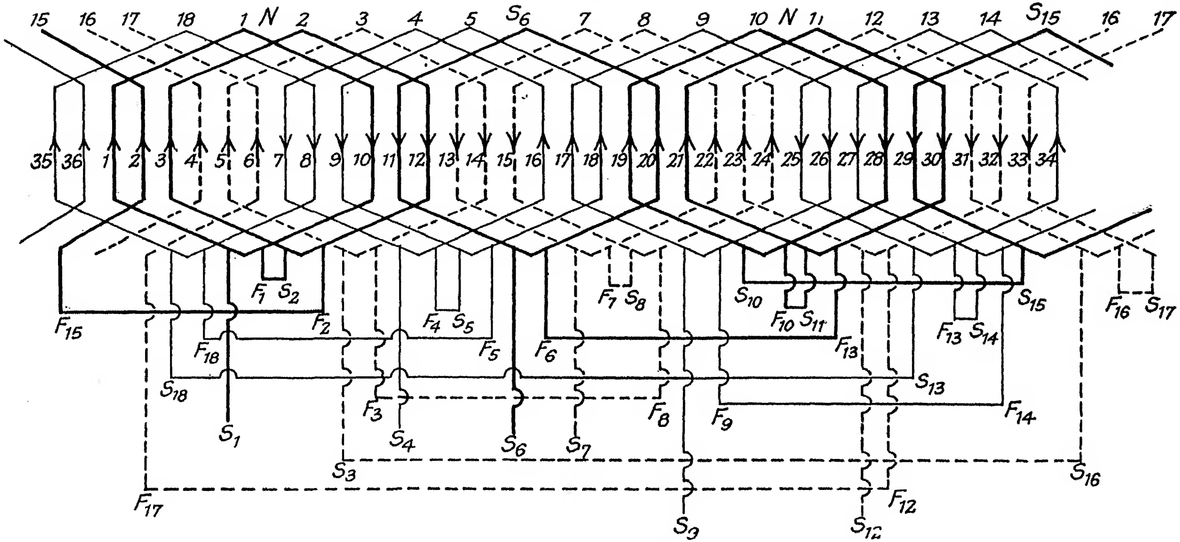


FIG. 92A.—THREE-PHASE MUSH WINDING

Then with 2 bars per slot, the number of slots and coils for a three-phase rotor $= 3pq$

The normal winding step $= 3q$

The abnormal winding step $= 3q - 1$

Then one splits the winding into six portions.

The number of coils in each portion $= \dfrac{3pq}{6} = \dfrac{pq}{2}$

With openings grouped together, the travel of the opening

$$= \frac{pq}{2} \div \frac{p}{2} = q$$

The openings on the upper bars will be—

Phase .	I	III′	II	I′	III	II′
Bar . .	1	$q + 1$	$2q + 1$	$3q + 1$	$4q + 1$	$5q + 1$

The corresponding openings (ends) on lower bars will be—

Bar . .	$4q$	$5q$	$6q$	$7q$	$8q$	$9q$

To obtain openings which are distributed round the rotor, it is necessary to displace Phases II and III by an integral number of pole pairs. In doing this, one should take note of the arms of the rotor centre carrying the leads to the slip-rings.

For a symmetrical arrangement, displace Phase II by nearest whole number of pole pairs to $p/6$, and Phase III by the nearest whole number of pole pairs to $p/3$.

Let Phase II be displaced by m-pole pairs, i.e. $6mq$ bars;

Phase III be displaced by n-pole pairs or $6nq$ bars.

Then the openings are—

	Phase I	Phase II
Upper bars . .	$1, 3q + 1$	$2q + 1 + 6mq$; $5q + 1 + 6mq$
Lower bars . .	$4q, 7q$.	$6q + 6mq$; $9q + 6mq$

	Phase III
Upper bars	$4q + 1 + 6nq$; $q + 1 + 6nq$
Lower bars	$8q + 6nq$; $5q + 6nq$

The phases can be displaced a different number of pole-pairs, if desired. The displacement must always be an integral number of pole-pairs. The lower bars with abnormal step will be $6(q - 1)$ in number and will be on the bars preceding the lower bar at which the connection is made.

$$\text{i.e. } 4q - (q - 1) \text{ to } 4q - 1, \text{ etc.}$$

It will be observed that, splitting the winding in six equal portions, the first section and the fourth section are 180° out of phase

with each other. If one wants to connect these two sections in series, then the beginning of the first section is connected to a slip-ring and the end of the first section is connected to the end of the fourth section, and the beginning of the fourth section to the star point for star connection.

Similarly, sections 3 and 6 and sections 5 and 2 are 180° out of phase, and one proceeds similarly with these sections. If one wishes two circuits in parallel per phase, then sections 1 and 4 are connected in parallel by connecting the end of section 1 to the beginning of section 4, and the beginning of section 1 to the end of section 4, and similarly for the other phases.

Let us take an example. A two-bar per slot wave winding has 16 poles and 144 slots. It is to be connected as a three-phase winding with one circuit per phase and star-connected.

Slots per pole per phase = 3
Number of bars = 288

144 upper bars numbered clockwise round the rotor as 1, 2, 3, etc.
144 lower bars numbered 1′, 2′, 3′, etc.

Step of winding front = 9 bars
,, ,, back = 9 ,,
Abnormal step = 8

Lower bars with abnormal step on front—

Phase I	Phase II	Phase III
10′ – 11′	106′ – 107′	58′ – 59′
19′ – 20′	115′ – 116′	67′ – 68′

Total abnormal lower bars $12 = 6(q-1)$.

Connections to slip-rings on upper bars—

Phase I	Phase II	Phase III
Bar 1	97	49

Connections to neutral on upper bars—

10 106 58

Cross-connections on lower bars—

12′ 21′ 108′ 117′ 60′ 69′

The winding proceeds thus—

i.e.

1–136′–127–118′–109–100′–91–82′–73–64′–55–46′–37–28′–19–10′–2,
and so on.

It is perhaps desirable in some cases to have a fractional number of slots per pole per phase, since in such cases the motor starts up more smoothly due to the absence of cogging points, and it gives a greater flexibility in the choice of slots. The common wave winding with one circuit per phase is quite suitable for this. The

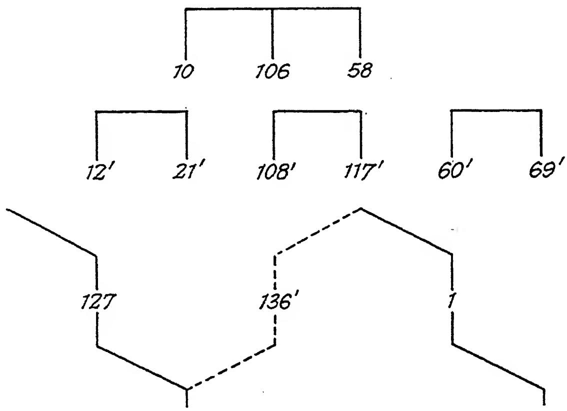


FIG. 93

winding is usually designed as a six-phase winding, opened out in six places, and each pair of opposite phases is connected in series or parallel.

Let a be any integer 1, 2, 3, etc.; and let it equal the number of similar parts or circuits in a phase of a symmetrical winding. With $a = 1$, it is only possible to get a symmetrical six-phase winding with machines having 2, 10, 14, and 22 poles. It is possible, however, to get a symmetrical three-phase system for all numbers of poles which are not a multiple of 3. In this case one can evolve a symmetrical three-phase winding from an unsymmetrical six-phase winding. The parallel connection is possible by making a wave winding with $a = 2$ which gives two similar parts or circuits per phase.

To Dr. S. P. Smith we are indebted for a very lucid description of such windings, and the table given on pp. 200–1 taken from his paper shows the numbers of slots and conductors per slot and numbers of poles possible for symmetrical wave windings.

If C = number of coils

p = number of pole-pairs

then the commutator pitch $= y_c = \dfrac{C \pm a}{p}$

the resultant winding pitch $y_r = 2y_c = y_b + y_f$

where y_b = back pitch in coil sides

y_f = front pitch in coil sides

The number of coils per phase in a six-phase winding $= \dfrac{C}{6}$

The openings may be found as follows—

The vectors of E.M.F. of consecutive coils are displaced in phase by a constant angle, assuming the coils to rotate in a pure sine wave of flux distribution. Then corresponding to any vector number, we have a certain coil.

Thus Vector No. .	1	2	3	x
Segments or joints	1	$1 + y_c$	$1 + 2y_c$	$1 + (x-1)y_c$
Top coil sides .	1	$1 + y_r$	$1 + 2y_r$	$1 + (x-1)y_r$
	$= 1$	$1 + 2y_c$	$1 + 4y_c$	$1 + (x-1)y_c$
Bottom coil sides.	$1 - y_f$	$1 + 2y_c - y_f$		$\{1 + (x-1)y_c\} - y_f$

Let the commutator bar (or joint) $1 + (x-1)y_c = b$

then corresponding top coil side $= 2b - 1$

" bottom coil side $= (2b - 1) - y_f$

The following example given by Dr. Smith is instructive :

$a = 1$, $C = 105$; pairs of poles $= 4$, slots $= 105$

$$\text{Commutator pitch} = \frac{C \pm a}{p} = \frac{105 - 1}{4} = 26 \text{ coils}$$

Resultant winding pitch $= 2y_c = y_r = 52 = y_b + y_f$

Back pitch in coil sides = 27

Front pitch in coil sides = 25

Coil span y_b in slots = slots 1 to 14

Coils per phase in a six-phase winding $= \dfrac{105}{6} = 17\frac{1}{2}$

Each of the six portions will consist of 17 and 18 coils alternately.

WAVE WINDINGS

Pole-pairs p'	Slots S'	Permissible Coil-sides per Slot u	Number of Similar Parts in Winding a	Number of Pole-pairs in Machine $p = p'a$	Number of Slots in Machine $S = S'a$	Number of Slots, S, in Machine when $N = 3$	$N = 4$	$N = 6$
1	n	any even number	1	1	n	$3n$	$4n$	$6n$
			p	p	pn	$3pn$	$4pn$	$6pn$
2	$2n \pm 1$	2, 6, 10, 14	1	2	$2n \pm 1$	$6n \pm 3$	—	—
			2	4	$4n \pm 2$	$12n \pm 6$	—	—
			3	6	$6n \pm 3$	$18n \pm 9$	—	—
			4	8	$8n \pm 4$	$24n \pm 12$	—	—
			5	10	$10n \pm 5$	$30n \pm 15$	—	—
			6	12	$12n \pm 6$	$36n \pm 18$	—	—
3	$3n \pm 1$	2, 4, 8, 10, 14, 16	1	3	$3n \pm 1$	—	$12n \pm 4$	—
			2	6	$6n \pm 2$	—	$24n \pm 8$	—
			3	9	$9n \pm 3$	—	$36n \pm 12$	—
			4	12	$12n \pm 4$	—	$48n \pm 16$	—
4	$4n \pm 1$	2, 6, 10, 14	1	4	$4n \pm 1$	$12n \pm 3$	—	—
			2	8	$8n \pm 2$	$24n \pm 6$	—	—
			3	12	$12n \pm 3$	$36n \pm 9$	—	—
5	$5n \pm 1$	2, 8, 12	1	5	$5n \pm 1$	$15n \pm 6$	$20n \pm 4$	$30n \pm 6$
			2	10	$10n \pm 2$	$30n \pm 12$	$40n \pm 8$	$66n \pm 12$
	$5n \pm 2$	4, 6, 14, 16	1	5	$5n \pm 2$	$15n \pm 3$	$20n \pm 8$	$30n \pm 12$
			2	10	$10n \pm 4$	$30n \pm 6$	$40n \pm 16$	$60n \pm 24$
6	$6n \pm 1$	2, 10, 14	1	6	$6n \pm 1$	—	—	—
			2	12	$12n \pm 2$	—	—	—

7	$7n \pm 1$	2, 12, 16	1	7	$7n \pm 1$	$21n \pm 6$	$28n \pm 8$	$42n \pm 6$
	$7n \pm 2$	6, 8	1	7	$7n \pm 2$	$21n \pm 9$	$28n \pm 12$	$42n \pm 12$
	$7n \pm 3$	4, 10	1	7	$7n \pm 3$	$21n \pm 3$	$28n \pm 4$	$42n \pm 18$
8	$8n \pm 1$	2, 14	1	8	$8n \pm 1$	$24n \pm 9$	—	—
	$8n \pm 3$	6, 10	1	8	$8n \pm 3$	$24n \pm 3$	—	—
9	$9n \pm 1$	2, 16	1	9	$9n \pm 1$	—	$36n \pm 8$	—
	$9n \pm 2$	8, 10	1	9	$9n \pm 2$	—	$36n \pm 16$	—
	$9n \pm 4$	4, 14	1	9	$9n \pm 4$	—	$36n \pm 4$	—
10	$10n \pm 1$	2	1	10	$10n \pm 1$	$30n \pm 9$	—	—
	$10n \pm 3$	6, 14	1	10	$10n \pm 3$	$30n \pm 3$	—	—
11	$11n \pm 1$	2	1	11	$11n \pm 1$	$33n \pm 12$	$44n \pm 12$	$66n \pm 12$
	$11n \pm 2$	10, 12	1	11	$11n \pm 2$	$33n \pm 9$	$44n \pm 20$	$66n \pm 24$
	$11n \pm 3$	8, 14	1	11	$11n \pm 3$	$33n \pm 3$	$44n \pm 8$	$66n \pm 30$
	$11n \pm 4$	6, 16	1	11	$11n \pm 4$	$33n \pm 15$	$44n \pm 4$	$66n \pm 18$
	$11n \pm 5$	4	1	11	$11n \pm 5$	$33n \pm 6$	$44n \pm 16$	$66n \pm 6$
12	$12n \pm 1$	2	1	12	$12n \pm 1$	—	—	—
	$12n \pm 5$	10, 14	1	12	$12n \pm 5$	—	—	—

Number of Coils in Winding : $C = u\,S/2$; Commutator Pitch : $y_c = (C \pm a)/p$; Potential Pitch : $y_p = C/a$
Phase Pitch : $y_{ph} = C/(a\,N)$; no idle coils permissible ; $n =$ any integer.

N.B.—$a = 1$ denotes the common wave winding : i.e. $y_c = (C \pm 1)/p$

$$\text{The starts and finishes of the 3 phases} = 2\pi \times \frac{35}{105}$$

$$= \frac{2\pi}{3} \text{ radians apart}$$

and the three-phase winding is thus symmetrical.

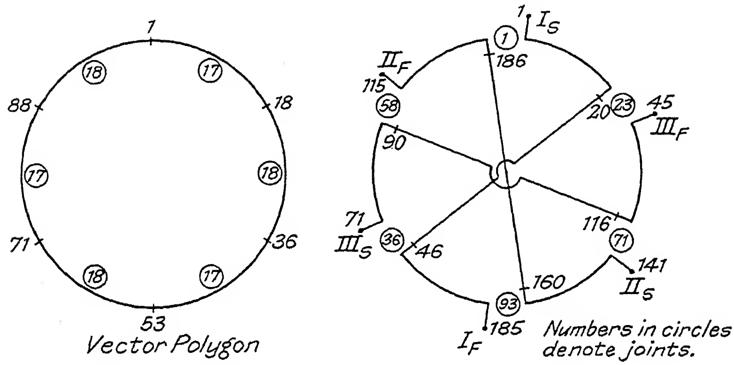


FIG. 94

The following table shows the joints to open and the corresponding coil sides—

Vector.	Joint. $b = 1 + (x - 1)y_c$	Top Coil Side $2b - 1$	Bottom Coil Side $(2b - 1) - y_f$
1	1	1 = top coil side in slot 1	186 = bottom side in slot 93
18	23	45 = ,, 23	20 = ,, 10
36	71	141 = ,, 71	116 = ,, 58
53	93	185 = ,, 93	160 = ,, 80
71	36	71 = ,, 36	46 = ,, 23
88	58	115 = ,, 58	90 = ,, 45

Figs. 95, 96, and 97 show the winding diagrams for double-layer rotor windings of the three-phase type. The diagrams are self-explanatory.

CHAPTER XIV

STARTING DEVICES

Small motors. For small motors up to about 5 b.h.p., it is generally permissible to switch the motor directly on to the line, but for larger machines this is seldom permissible on account of the large wattless current drawn from the line. To reduce this current and also to provide means for controlling the accelerating torque, various devices are in common use.

Auto-transformers. For large squirrel-cage motors, auto-transformers provided with suitable taps to give 50 per cent, 60 per cent, and 70 per cent of full voltage are generally used. The starting torque for each tap will be proportional to the square of the voltage corresponding to that tap. Thus with a 50 per cent tap, the starting torque will be $\frac{1}{4}$ of the starting torque with full voltage applied. With full voltage applied, the machine would probably take 4 to 6 times full-load current from the line.

Assuming the motor takes 5 times full-load current at full voltage, with a 50 per cent tap the motor will take 2·5 full-load current, and the current from the line will be 1·25 times full-load current, and similarly for any other tapping point.

The auto-transformer start is better than the method of starting by using an adjustable resistance in each stator phase.

We have seen that the $\dfrac{\text{starting torque}}{\text{full-load torque}}$

$$= \left(\frac{\text{starting current in the motor}}{\text{full-load motor current}}\right)^2 \times \text{full-load slip} \quad (611)$$

It is clear that the same starting torque will be obtained in each case for the *same current* in the motor, but for the same current in the motor the current drawn from the line will be considerably less with the auto-transformer start; or, in other words, for the same current from the line, a very much greater torque is obtainable with the auto-transformer start. Thus assume that the short-circuit current of the motor is 5 times full-load current and the slip at full load 5 per cent.

Stator resistance method. With the resistance in the stator

method, and with a starting current in the motor of twice full-load current, the starting torque

$$= 20\% = \left(\frac{I_s}{I_f}\right)^2 \times s = 2^2 \times 5\% \quad . \quad . \quad . \quad (612)$$

Now with a $63\frac{1}{4}$ per cent tap on the auto-transformer, we shall get twice full-load current from the line and a starting torque of $\left(\frac{63\frac{1}{4}}{100}\right)^2 \times 1{\cdot}25$, i.e. 50 per cent, that is, for twice full-load current from the line, the starting torque with auto-transformer is $2\frac{1}{2}$ times that with a resistance starter.

The star-delta starter. A favourite method of start for squirrel-cage machine is that known as the star-delta start. On starting up, the winding is first connected in star, and when up to speed the connection is changed to delta. With the windings in star, the voltage per phase

$$= \frac{\text{line voltage}}{\sqrt{3}} \text{ and the starting torque} \quad . \quad . \quad (613)$$

being proportional to the square of the applied voltage per phase

$$= \tfrac{1}{3} \text{ of that at full voltage} \quad . \quad . \quad . \quad . \quad (614)$$

At starting, the phase current is only $\frac{1}{\sqrt{3}}$ of the phase current when connected in delta. With star-connection, the phase current = line current. With delta-connection, the line current = $\sqrt{3}$ times the phase current. Consequently the line-starting current = $\frac{1}{3}$ of the short-circuit current of the motor.

For slip-ring motors, the usual method of start is to connect resistance in the rotor circuits.

We have seen that the rotor current

$$I_r = \frac{E_r}{\sqrt{\left(\frac{R_r}{s}\right)^2 + L_r^2\omega^2}} \quad . \quad . \quad . \quad (615)$$

where E_r = volts per phase at standstill
R_r = rotor resistance per phase
$L_r\omega$ = rotor reactance at standstill

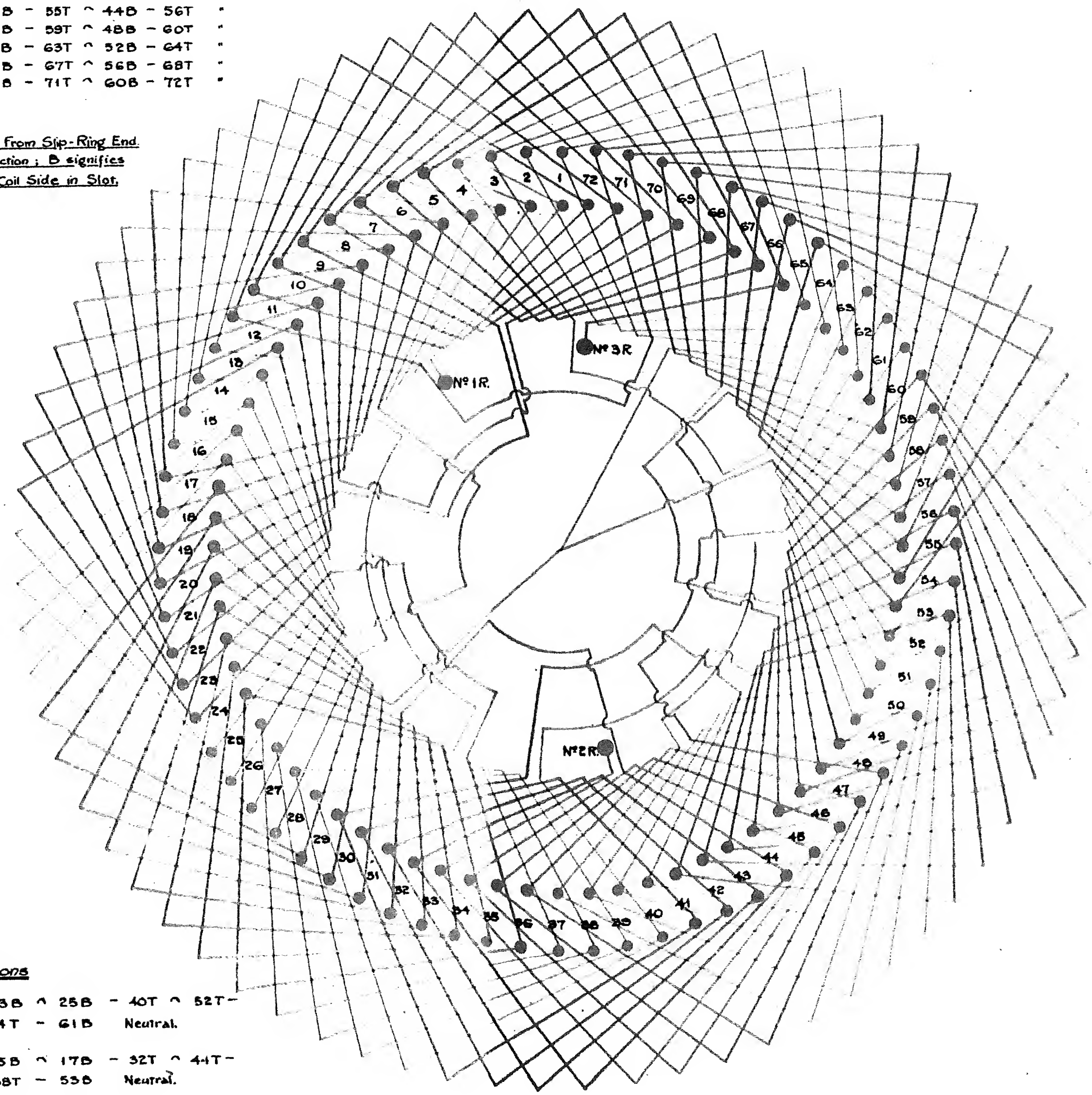
If the rotor current, and hence the stator current, is to remain constant during the starting period, then clearly R_r must vary proportionately with the slip. This is possible with a liquid rheostat.

3 PHASE LAP TYPE
STAR CONNECTED ROTOR WINDING
6 POLE 72 SLOTS

Coil Connections.

End	61B − 1T ^	62B − 2T ^	63B − 3T ^	64B − 4T	End
"	65B − 5T ^	66B − 6T ^	67B − 7T ^	68B − 8T	"
"	69B − 9T ^	70B − 10T ^	71B − 11T ^	72B − 12T	"
"	1B − 13T ^	2B − 14T ^	3B − 15T ^	4B − 16T	"
"	5B − 17T ^	6B − 18T ^	7B − 19T ^	8B − 20T	"
"	9B − 21T ^	10B − 22T ^	11B − 23T ^	12B − 24T	"
"	13B − 25T ^	14B − 26T ^	15B − 27T ^	16B − 28T	"
"	17B − 29T ^	18B − 30T ^	19B − 31T ^	20B − 32T	"
"	21B − 33T ^	22B − 34T ^	23B − 35T ^	24B − 36T	"
"	25B − 37T ^	26B − 38T ^	27B − 39T ^	28B − 40T	"
"	29B − 41T ^	30B − 42T ^	31B − 43T ^	32B − 44T	"
"	33B − 45T ^	34B − 46T ^	35B − 47T ^	36B − 48T	"
"	37B − 49T ^	38B − 50T ^	39B − 51T ^	40B − 52T	"
"	41B − 53T ^	42B − 54T ^	43B − 55T ^	44B − 56T	"
"	45B − 57T ^	46B − 58T ^	47B − 59T ^	48B − 60T	"
"	49B − 61T ^	50B − 62T ^	51B − 63T ^	52B − 64T	"
"	53B − 65T ^	54B − 66T ^	55B − 67T ^	56B − 68T	"
"	57B − 69T ^	58B − 70T ^	59B − 71T ^	60B − 72T	"

Slots Numbered Anti-Clockwise Looking From Slip-Ring End.
− signifies Coil ; ^ signifies Soldered Connection ; B signifies Bottom Coil Side in Slot ; T signifies Top Coil Side in Slot.



Inter - Coil Connections

Nº 1 Slip Ring 1B − 16T ^ 28T − 13B ^ 25B − 40T ^ 52T −
− 37B ^ 49B − 64T ^ 4T − 61B Neutral.

Nº 3 Slip Ring 65B − 8T ^ 20T − 5B ^ 17B − 32T ^ 44T −
− 29B ^ 41B − 56T ^ 68T − 53B Neutral.

Nº 2 Slip Ring 33B − 48T ^ 60T − 45B ^ 57B − 72T ^ 12T −
− 69B ^ 9B − 24T ^ 36T − 21B Neutral.

With a rotor starter constructed of metal constant current is impossible, and the current is allowed to vary between a maximum and minimum value. Let there be n steps in the controller.

On the first notch initially,

$$I_1 = \frac{E_r}{\sqrt{\left(\frac{R_1}{s_1}\right)^2 + L_r^2\omega^2}} \quad . \quad . \quad . \quad (616)$$

where R_1 = total resistance in series with the rotor

$s_1 = 1$ at the start

As the speed rises, the current will fall; let the slip be s_2 when the current has fallen to I_2

$$\text{then} \quad I_2 = \frac{E_r}{\sqrt{\left(\frac{R_1}{s_2}\right)^2 + L_r^2\omega^2}} \quad . \quad . \quad . \quad (617)$$

The controller is now moved to the next notch and the current rises to I_1

$$\text{then} \quad I_1 = \frac{E_r}{\sqrt{\left(\frac{R_2}{s_2}\right)^2 + L_r^2\omega^2}} \quad . \quad . \quad . \quad (618)$$

the speed continues to rise and the slip falls until I_2 is reached,

$$\text{and we have} \quad I_2 = \frac{E_r}{\sqrt{\left(\frac{R_2}{s_3}\right)^2 + L_r^2\omega^2}} \quad . \quad . \quad . \quad (619)$$

It is clear therefore that for the $n + 1$ notches, we have

$$\left(\frac{R_1}{s_1} = \frac{R_2}{s_2} = \frac{R_3}{s_3} = \ldots = \frac{R_{n+1}}{s_{n+1}} \quad . \quad . \quad (620)\right.$$

$$\text{or} \quad \frac{s_2}{s_1} = \frac{s_3}{s_2} = \frac{s_4}{s_3} = \frac{s_{n+1}}{s_n} = \frac{R_2}{R_1} = \frac{R_3}{R_2} = \frac{R_{n+1}}{R_n} = a \quad . \quad (621)$$

Now $s_1 = 1$

$$\therefore R_2 = a\,R_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (622)$$

$$R_3 = a\,R_2 = a^2 R_1 \quad . \quad . \quad . \quad . \quad . \quad (623)$$

$$R_n = a\,R_{n-1} = a^{n-1} R_1 \quad . \quad . \quad . \quad . \quad (624)$$

$$R_{n+1} = a\,R_n = a^n R_1 \quad . \quad . \quad . \quad . \quad . \quad (625)$$

where R_{n+1} = rotor resistance with rings short-circuited

$$R_{n+1} = a^n R_1 = a^n \frac{R_{n+1}}{s_{n+1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (626)$$

s_{n+1} = slip with the rotor short-circuited, i.e. with all external resistance cut out,

$$\text{thus } R_1 = \frac{R_{n+1}}{s_{n+1}} \qquad . \quad . \quad . \quad . \quad . \quad (627)$$

$$\text{and } a^n = s_{n+1} \qquad . \quad . \quad . \quad . \quad . \quad (628)$$

the steps in the rotor resistance must be

$$r_1 = R_1 - R_2 = R_1 (1 - a) \qquad . \quad . \quad . \quad (629)$$

$$r_2 = R_2 - R_3 \qquad . \quad . \quad . \quad . \quad . \quad (630)$$

$$= R_1 (a - a^2) = a\, r_1 \qquad . \quad . \quad . \quad (631)$$

$$r_3 = R_3 - R_4 = a\, r_2 \qquad . \quad . \quad . \quad . \quad (632)$$

and so on.

It is also clear that approximately $\dfrac{I_2}{I_1} = a$ . . . (633)

CHAPTER XV

DESIGN OF INDUCTION MOTORS

Principles—Output equation.

Let E = counter E.M.F. per phase in volts

V = applied P.D. per phase in volts

Z = conductors in series per phase

f = supply frequency

ϕ = flux per pole in C.G.S. lines

I = current in amperes per phase

p = number of poles

L = core length in centimetres

B = average flux density in the air-gap in C.G.S. lines per sq. cm.

θ = angle of lag of the stator current behind the applied voltage vector

q = ampere-conductors per cm. of stator periphery

= specific electric loading

η = efficiency

D = diameter of stator core in cms.

Three-phase machines.

Input in watts $= 3\,VI \cos\theta$ . . . . . . (634)

Assuming $V = 1{\cdot}05\,E$

$$\text{Input} = 3 \times 1{\cdot}05\,EI \cos\theta \qquad (635)$$

$$\text{Output} = \frac{3 \times 1{\cdot}05\,EI \cos\theta\eta}{746}\ \text{(B.H.P.)} \qquad (636)$$

Now with a distributed winding of full coil span

$$E = 2{\cdot}12 \times \phi \times Z \times f \times 10^{-8} \qquad (637)$$

$$\text{and } \phi = \frac{\pi D}{p} \times L \times B\,; \quad f = \frac{p}{2} \times \frac{\text{R.P.M.}}{60} \qquad (638)$$

$\therefore$ B.H.P.

$$= 3 \times 1{\cdot}05 \times 2{\cdot}12 \times \frac{\pi D}{p} \times L \times B \times \frac{p}{2} \times \frac{\text{R.P.M.}}{60} \times \frac{ZI \cos\theta \times \eta}{746 \times 10^8}$$

R.P.M. = revolutions per minute . . . (639)

$$\text{Also } 3\,ZI = \pi Dq \quad \therefore\ ZI = \frac{\pi Dq}{3} \qquad (640)$$

$$\therefore\ \text{B.H.P.} = \frac{1{\cdot}05 \times 2{\cdot}12 \times \pi^2 D^2 L \times B \times q \times \text{R.P.M.} \times \cos\theta \times \eta}{2 \times 60 \times 746 \times 10^8} \qquad (641)$$

$$\text{and } D^2L \text{ (cu. cms.)} = \frac{4{\cdot}06 \times 10^{11} \times \text{B.H.P.}}{B \times q \times \text{R.P.M.} \times \cos\theta \times \eta} \qquad (642)$$

Two-phase induction motors.

$$\text{Input} = 2\,VI\cos\theta \qquad (643)$$

$$= 2 \times 1{\cdot}05\,EI\cos\theta \qquad (644)$$

$$\text{Output (B.H.P.)} = \frac{2 \times 1{\cdot}05\,EI\cos\theta \times \eta}{746} \qquad (645)$$

B.H.P. = brake horse-power

$$E = 2 \times \phi \times Z \times f \times 10^{-8} \qquad (646)$$

$$= 2 \times \frac{\pi D}{p} \times L \times B \times Z \times \frac{p}{2} \times \frac{\text{R.P.M.}}{60} \times 10^{-8} \qquad (647)$$

$$\therefore\ \text{B.H.P.} = \frac{2 \times 1{\cdot}05 \times 2 \times \pi D \times L \times B \times \text{R.P.M.} \times ZI\cos\theta \times \eta}{2 \times 60 \times 10^8 \times 746} \qquad (648)$$

$$2\,ZI = \pi Dq \qquad (649)$$

$$\therefore\ \text{B.H.P.} = \frac{1{\cdot}05 \times 2 \times \pi^2 D^2 \times L \times q \times B \times \text{R.P.M.} \times \cos\theta \times \eta}{2 \times 60 \times 10^8 \times 746} \qquad (650)$$

$$\text{and } D^2L \text{ (cu. cms.)} = \frac{4{\cdot}3 \times 10^{11} \times \text{B.H.P.}}{q \times B \times \text{R.P.M.} \times \cos\theta \times \eta} \qquad (651)$$

It will be noticed that the D^2L for two-phase machines is about 6 per cent greater than that for three-phase machines for the same output.

Single-phase motors.

$$\text{Input} = VI\cos\theta \qquad (652)$$

$$= 1{\cdot}05\,EI\cos\theta \qquad (653)$$

$$\text{B.H.P.} = \frac{1{\cdot}05\,EI\cos\theta \times \eta}{746} \quad . \quad . \quad . \quad (654)$$

Now with all the slots wound,

$$E = 1{\cdot}41 \times \phi \times Z \times f \times 10^{-8} \quad . \quad . \quad . \quad . \quad (655)$$

$$= 1{\cdot}41 \times \frac{\pi D}{p} \times L \times B \times Z \times f \times 10^{-8} \quad . \quad (656)$$

$\therefore$ B.H.P.

$$= 1{\cdot}41 \times \pi D \times L \times \frac{B}{2} \times \frac{\text{R.P.M.}}{60} \times \frac{1}{10^8} \times \frac{ZI\cos\theta \times \eta}{746} \times 1{\cdot}05 \quad (657)$$

$$\text{and } ZI = \pi D q \quad . \quad . \quad . \quad . \quad . \quad (658)$$

$\therefore$ B.H.P.

$$= \frac{1{\cdot}05 \times 1{\cdot}41 \times \pi^2 D^2 \times L \times q \times B \times \text{R.P.M.} \times \cos\theta \times \eta}{2 \times 60 \times 10^8 \times 746} \quad (659)$$

$$\therefore \underset{\text{(cu. cms.)}}{D^2L} = \frac{6{\cdot}1 \times 10^{11} \times \text{B.H.P.}}{q \times B \times \text{R.P.M.} \times \cos\theta \times \eta} \quad . \quad (660)$$

It will be observed that the D^2L, for single-phase machines, is 50 per cent greater than for three-phase machines for the same output with all the slots wound; or, in other words, the output for the single-phase machine is $\frac{2}{3}$ of that of the three-phase machine for the same D^2L.

In practice, only $\frac{2}{3}$ of the slots are utilized for the main winding, and the remaining slots are taken up for the auxiliary starting winding. In this case the output is only 50 per cent of the three-phase output for the same D^2L.

In order to use our D^2L equation, we require to know appropriate values for B, q, $\cos\theta$, and η. Usually values for these quantities are assumed in a rather arbitrary manner, and one is left in the dark as to how these various quantities are related to the performance of the machine. It is true that an experienced designer can usually fix on suitable values based on experience with existing machines, but it does not follow that his values are the best for best performance.

An attempt will be made to put this matter on a firm scientific basis. Again, when suitable values for the various quantities have been obtained, the designer is faced with the problem of splitting up the D^2L into its component parts. It has been stated that there is no simple method of effecting this, and that the only method is to work out several designs and select that which gives

the best performance. Such is not the case. It will be shown that there is a very definite relation between D and L for best performance.

The first consideration, which we shall regard as essential in each machine, is that it shall have an overload capacity equal to approximately twice its normal output.

Let us turn to Fig. 14 and see what this involves. We saw that the maximum

$$\text{H.P.} = \frac{m \times V \times JF}{2\,(1 + \cos\phi) \times 746} \quad . \quad . \quad . \quad . \quad . \quad (661)$$

where m = number of phases in the stator

JF = actual rotor short-circuit current per phase referred to the stator

Also $JF = JK \sin\phi$

where JK = ideal rotor short-circuit current per phase referred to the stator

and ϕ = angle of lag of the actual short-circuit rotor current behind the applied voltage vector, or rather more strictly behind the counter E.M.F. vector

$$\therefore \text{ max. H.P.} = \frac{m \times V \times JK \sin\phi}{2\,(1 + \cos\phi) \times 746} \quad . \quad . \quad . \quad . \quad (662)$$

$$= \frac{m \times V \times (OB - OA) \sin\phi}{2\,(1 + \cos\phi) \times 746} \quad . \quad . \quad . \quad (663)$$

where OB = ideal stator short-circuit current per phase, i.e. the current that would flow in the stator at short circuit if no resistance were present

and OA = magnetizing current per phase

Now since it is desirable that the max. h.p. shall be twice the normal b.h.p., we have

$$2\ \text{B.H.P.} = \frac{m \times V \times (OB - OA) \sin\phi}{2\,(1 + \cos\phi) \times 746} \quad . \quad . \quad . \quad (664)$$

For *three-phase machines*,

$$\text{B.H.P.} = \frac{3\,V \times (OB - OA) \sin\phi}{4\,(1 + \cos\phi) \times 746} \quad . \quad . \quad . \quad (665)$$

For *two-phase machines*,

$$\text{B.H.P.} = \frac{V \times (OB - OA) \times \sin\phi}{2\,(1 + \cos\phi) \times 746} \quad . \quad . \quad . \quad (666)$$

For *three-phase machines,*

$$OB - OA = \frac{4 \text{ B.H.P. } (1 + \cos\phi) \times 746}{3 V \sin\phi} \quad . \quad . \quad (667)$$

For *two-phase machines,*

$$OB - OA = \frac{2 \text{ B.H.P. } (1 + \cos\phi) \times 746}{V \sin\phi} \quad . \quad . \quad (668)$$

Now for machines up to about 50 b.h.p. in output,

$$\cos\phi \text{ at short circuit} = 0{\cdot}5 \text{ (approx)}$$
$$\text{and } \phi = 60^\circ$$
$$\sin\phi = 0{\cdot}866$$

∴ for machines up to 50 b.h.p. (approx.), we have

For *three-phase machines,*

$$OB - OA = \frac{4 \text{ B.H.P. } (1 + 0{\cdot}5)\, 746}{3 V \times 0{\cdot}866} \quad . \quad . \quad . \quad (669)$$

$$= \frac{1725 \text{ B.H.P.}}{V} \quad . \quad . \quad . \quad . \quad . \quad (670)$$

For *two-phase machines* up to about 50 h.p.

$$OB - OA = \frac{2580 \text{ B.H.P.}}{V} \quad . \quad . \quad . \quad . \quad . \quad (671)$$

For large machines, $\cos\phi$ at short circuit $= 0{\cdot}25$ approx.

and $\sin\phi = 0{\cdot}967$

∴ for large *three-phase motors,*

$$OB - OA = \frac{4 \text{ B.H.P. } \times 1{\cdot}25 \times 746}{3 V \times 0{\cdot}967}$$

$$= \frac{1285 \text{ B.H.P.}}{V} \quad . \quad . \quad . \quad . \quad . \quad (672)$$

for large *two-phase motors,*

$$OB - OA = \frac{1930 \text{ B.H.P.}}{V} \quad . \quad . \quad . \quad . \quad . \quad (673)$$

It is clear that the ideal short-circuit current is definitely fixed if this condition is to be satisfied.

Now, the ideal short-circuit current per phase

$$= \frac{\text{applied E.M.F. per phase}}{\text{equivalent leakage reactance per phase at standstill}} \quad (674)$$

$$\therefore OB = \frac{E}{L \times 2\pi f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (675)$$

where L = equivalent coefficient of self-inductance per phase in henries

$$\text{Now } E = \sqrt{2} \times \pi \times \phi \times \frac{Z}{2} \times f \times 10^{-8} \times \text{breadth factor}$$

$$\text{and } Z = p \times q_1 \times z_1 \quad (676)$$

where p = number of poles

q_1 = slots per pole per phase

z_1 = conductors in series per slot

$$\therefore \frac{E}{f} = \phi \times \sqrt{2} \times \pi \times \frac{p}{2} \times q_1 \times z_1 \times \frac{1}{10^8} \times \text{breadth factor} \quad (677)$$

$$\text{and } L = \frac{4\pi \times (q_1 z_1)^2 \times \lambda \times p}{10^9} \text{ henries} \quad . \quad . \quad . \quad (678)$$

where λ = magnetic conductivity of the leakage flux path encircled by the amp.-turns

$$\therefore OB = \frac{\phi \times \sqrt{2} \times \pi \times p \times q_1 \times z_1 \times 10^9}{4\pi \times 2p \times 2\pi \times \lambda \times (q_1 \times z_1)^2 \times 10^8} \quad . \quad (679)$$

$$= \frac{\phi}{0{\cdot}4\pi \times 2\sqrt{2} \times \lambda \times q_1 \times z_1} \quad . \quad . \quad . \quad (680)$$

i.e. the ideal short-circuit current per phase

$$= \frac{\text{working flux per pole}}{\text{leakage flux per pole per ampere}} \quad . \quad . \quad . \quad (681)$$

Now λ the magnetic permeance of the leakage paths can be split up into the following quantities—

λ_L = magnetic permeance of *slot* stray flux path

λ_T = magnetic permeance of the stray flux path of the overhang

λ_Z = magnetic permeance of the so-called zigzag leakage path

Expressions for these various quantities will be developed later.

$$\text{Now } \lambda = \lambda_L + \lambda_T + \lambda_Z \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (682)$$

$$\therefore OB = \frac{\phi}{3{\cdot}55 \times \lambda \times q_1 \times z_1} \quad . \quad . \quad . \quad . \quad . \quad (683)$$

$$\text{Also } OB - OA = \frac{\text{constant} \times \text{B.H.P.}}{V} \quad . \quad . \quad . \quad . \quad (684)$$

$$\therefore \ 1 - \frac{OA}{OB} = \frac{\text{constant} \times \text{B.H.P.} \times 3{\cdot}55\,\lambda \times q_1 z_1}{V \times \phi} \quad . \quad . \quad (685)$$

$$\text{Now B.H.P.} = m\,VI \cos\theta\,\eta \quad . \quad . \quad . \quad . \quad . \quad . \quad (686)$$

$$\therefore \ 1 - \frac{OA}{OB} = \frac{\text{constant} \times m\,I \cos\theta\,\eta \times 3{\cdot}55\,\lambda\,q_1 z_1}{\times \phi} \quad . \quad . \quad (687)$$

$$= \frac{\text{constant} \times m\,q_1 z_1 \times p \times 3{\cdot}55\,\lambda}{p\phi} \times \cos\theta \times \eta \times I \quad . \quad . \quad (688)$$

$$= \frac{\text{constant} \times 3{\cdot}55 \times \lambda \times \cos\theta \times \eta \times \text{total amp.-conductors}}{\text{total flux}} \quad (689)$$

since $m\,q_1 z_1 \times p \times I = \text{total ampere-conductors}$

and $p\phi = \text{total flux}$

$\therefore \ 1 - \sigma$

$$= \frac{\text{constant} \times 3{\cdot}55 \times \lambda \times \cos\theta \times \eta \times \text{total amp.-conductors}}{\text{total flux}} \quad (690)$$

$$\frac{OA}{OB} = \text{dispersion coefficient} \quad . \quad . \quad (691)$$

$$\therefore \ \frac{\text{total amp.-conductors}}{\text{total flux}} = \frac{1 - \sigma}{\text{constant} \times 3{\cdot}55 \times \lambda \times \cos\theta \times \eta} \quad (692)$$

The constant is taken from the appropriate equation already given in preceding pages. This is the relation which must hold between the electric and magnetic loadings, in order that the overload capacity required may be obtained. This is an important relation and will serve as a useful guide in design work. Now generally σ increases as λ increases, and we see therefore that for machines having large values of σ, viz., machines of many poles, that the ratio of electric loading to magnetic loading becomes smaller and smaller. In such machines, as a general rule, overload capacity must be sacrificed if reasonably high power factors are to be obtained, and the working flux must be reduced below the value required for an overload capacity of twice the normal.

Now the greater the specific electric and magnetic loadings, the smaller the D^2L of the machine. From the commercial standpoint, it is desirable to push these two quantities to their limits. The question arises therefore as to what are their limits and how are they determined? It may be taken as axiomatic that a high power factor is the first desideratum and, secondly, that the

efficiency shall be as high as is consistent with the machine reaching its permissible temperature rise in all its parts.

The temperature rise is fixed by the magnetic and electric loadings, for the copper loss is proportional to the ampere-conductors with constant current density, and the iron loss is a function of the magnetic loading.

Power factor considerations dictate that the specific magnetic loading, viz., B, shall be as small as possible. There should be no saturation in teeth or in core if high power factor is to be obtained, and indeed at 50 cycles, this more or less agrees with the heating requirement. The maximum tooth density, at minimum section, should not exceed 16,000 lines per sq. cm. at 50 ~. This corresponds to 10,200 lines average per sq. cm. at minimum tooth section.

If we assume a width of tooth equal to half the slot pitch, this gives an average flux density in the air-gap of 5000 lines per sq. cm. This refers to the density which is obtained when the slotting in stator and rotor is taken into account. This is the magnetic loading which gives us economical use of the material and, at the same time, requires reasonably small M.M.F. for the iron parts.

Again, it may be said that, for a machine of given diameter, the electric loading is proportional to the width of slot, and the magnetic loading to the width of tooth. For a given diameter the sum is constant, and the product is greatest when they are equal.

Maximum output is obtained therefore when the width of slot equals the width of tooth at minimum section.

We have now to investigate how the heating limit affects the electric loading of the machine.

The permissible watts per sq. cm. of barrel surface at the gap of the stator for protected type machines, fitted with fans, varies with peripheral speed according to the curve shown in Fig. 98.

This is an experimental curve taken from test results on a large range of machines.

$$\text{The core barrel surface} = \pi D \times L$$

The curves for iron loss at different frequencies, obtained also from test results, are shown in Fig. 99.

Analysis of the curves for iron losses gives the following formula—

Iron loss per kilogramme

$$= 0{\cdot}000227 \times B^{1{\cdot}8} \times f^{1{\cdot}6} \text{ watts} \qquad (693)$$

where B = maximum density in thousands of lines per sq. cm.

f = supply frequency

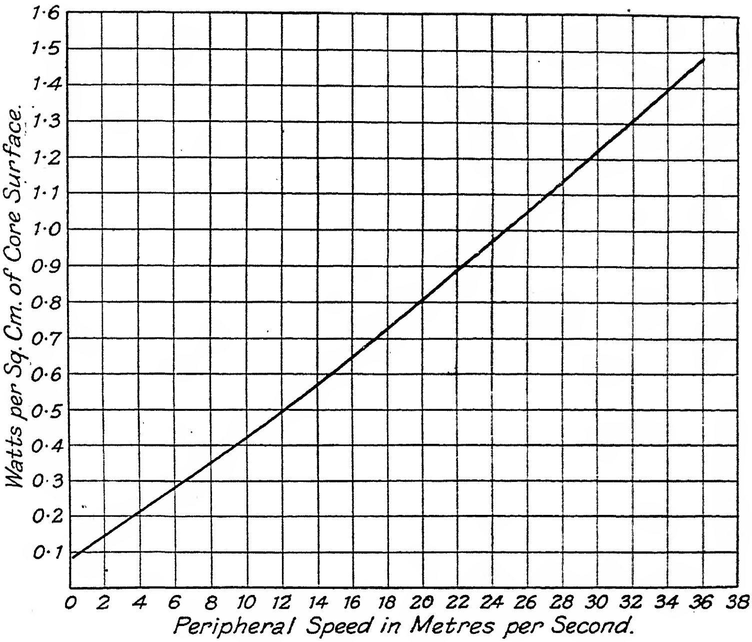


FIG. 98.—CURVE SHOWING THE RELATION BETWEEN PERMISSIBLE WATTS IN STATOR CORE SURFACE, AND PERIPHERAL SPEED

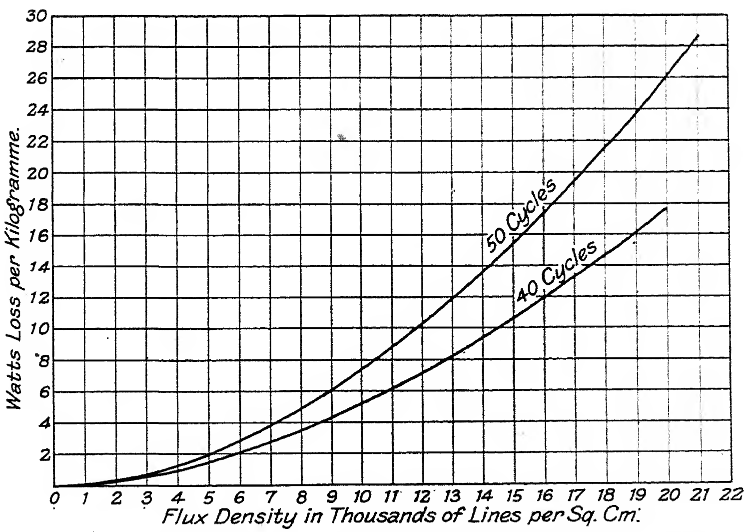


FIG. 99.—CURVES SHOWING THE IRON LOSSES IN INDUCTION MOTORS

Experimental curves

Since 1 cu. cm. of iron weighs 7·9 grammes,

$$1 \text{ kg. of iron} = \frac{1000}{7{\cdot}9} = 126{\cdot}5 \text{ cu. cms.}$$

and ∴ iron loss in watts per cubic centimetre

$$= 0{\cdot}00000179\, B^{1.8} \times f^{1.6} \quad . \quad . \quad . \quad (694)$$

where B is in kilolines per sq. cm.

Analysis of the permissible watts curve shows that, below a peripheral speed of 12 metres per second,

permissible watts per sq. cm. of stator barrel surface at the gap = 0·033 × peripheral speed in metres per second + 0·08

above 12 metres per second = 0·0408 × peripheral speed in metres per second

Now, if z_1 = conductors in series per slot

I = current per conductor in amperes

h_s = height of the slot in cms.

ω_s = width of the slot in cms.

s = space factor of slot

A = area of conductor in sq. cms.

Δ = current density in amps. per sq. cm.

Considering unit length of slot, i.e. 1 cm. length,

$$\text{the copper loss per cm.} = \frac{I^2 z_1 \rho}{A} \quad . \quad . \quad . \quad (695)$$

where ρ = specific resistance of copper at the temp. considered

$$\text{copper loss per cm.} = \frac{I}{A} I z_1 \rho$$

$$= \Delta\, I z_1 \rho \quad . \quad . \quad . \quad (696)$$

$$\text{Now } q = \frac{I z_1 \times \text{number of slots}}{\pi D} \quad . \quad . \quad . \quad (697)$$

$$= \frac{I z_1}{\text{slot pitch}} = \frac{I z_1}{K \omega_s} \quad . \quad . \quad . \quad . \quad (698)$$

where K is some constant

$$\therefore \frac{I z_1}{\omega_s} = q \times K \quad . \quad . \quad . \quad . \quad . \quad .$$

Now the copper loss per sq. cm. at the gap surface

$$= \frac{\Delta\, I z_1 \rho}{\omega_s} = \Delta\, \rho \,.\, q \times K \quad . \quad . \quad . \quad (700)$$

Also the iron loss in the teeth at

$B = 16 \times 10^2$ and $f = 50$ per cu. cm.

$$= 0{\cdot}00000179 \times 16^{1{\cdot}8} \times 50^{1{\cdot}6}$$

$$= 0{\cdot}135 \text{ watt} \quad . \quad . \quad . \quad (701)$$

$\therefore$ the iron loss per sq. cm. at the gap, assuming all the tooth loss to be dissipated at the gap surface

$$= 0{\cdot}135 \times h_s \text{ watts} \quad . \quad . \quad (702)$$

$\therefore$ we have the following equations—

Up to 12 metres per second,

$$\Delta\, \rho \,.\, qK + 0{\cdot}135\, h_s = 0{\cdot}033 V_s + 0{\cdot}08 \quad . \quad . \quad . \quad (703)$$

Above 12 metres per second,

$$\Delta\, \rho q K + 0{\cdot}135\, h_s = 0{\cdot}0408 V_s \quad . \quad . \quad . \quad . \quad (704)$$

From these two equations we can draw a series of curves for several values of h_s giving the relation between q and V_s, i.e. the relation between q and peripheral speed in metres per second.

It will be seen from the above that the product of current density and specific electric loading is sensibly constant for a given peripheral speed and depth of slot.

If we take a value for Δ of 350 amp. per sq. cm. (a very usual value) and calculate q for several values of h_s, we get the following series of curves. (Fig. 100.)

On 25 ~ machines, the iron loss is considerably reduced; it follows therefore that on 25 ~ machines, we can increase the electric loading to a certain extent.

The iron loss at 25 ~ per cu. cm. at B 16,000 = 0·049. Our equation for 25 ~ is therefore

$$\Delta\, \rho q K + 0{\cdot}049\, h_s = 0{\cdot}0408\, V_s \text{ (above 12 metres per sec.)} \quad (705)$$

and $\Delta\, \rho q K + 0{\cdot}049\, h_s = 0{\cdot}033 V_s + 0{\cdot}08$ . . . . (706)

It must be clearly understood that the values of q given in the curves above are based on a current density of 3·5 amp. per sq. mm., and a value of $K = 2$

If the current density is lower and K less than 2, then q is increased correspondingly.

Further, in deriving the relation, it has been assumed that the whole of the copper loss and tooth loss is dissipated at the gap surface. It is well known, of course, that a certain amount of heat is dissipated from the ducts and end plates, and it would appear that the values of q may be increased above the values given in the curves by about 10 per cent. Such curves are by

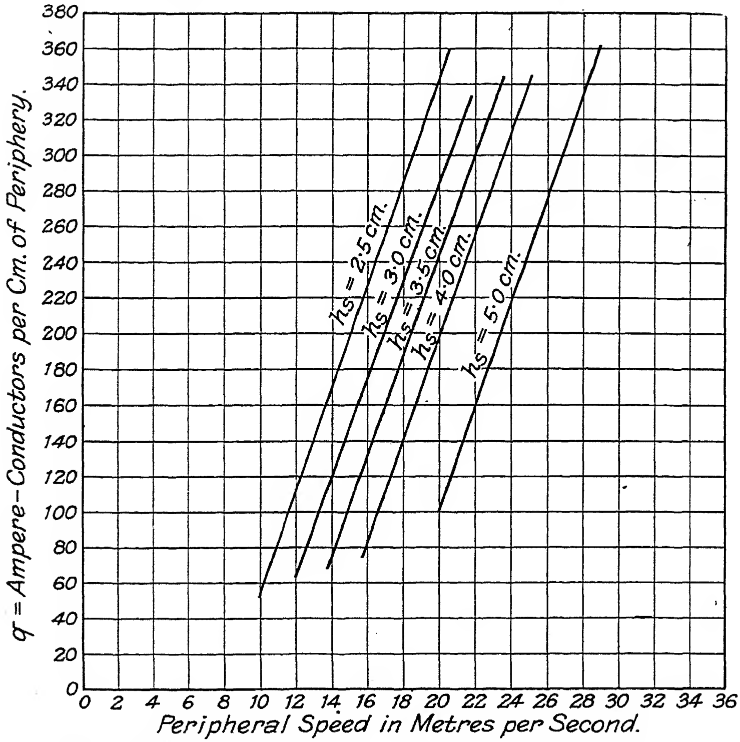


FIG. 100.—RELATION BETWEEN q AND PERIPHERAL SPEED FOR DIFFERENT SLOT DEPTHS

nature approximate only, but they do give a suitable working basis for arriving at suitable values for q. Incidentally it shows the effect of slot depth on the value of q.

We have now determined the specific electric and magnetic loadings, and it now remains to determine the power factors and efficiencies which may reasonably be expected from a well-designed line of machines.

We shall investigate, first, the relation which exists between D and L to give the best power factor.

As is well known, the ratio

$$\frac{\text{magnetizing current per phase}}{\text{ideal short-circuit current per phase}} = \sigma \quad . \quad . \quad (707)$$

which is called the dispersion coefficient, is fundamental in determining the power factor. The smaller this ratio, the higher the power factor.

In calculating the magnetizing current per phase, we shall use the method due to Dr. Max Kloss. He has shown that the ratio of the flux density in the air-gap, at a point one-third of the pole pitch from the zero value, to the average density is nearly constant, and indeed, in calculating the magnetizing current, we shall take all densities at 60° along the pole pitch.

For a three-phase machine $\frac{B60}{B_{av}} = 1{\cdot}28$ . . . (708)

For a two-phase motor $\frac{B60}{B_{av}} = 1{\cdot}18$ . . . (709)

For a single-phase motor $\frac{B60}{B_{av}} = 1{\cdot}57$ . . . (710)

Now the resultant M.M.F. for the machine at 60°

$= 2{\cdot}12 \times I \times q_1 \times z_1$ for a three-phase motor . . (711)

$= 1{\cdot}25 \times I \times q_1 \times z_1$ for a two-phase motor . . (712)

$= 0{\cdot}74 \times I \times q_1 \times z_1$ for a single-phase motor . . (713)

where q_1 = slots per pole per phase in the stator

z_1 = conductors in series per slot

I = current in amperes per conductor

Also the resultant M.M.F. at 60° for a complete magnetic circuit

$$= \frac{1}{4\pi} \times B_{60} \times 2\delta$$

$$= 0{\cdot}796 \times B_{60} \times 2\delta \quad . \quad . \quad . \quad (714)$$

B_{60} = flux density at 60° along the pole pitch measured from zero value

δ = equivalent air-gap length, which includes increase due to slot openings, and also increase due to saturation

$\delta = \delta_1 + \delta_t + \delta_c$

δ_1 = actual gap length increased by Carter's factor

δ_t = equivalent increase in gap length due to tooth saturation

δ_c = increase in gap length due to core saturation

Considering first three-phase machines: since $B_{60} = 1{\cdot}28\, B_{av}$

Magnetizing current per phase in amperes

$$I\mu = \frac{B_{av} \times 1{\cdot}28 \times 2\delta}{0{\cdot}4\pi \times 2{\cdot}12 \times q_1 \times z_1} \quad . \quad . \quad . \quad . \quad (715)$$

The ideal short-circuit current per phase

$$I_{sc} = \frac{\phi}{0{\cdot}4\pi 2\sqrt{2} \times \lambda \times q_1 \times z_1} \quad . \quad . \quad . \quad (716)$$

$$= \frac{\pi D}{p} \times \frac{L \times B_{av}}{0{\cdot}4\pi \times 2\sqrt{2} \times \lambda \times q_1 \times z_1} \quad . \quad (717)$$

$\therefore$ for three-phase machines

$$\sigma = \frac{I\mu}{I_{sc}} = \frac{B_{av} \times 1{\cdot}28 \times 2\delta \times p \times 0{\cdot}4\pi \times 2\sqrt{2} \times \lambda \times q_1 z_1}{0{\cdot}4\pi \times \pi D \times L \times B_{av} \times 2{\cdot}12 \times q_1 \times z_1} \quad (718)$$

$$= \frac{1{\cdot}09 \times p \times \lambda \times \delta}{D \times L} \quad . \quad . \quad . \quad . \quad (719)$$

$$\text{Now } \lambda = \lambda_L + \lambda_T + \lambda_Z \quad . \quad . \quad . \quad . \quad . \quad (720)$$

$$\text{and } \lambda_L = \left\{\frac{\lambda_1}{q_1} + \frac{\lambda_2}{q_2}\right\} \times L \quad . \quad . \quad . \quad . \quad . \quad (721)$$

$$\text{where } \lambda_1 = \frac{h_s}{3\omega_s} \text{ and } \lambda_2 = \frac{h_r}{3\omega_r} \text{ (approx.)}$$

h_s = height of stator slot; ω_s = width of stator slot

h_r = height of rotor slot; ω_r = width of rotor slot

$$\text{Also } \lambda_T = 0{\cdot}73\,(0{\cdot}36\tau + A) \quad . \quad . \quad . \quad . \quad . \quad . \quad (722)$$

τ = pole pitch (*see* section on calculation of short-circuit current)

$$\text{and } \lambda_Z = \frac{L \times \varepsilon \times \tau}{48 \times \delta \times q_1 \times q_2} \quad . \quad . \quad . \quad . \quad . \quad (723)$$

A = length of overhang – pole pitch

$$\therefore \quad \lambda = \left[\frac{h_s}{3\omega_s q_1} + \frac{h_r}{3\omega_r q_2}\right] \times L + 0{\cdot}73(0{\cdot}36\tau + A)$$

$$+ \frac{L\varepsilon\tau}{48 \times \delta \times q_1 \times q_2} \quad . \quad . \quad . \quad . \quad (724)$$

Now since our purpose is to obtain the best ratio of D to L, and not so much to estimate λ with great accuracy, we may make the following simplifications—

(a) h_s and h_r will be assumed equal and $= h$

(b) width of tooth equals width of slot

(c) the overhang permeance will be replaced by $m\tau$

$$3q_1\omega_s \text{ and } 3\omega_r q_2 = \text{slot space per pole} = \frac{\pi D}{2p}$$

Also if $\tau_1 =$ stator slot pitch

$\tau_2 =$ rotor slot pitch

we have $3q_1 p\tau_1 = \pi D = 3q_2\tau_2 p$ . . . . . (725)

$$\therefore q_1 = \frac{\pi D}{3p\tau_1};\ q_2 = \frac{\pi D}{3p\tau_2} \qquad (726)$$

$$\therefore \lambda = \frac{4hpL}{\pi D} + \frac{m\pi D}{p} + \frac{L\varepsilon\pi D \times \tau_1 \times \tau_2 \times 9p^2}{48 \times p \times \delta \times \pi^2 D^2} \qquad (727)$$

$$= \frac{4hpL}{\pi D} + \frac{m\pi D}{p} + \frac{L\varepsilon \times \tau_1\tau_2 \times p \times 9}{\delta \times \pi \times D \times 48} \qquad (728)$$

$$\therefore \sigma = \frac{1{\cdot}09 \times p \times \delta}{D \times L}\left[\frac{4hpL}{\pi D} + \frac{m\pi D}{p} + \frac{9 \times \varepsilon \times \tau_1 \times \tau_2 \times p \times L}{48 \times \pi \times D \times \delta}\right] \qquad (729)$$

$$\therefore \sigma = \frac{1{\cdot}09 \times p \times \delta}{D^2 L}\left[\left(\frac{4hp}{\pi} + \frac{9 \times \varepsilon \times \tau_1\tau_2 p}{48 \times \pi \times \delta}\right)L + \frac{m\pi D^2}{p}\right] \qquad (730)$$

Now D^2L is a constant for any given output and speed

$= \beta$, say,

and the expression in brackets $= aL + bD^2$ . . . (731)

$= y$ suppose

$$\therefore y = \frac{a\beta}{D^2} + bD^2 \qquad (732)$$

$$= \frac{a\beta + bD^4}{D^2} \qquad (733)$$

$$\frac{1}{y} = \frac{D^2}{a\beta + bD^4} \qquad (734)$$

$$\frac{d\left(\frac{1}{y}\right)}{d(D)} = \frac{2D(a\beta + bD^4) + 4bD^5}{(a\beta + bD^4)^2} \qquad (735)$$

For a maximum this must be zero,

$$\therefore\ a\beta + bD^4 = 2bD^4 \quad . \quad . \quad . \quad . \quad . \quad . \quad (736)$$

$$\therefore\ a\beta = bD^4 \quad . \quad . \quad . \quad . \quad . \quad . \quad (737)$$

$$\therefore\ \frac{a\beta}{D^2} = bD^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (738)$$

$$\text{i.e.}\quad aL = bD^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (739)$$

i.e. y is a minimum when $\frac{1}{y}$ is a maximum,

and we arrive at the conclusion that minimum value of σ occurs, or maximum power factor, *when the slot leakage flux equals the end connection leakage flux,* for the two expressions represent the slot permeance and overhang permeance respectively.

This then gives us the best ratio of D to L

$$\frac{m\pi D^2}{p} = \left[\frac{4hp}{\pi} + \frac{9\varepsilon\tau_1\tau_2 p}{48 \times \pi \times \delta}\right] \times L \quad . \quad . \quad (740)$$

$$\therefore\ \frac{D^2}{L} = \frac{p^2}{m\pi^2}\left[4h + \frac{9\varepsilon\tau_1\tau_2}{48 \times \delta}\right] \quad . \quad . \quad (741)$$

$$\therefore\ \frac{1}{L} = \frac{p^2}{m\pi^2 D^2}\left[4h + \frac{9}{48}\frac{\varepsilon\tau_1\tau_2}{\times \delta}\right] \quad . \quad . \quad . \quad (742)$$

$$= \frac{1}{m\tau^2}\left[4h + \frac{9}{48}\frac{\varepsilon\tau_1\tau_2}{\times \delta}\right] \quad . \quad . \quad . \quad (743)$$

$$\therefore\ \frac{\tau}{L} = \frac{1}{\tau}\left[\frac{4h}{m} + \frac{9}{48}\frac{\varepsilon\tau_1\tau_2}{\delta \times m}\right] \quad . \quad . \quad . \quad (744)$$

We will take average values for the various quantities, viz., $h = 2{\cdot}5$ cm.; $m = 1{\cdot}5$, $\tau_1 = \tau_2 = 1{\cdot}5$ cm.; and $\delta = 0{\cdot}035$ cm. (minimum value);

$$\text{then}\ \frac{\tau}{L} = \frac{1}{\tau}\left[\frac{10}{1{\cdot}5} + \frac{9 \times 1{\cdot}5 \times 1{\cdot}5 \times 1{\cdot}6}{48 \times 0{\cdot}035 \times 1{\cdot}5}\right]. \quad . \quad (745)$$

$$= \frac{19{\cdot}47}{\tau}$$

This relation will hold for small machines;

for large machines, $h_s = 4{\cdot}5$ and $\delta = 0{\cdot}09$ cm.

$$\therefore\ \frac{\tau}{L} = \frac{1}{\tau}\left[\frac{18}{1{\cdot}5} + \frac{9 \times 1{\cdot}6 \times 1{\cdot}5 \times 1{\cdot}5}{48 \times 0{\cdot}09 \times 1{\cdot}5}\right] \quad (746)$$

$$= \frac{17}{\tau} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (747)$$

We shall not be far from the truth if we take an average value of $\frac{\tau}{L} = \frac{18}{\tau}$, and this relation is given in the curve below. Fig. 101.

It will be observed that the relation between $\frac{\text{pole pitch}}{\text{core length}}$ and pole pitch is a rectangular hyperbola, the equation to which is yx constant.

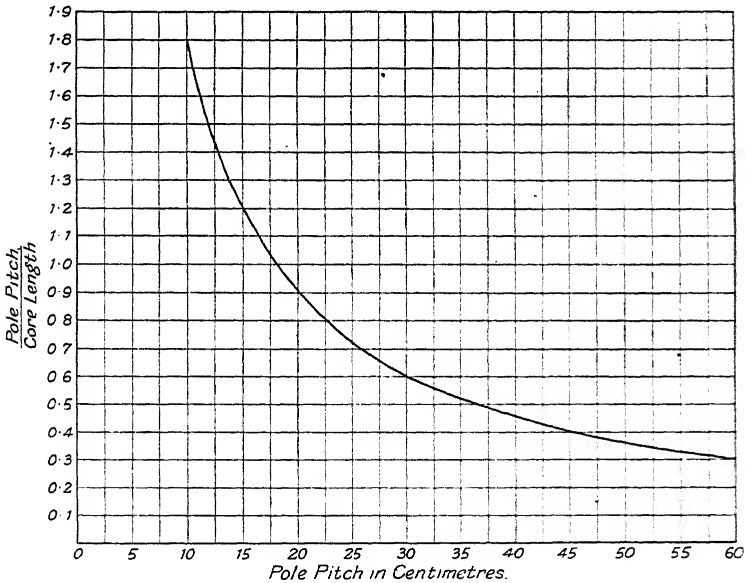


FIG. 101.—RELATION OF $\frac{\text{POLE PITCH}}{\text{CORE LENGTH}}$ AND POLE PITCH FOR BEST POWER FACTOR $\frac{\tau}{L} = \frac{18}{\tau}$

It shows clearly that the larger the pole pitch, the smaller the ratio of D to L becomes, and the smaller the pole pitch, the larger the ratio of D to L.

$$\text{Now since } \frac{\tau}{L} = \frac{18}{\tau} \qquad . \quad . \quad . \quad . \quad (748)$$

$$\frac{\tau^2}{L} = 18 \qquad . \quad . \quad . \quad . \quad . \quad (749)$$

$$\text{and since } \tau = \frac{\pi D}{p} \quad \therefore \; \tau^2 = \frac{\pi^2 D^2}{p^2} \quad . \quad (750)$$

$$\therefore \; \frac{\pi^2 D^2}{p^2 L} = 18 \quad . \quad . \quad . \quad . \quad . \quad (751)$$

$$\therefore \; D^2 = \frac{18 p^2 L}{\pi^2} = 1{\cdot}82 p^2 L \quad . \quad . \quad (752)$$

$$\text{and} \; \therefore \; D = 1{\cdot}35 p \sqrt{L} \quad . \quad . \quad . \quad . \quad (753)$$

The following curves show this relation for various numbers of poles.

It is now our business to determine the values of the dispersion coefficient σ for different values of D and L, and for different

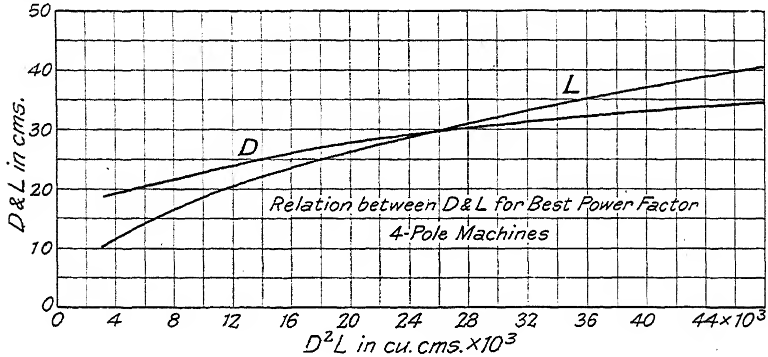


FIG. 102

numbers of poles, and from this value we shall deduce the power-factors which can be expected at full load. In order to do this with the greatest possible accuracy, we require to determine the relation between diameter and length of core and length of gap. As is well known, the shorter the air-gap the better, and this is determined by the safe mechanical clearance. The values of σ are calculated from the following equation—

$$\sigma = \frac{1{\cdot}09 \times p \times \delta}{D^2 L} \left[\left\{ \frac{4hp}{\pi} + \frac{9\varepsilon\tau_1\tau_2 p}{\delta \times 48 \times \pi} \right\} L + \frac{m\pi D^2}{p} \right] \quad (754)$$

The following are the smallest mechanical clearances for four-pole machines—

D_{cm}	δ_{cm}	D_{cm}	δ_{cm}
15	0·035	45	0·13
21	0·05	55	0·18
25	0·06	65	0·25
30	0·07	80	0·4
40	0·12		

Now we have seen that for minimum value of σ

$$aL = bD^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (755)$$

$$\therefore \sigma = \frac{1{\cdot}09 \times p \times \delta}{D^2L} \times \frac{2m\pi D^2}{p} \quad . \quad . \quad . \quad (756)$$

$$\therefore \sigma = \frac{1{\cdot}09 \times \delta \times 2m\pi}{L} \quad . \quad . \quad . \quad . \quad . \quad (757)$$

It would appear therefore that, whatever the number of poles, if

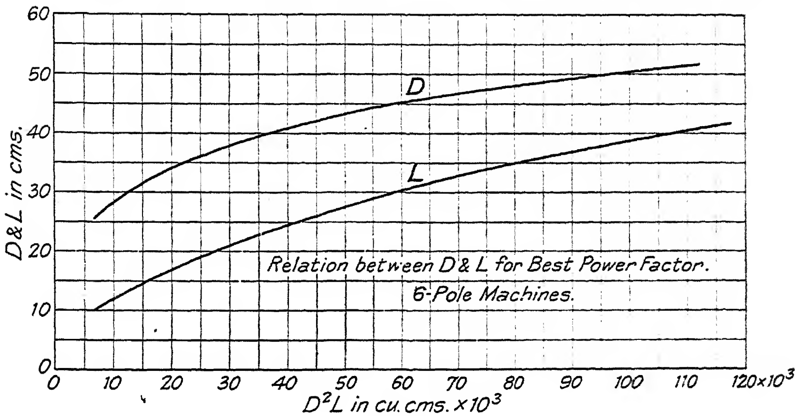


FIG. 103

the ratio of D to L were fixed to give minimum value of σ, that value would vary between the limits of

$$\sigma = \frac{1{\cdot}09 \times 0{\cdot}035 \times 2 \times 1{\cdot}5 \times \pi}{L} = \frac{1{\cdot}09 \times 0{\cdot}035 \times 3 \times \pi}{10} \quad (758)$$

(taking L as 10 cm. as minimum core length)

$$= 0{\cdot}036$$

$$\text{and } \sigma = \frac{1{\cdot}09 \times 0{\cdot}4 \times 3 \times \pi}{80} = 0{\cdot}041 \quad . \quad . \quad (759)$$

taking extreme limits for σ of 0·035 and 0·4, and limits for L = 10 and 80 cm. respectively.

If such conditions could be realized in a standard line of machines, we should have power factors at full load of over 90 per cent, and the much-debated question of power-factor correction would be dismissed for ever. Alas ! such conditions cannot be realized. One has to build on *one* frame all numbers of poles up to the limit of that frame.

It is true, even with one frame, that the diameter of the stator

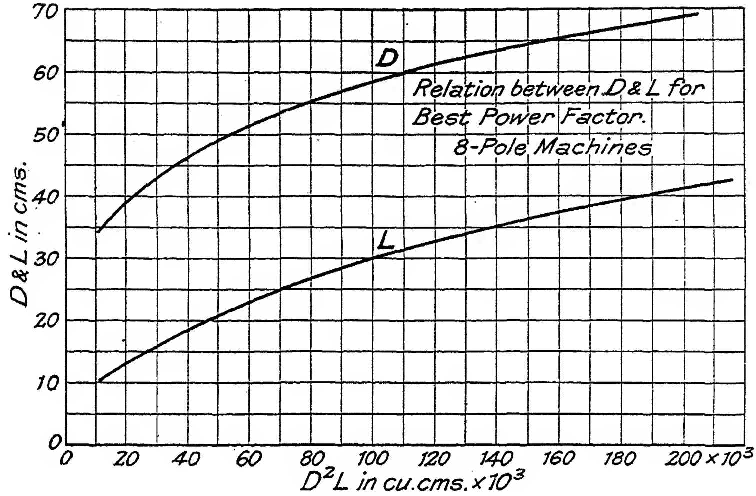


FIG. 104

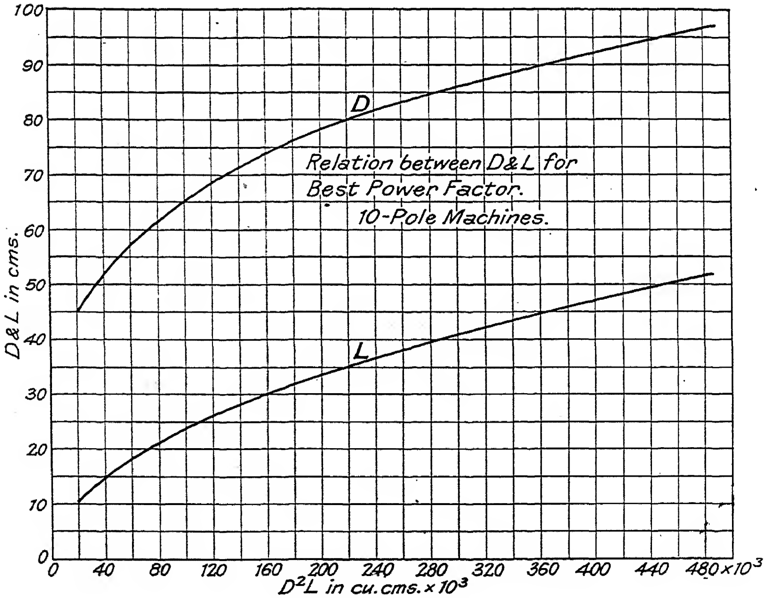


FIG. 105

bore is increased as the number of poles is increased, but the increase can only be slight.

If one chooses, for a given frame, the most suitable ratio of D to L for 6 poles or 8 poles, then one gets much poorer power factor on the other numbers of poles.

Now a line of machines is invariably standardized for numbers

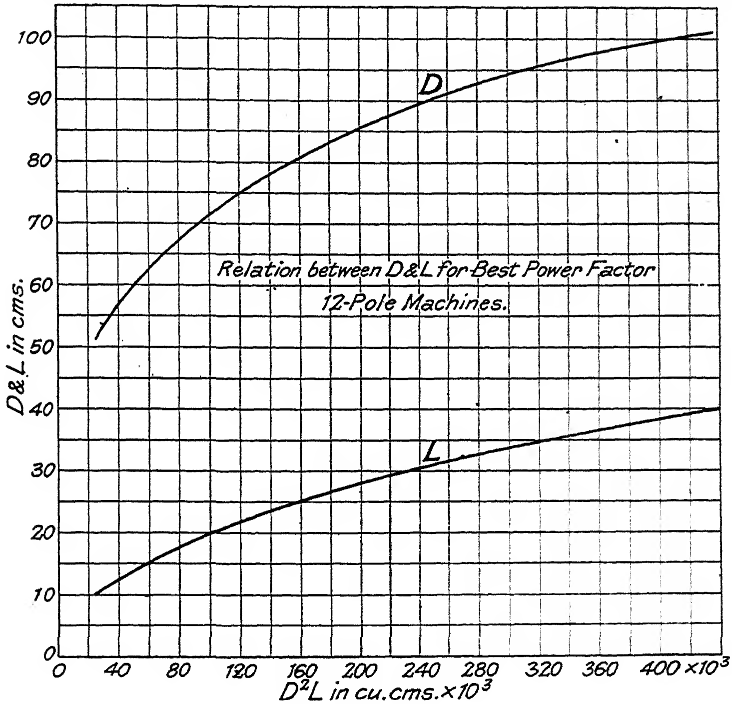


FIG. 106

of poles up to 12 and for outputs up to about 200 h.p. Beyond this range, one is fairly free to choose a ratio of D to L which is best from power-factor considerations, always paying regard to the question of cost.

It is an instructive exercise to choose the relation of D to L for best power factor for 6 poles, and work out the corresponding values of σ and the power factor at full load. This can readily be done from the equation

$$\sigma = \frac{1\cdot 09 \times p \times \delta}{D^2L}\left[\left(\frac{4hp}{\pi} + \frac{9\varepsilon\tau_1\tau_2 p}{48 \times \pi \times \delta}\right)L + \frac{m\pi D^2}{p}\right] \quad . \quad (760)$$

and putting in the appropriate values for the various quantities. The power factors at any load can be found from the power-factor curves below, which are taken from Miles-Walker's book.

The following values are calculated for a ratio of D to L for 8 poles for best power factor and with suitable air-gap lengths.

For 4-pole machines—

D	$\cos\phi$	D	$\cos\phi$
15	79½%	40	90¾%
20	86 %	45	92 %
25	88 %	55	94 %
30	90½%	80	93 %

Power factor for 6 poles. D to L chosen for 8 poles—

D	$\cos\phi$	D	$\cos\phi$
15	74 %	40	89½%
20	81 %	50	90·5%
25	86 %	60	92·5%
30	88½%	80	92 %

Power factor for 8 poles. D to L chosen for 8 poles—

D	$\cos\phi$	D	$\cos\phi$
15	68½%	50	91 %
20	74½%	60	92 %
25	79 %	80	94 %
30	84 %	90	94 %
35	87½%		

10 poles. D to L chosen for 8 poles—

D	$\cos\phi$	D	$\cos\phi$
30	83 %	60	93 %
40	90½%	80	93·2%
45	91 %	90	93 %

It is not suggested that the ratio of D to L should be chosen for 8 poles for best power factor for a given frame, for indeed the core lengths would be excessively small on 4 and 6 poles, but the tables above show clearly what power factors may be expected. If, as is usually the case, the ratio of D to L is chosen for best power factor for 4 or 6 poles, then we should expect much lower power factors on 8 and 10 poles.

The efficiencies which may be expected on a range of induction motors are given in the following series of curves.

We will now develop an expression for the power factor of an induction motor at any load. (*See* Fig. 112.)

The line KC represents the output line.

$$\text{The max. H.P.} = \frac{mV \times EF}{746} \qquad (761)$$

where m = number of phases

V = applied volts per phase

$$\text{Now } EF = \frac{KC}{2(1 + \cos CKY)} = \frac{KC}{2(1 + \sin CKX)} \qquad (762)$$

Let us take our origin at K and the axis of x along OX. We

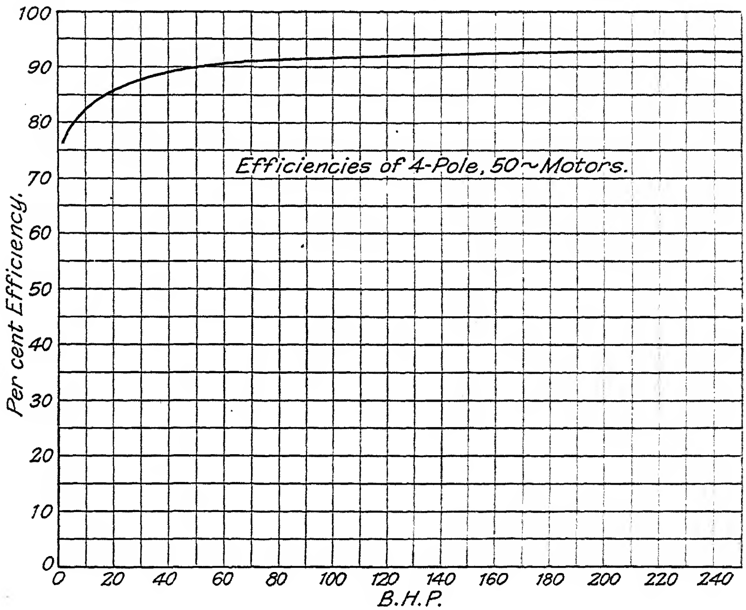


FIG. 107

require to determine the power factor at any given load, which is a definite fraction of the maximum load.

The primary current is OH, and the power factor $= \dfrac{HL}{OH}$

$$= \frac{HL}{\sqrt{HL^2 + OL^2}} \quad . \quad . \quad . \quad . \quad . \quad (763)$$

In the first place, we determine the x co-ordinate of EG.

The equation to the circle is $(x - r)^2 + y^2 = r^2$ . . (764)

$$\text{i.e. } y^2 = 2rx - x^2 \quad . \quad . \quad . \quad . \quad . \quad (765)$$

$$y = \pm\sqrt{2rx - x^2} \quad . \quad . \quad . \quad . \quad . \quad (766)$$

Also $FG = y_1 = m_1 x$, where m_1 is the tangent of the angle CKD.

$$\therefore\ EF = y - y_1 = \sqrt{2rx - x^2} - m_1 x \qquad . \qquad . \qquad (767)$$

This is a maximum when $\dfrac{d\,(y - y_1)}{dx} = 0$ . . . (768)

$$\frac{d}{dx}(y - y_1) = \frac{r - x}{\sqrt{2rx - x^2}} - m_1 \qquad . \qquad . \qquad . \qquad . \qquad (769)$$

for a max. $\therefore\ r - x = m_1 \sqrt{2rx - x^2}$ . . . . (770)

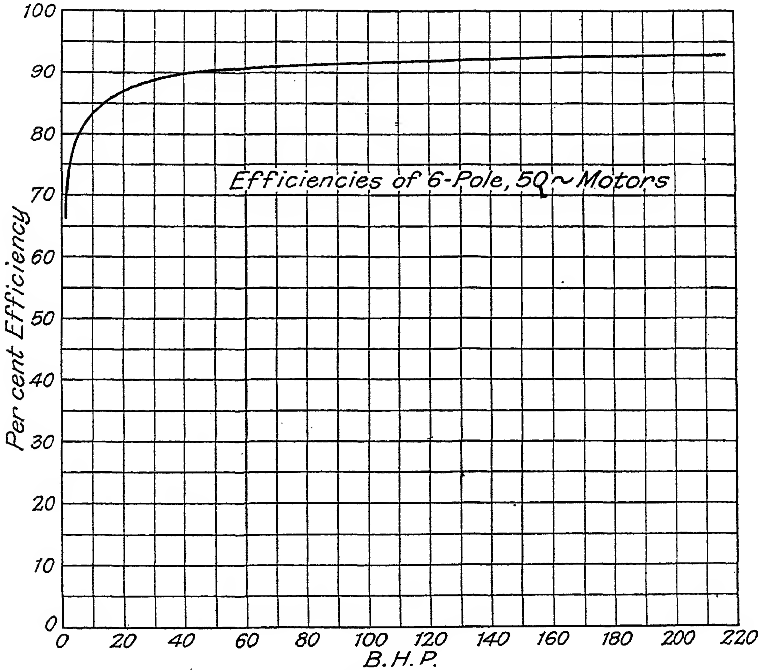


FIG. 108

$$\therefore\ r^2 - 2rx + x^2 = m_1^2\, 2rx - m_1^2 x^2 \qquad . \qquad . \qquad . \qquad (771)$$

i.e. $x^2(1 + m_1^2) - x(2rm_1^2 + 2r) + r^2 = 0$ . . (772)

$$\therefore\ x = r\left[1 \pm \frac{m_1}{\sqrt{1 + m_1^2}}\right] \qquad . \qquad . \qquad . \qquad (773)$$

and since it is obviously less than r, we must take the negative sign

$$\therefore\ x = r\left[1 - \frac{m_1}{\sqrt{1 + m_1^2}}\right] \qquad . \qquad . \qquad . \qquad (774)$$

Also since the angle in a semicircle is a right angle, we have

$$EG^2 = KG \times GX = x(2r - x) \quad . \quad . \quad . \quad . \quad (775)$$

Now let $x_1 = x$ co-ordinate of primary current vector OH.

Also $HJ^2 = KJ \times JX = x_1(2r - x_1) \quad . \quad . \quad . \quad . \quad (776)$

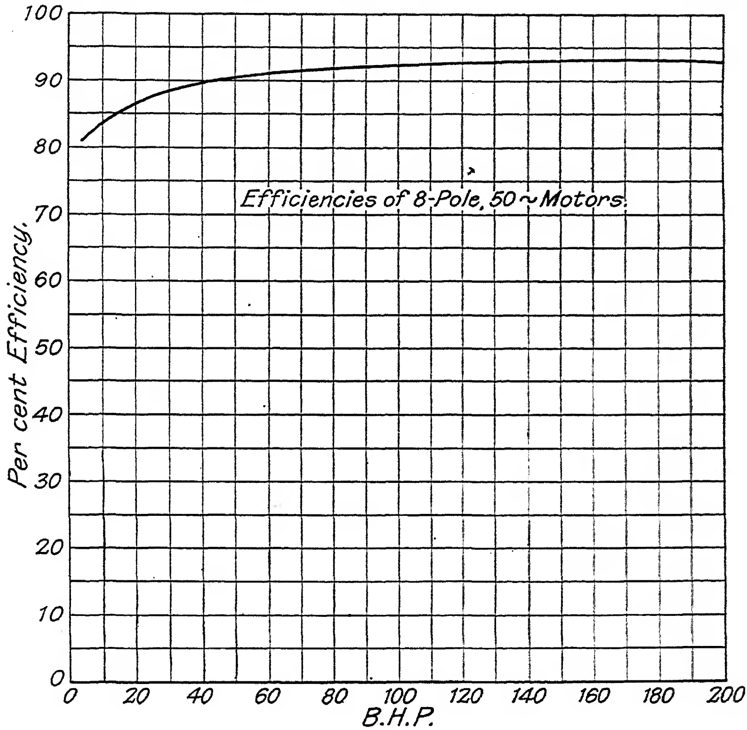


FIG. 109

$$EG^2 = x(2r - x) \quad . \quad . \quad . \quad . \quad . \quad . \quad (777)$$

$$= r\left(1 - \frac{m_1}{\sqrt{1 + m_1^2}}\right)\left(2r - r + \frac{rm_1}{\sqrt{1 + m_1^2}}\right) \quad . \quad (778)$$

$$= \frac{r^2}{1 + m_1^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (779)$$

It will simplify our calculations if we put in the values of m_1 with which we have to deal.

For small motors, $m_1 = \tan 30° \quad = 0{\cdot}5774$

,, large ,, $m_1 = \tan 14° \, 30' = 0{\cdot}2586$

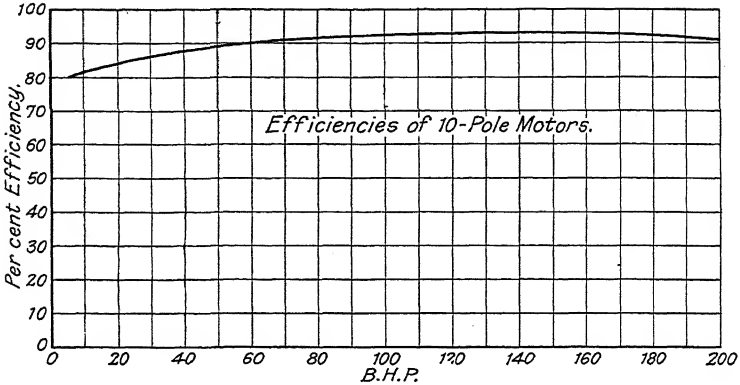


FIG. 110

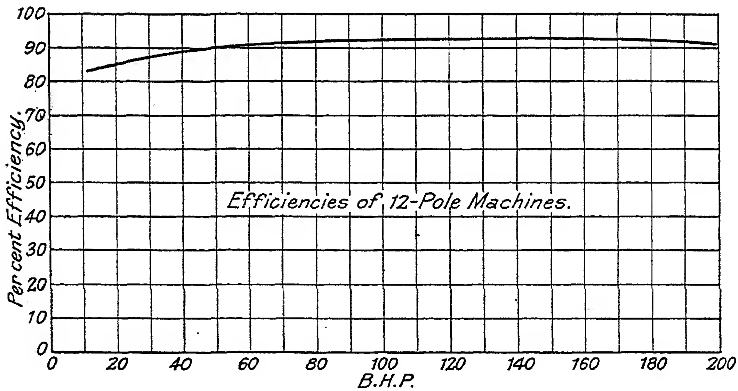


FIG. 111

$\therefore$ for small machines $EG = \dfrac{r}{\sqrt{1 + m_1^2}} = \dfrac{r}{\sqrt{1 + 0{\cdot}334}} = \dfrac{r}{1{\cdot}152}$

$$= 0{\cdot}866\, r \quad . \quad . \quad . \quad . \quad (780)$$

and for large machines $EG = 0{\cdot}967\, r \quad . \quad . \quad . \quad . \quad (781)$

$$\text{Also } EF = \frac{KC}{2(1 + \sin CKD)} = \frac{2r\cos CKD}{2(1 + \sin CKD)} \quad . \quad . \quad (782)$$

$$\cos CKD = \frac{1}{\sqrt{1 + m_1^2}}; \quad \sin CKD = \frac{m_1}{\sqrt{1 + m_1^2}} \quad (783)$$

$$\therefore \ EF = \frac{r}{m_1 + \sqrt{1 + m_1^2}} \quad . \quad . \quad (784)$$

and putting in the values of m_1,

we have for small machines, $EF = 0{\cdot}58\,r$ . . . (785)

" large " $EF = 0{\cdot}775\,r$ . . (786)

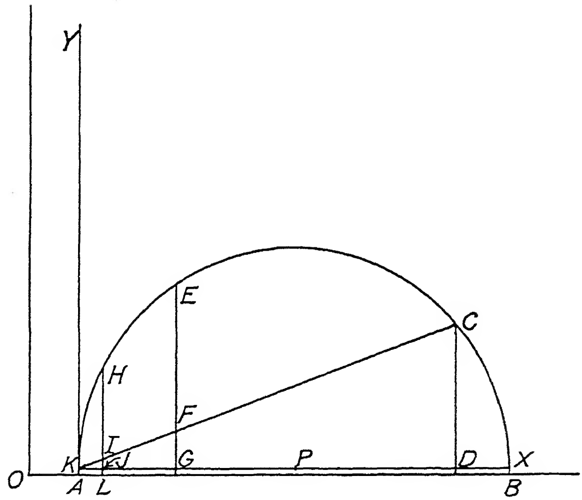


FIG. 112

Also for small machines,

$$FG = EG - EF = 0{\cdot}866\,r - 0{\cdot}58\,r = 0{\cdot}286\,r \quad . \quad . \quad (787)$$

For large machines, $FG = 0{\cdot}192\,r$ . . . . (788)

Now let $HI = KEF$, i.e. let us require to find the power factor at a load which is K times the maximum.

In practice we shall require the values of $\cos \phi$ chiefly at full load, $\frac{3}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$ full load.

If $\dfrac{\text{full load}}{\text{max. load}} = \frac{1}{2}$, then at full load $K = 0{\cdot}5$

$\frac{3}{4}$ " $K = 0{\cdot}375$

$\frac{1}{2}$ " $K = 0{\cdot}25$

$\frac{1}{4}$ " $K = 0{\cdot}125$

$$\text{Now } HJ = HI + IJ \quad . \quad . \quad . \quad . \quad (789)$$

$$= KEF + FG \times \frac{KJ}{KG} \quad . \quad . \quad (790)$$

$$= KEF + FG \times \frac{x_1}{x} \quad . \quad . \quad . \quad (791)$$

$$\text{Also } HJ^2 = KJ \times JX = x_1 (2r - x_1) \quad . \quad . \quad (792)$$

For small machines,

$$HJ = K \times 0{\cdot}58\, r + 0{\cdot}286\, r \times \frac{x_1}{r\left(1 - \frac{m_1}{\sqrt{1 + m_1^2}}\right)} \quad . \quad (793)$$

$$= 0{\cdot}58\, Kr + \frac{0{\cdot}286\, x_1}{1 - \frac{0{\cdot}5774}{1{\cdot}152}} = 0{\cdot}58\, Kr + 0{\cdot}572\, x_1 \quad . \quad (794)$$

$$HJ^2 = 0{\cdot}336\, K^2r^2 + 0{\cdot}663\, Krx_1 + 0{\cdot}327\, x_1^2 \quad . \quad . \quad (795)$$

$$\therefore\ 0{\cdot}336\, K^2r^2 + 0{\cdot}663\, Krx_1 + 0{\cdot}327\, x_1^2 = 2rx_1 - x_1^2 \quad . \quad (796)$$

$$\therefore\ 1{\cdot}327\, x_1^2 + x_1 (0{\cdot}663\, Kr - 2r) + 0{\cdot}336\, K^2r^2 = o \quad . \quad (797)$$

$$\therefore\ x_1 = r\left[\frac{2 - 0{\cdot}663\, K \pm \sqrt{4 - 2{\cdot}65\, K - 1{\cdot}35\, K^2}}{2{\cdot}654}\right]$$

If we substitute in this expression the various values for K, viz., 0·5, 0·375, 0·25, 0·125, we get for x_1 the following series of values—

K	x_1	$HJ = 0{\cdot}58\, Kr + 0{\cdot}572\, x_1$
0·5	$0{\cdot}054\, r$	$= 0{\cdot}3209\, r$
0·375	$0{\cdot}0272\, r$	$= 0{\cdot}233\, r$
0·25	$0{\cdot}0111\, r$	$= 0{\cdot}151\, r$
0·125	$0{\cdot}0035\, r$	$= 0{\cdot}074\, r$

The value of $\cos \phi$ at any load

$$= \frac{HJ + JL}{\sqrt{(HJ + JL)^2 + (OA + x_1)^2}} \quad . \quad . \quad (798)$$

In any given case, let $HJ = ar$ and $x_1 = \beta r$

$$\text{and let } JL = \frac{1}{20} OA$$

$$a = 0{\cdot}58 K + 0{\cdot}572 \left[\frac{2 - 0{\cdot}633 K - \sqrt{4 - 2{\cdot}65 K - 1{\cdot}35 K^2}}{2{\cdot}654}\right] \quad (799)$$

$$\beta = \left[\frac{2 - 0{\cdot}663 K - \sqrt{4 - 2{\cdot}65 K - 1{\cdot}35 K^2}}{2{\cdot}654}\right] \quad . \quad . \quad . \quad (800)$$

$\cos\phi$ at any load

$$= \frac{ar + \frac{1}{20}OA}{\sqrt{\left(ar + \frac{1}{20}OA\right)^2 + (OA + \beta r)^2}} \quad . \quad . \quad . \quad . \quad (801)$$

$$= \frac{ar + \frac{1}{20}OA}{\sqrt{r^2(\alpha^2 + \beta^2) + r\,OA\left(2\alpha\frac{1}{20} + 2\beta\right) + OA^2\left(\frac{1}{400} + 1\right)}}$$

(802)

Now $\sigma = \dfrac{OA}{OA + 2r} = \dfrac{1}{1 + \dfrac{2r}{OA}}$

$$1 + \frac{2r}{OA} = \frac{1}{\sigma}$$

$$\therefore \frac{r}{OA} = \left(\frac{1}{\sigma} - 1\right) \times \frac{1}{2}$$

$\cos\phi$ at any load

$$= \frac{\alpha\frac{r}{OA} + \frac{1}{20}}{\sqrt{\frac{r^2}{OA^2}(\alpha^2 + \beta^2) + \frac{r}{OA}\left(2\alpha\frac{1}{20} + 2\beta\right) + \frac{1}{400} + 1}} \quad (803)$$

$\therefore$ $\cos\phi$ at any load

$$= \frac{\alpha\left\{\frac{1}{2}\left(\frac{1}{\sigma} - 1\right)\right\} + \frac{1}{20}}{\sqrt{(\alpha^2 + \beta^2)\left\{\frac{1}{2}\left(\frac{1}{\sigma} - 1\right)\right\}^2 + \frac{1}{2}\left(\frac{1}{\sigma} - 1\right)\left\{2\alpha\frac{1}{20} + 2\beta\right\} + \frac{1}{400} + 1}}$$

(804)

We will now tabulate the various curves of the power factor at full, $\frac{3}{4}$ full, $\frac{1}{2}$ full, and $\frac{1}{4}$ full load for various values of σ. This set applies to small motors where the power factor at short circuit $= 0{\cdot}5$.

The curves shown in Fig. 113 are extremely useful in enabling one to predict with fair accuracy the power factor at any given load for various values of σ.

We will now extend our investigations as regards the ratio of D to L for two-phase motors.

σ	Full Load.	Cos ϕ ¾ Full Load.	½ Full Load.
0·03	94%	93%	90%
0·04	93%	91·5%	86·5%
0·06	89%	86%	78%
0·07	86·5	83%	73·5%
0·08	84·25%	79·5	69%
0·09	81·75%	76·5%	64·5%
0·1	80%	73·5%	60·5%
0·12	75·5%	69%	54%
0·14	71%	63%	49·5%
0·16	66%	58%	43·5%

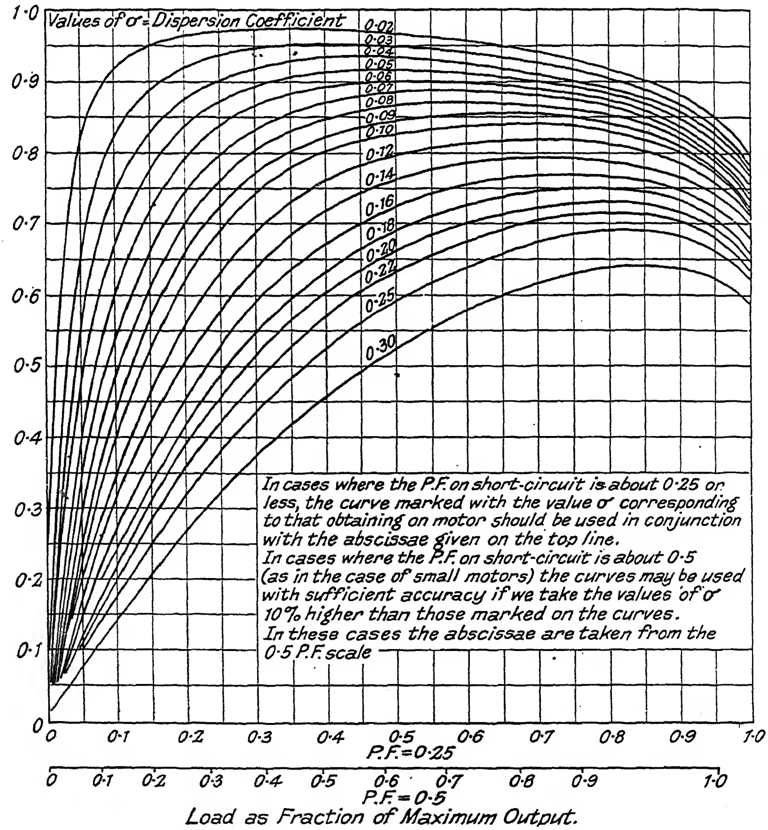


FIG. 113

We have $$I\mu = \frac{B_{av} \times 1{\cdot}18 \times 2\delta}{0{\cdot}4\pi \times 1{\cdot}25 \times q_1 \times z_1} \quad . \quad . \quad . \quad (805)$$

$$I_{sc} = \frac{\phi}{0{\cdot}4\pi \times 2\sqrt{2} \times \lambda \times q_1 \times z_1} \quad . \quad . \quad . \quad (806)$$

$$= \frac{\pi D}{p} \times \frac{L \times B_{av}}{0{\cdot}4\pi 2\sqrt{2} \times \lambda \times q_1 \times z_1} \quad . \quad . \quad (807)$$

$$\therefore \frac{I\mu}{I_{sc}} = \frac{1{\cdot}709\, p \times \lambda \times \delta}{D \times L} \quad . \quad . \quad . \quad . \quad (808)$$

Also $$\lambda = \left[\frac{h_s}{3\omega_s q_1} \times \frac{h_r}{3\omega_r q_2}\right] \times L + \frac{m\pi D}{p} + \frac{L\varepsilon\tau}{32 \times \delta \times q_1 q_2} \quad (809)$$

Assuming width of slot = width of tooth,

$$2\omega_s q_1 = \frac{\pi D}{2p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (810)$$

$$\therefore 3\omega_s q_1 = \frac{3}{4}\frac{\pi D}{p} \quad . \quad . \quad . \quad . \quad . \quad . \quad (811)$$

Using a three-phase rotor, which is usual,

$$3q_2\omega_r = \frac{\pi D}{2p} = \frac{2\pi D}{4p} \quad . \quad . \quad . \quad . \quad . \quad (812)$$

Also assuming $h_s = h_r = h$,

$$q_1 = \frac{\pi D}{2p\tau_1},\ q_2 = \frac{\pi D}{3p\tau_2} \quad . \quad . \quad . \quad . \quad . \quad (813)$$

and $$\lambda = \frac{4hp}{\pi D}\left(\frac{1}{3} + \frac{1}{2}\right) L + \frac{m\pi D}{p} + \frac{L\varepsilon\pi D \times 2p\tau_1 3p\tau_2}{32 \times \delta \times p \times \pi^2 D^2} \quad (814)$$

$$= \frac{5}{6}\frac{4hp}{\pi D} L + \frac{m\pi D}{p} + \frac{6}{32} \times \frac{\varepsilon\tau_1\tau_2 \times p \times L}{\delta \times \pi D} \quad . \quad . \quad (815)$$

$$\therefore \sigma = \frac{1{\cdot}709 \times p \times \delta}{D^2 L}\left[\left(\frac{5}{6} \times \frac{4hp}{\pi} + \frac{6}{32}\frac{\varepsilon\tau_1\tau_2 p}{\delta \times \pi}\right) L + \frac{m\pi D^2}{p}\right] \quad (816)$$

As proved before, this is a minimum

$$\text{when } aL = bD^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (817)$$

where $$a = \frac{5}{6} \times \frac{4hp}{\pi} + \frac{6}{32}\frac{\varepsilon\tau_1\tau_2 \times p}{\delta \times \pi} \text{ and } b = \frac{\pi m}{p} \quad . \quad . \quad (818)$$

$$\therefore \frac{D^2}{L} = \frac{p^2}{m\pi^2}\left[\frac{5}{6} \times 4h + \frac{6}{32}\frac{\varepsilon\tau_1\tau_2}{\delta}\right] \quad . \quad . \quad . \quad . \quad (819)$$

$$\text{i.e. } \frac{\tau}{L} = \frac{1}{\tau}\left[\frac{5}{6} \times \frac{4h}{m} + \frac{6}{32}\frac{\varepsilon\tau_1\tau_2}{\delta \times m}\right] \quad . \quad . \quad . \quad . \quad (820)$$

$$= \frac{1}{\tau} \times 18{\cdot}37 \text{ for small machines approx.} \quad . \quad . \quad (821)$$

$$= \frac{1}{\tau} \times 15 \text{ for large machines} \quad . \quad . \quad . \quad . \quad (822)$$

so that practically the same values hold as for three-phase machines.

From the above equation, σ can be estimated readily; and by using the expression, already deduced, the corresponding power factor at any load.

Before proceeding with the design proper, we shall deal with the calculation of the magnetizing current and the ideal short-circuit current.

The magnetizing current. The flux per pole can be found from the E.M.F. equation,

$$E = K \times \phi \times Z \times f \times 10^{-8} \times \sin \psi/2$$

$$\phi = \frac{E \times 10^8}{K \times Z \times f \times \sin \psi/2} \quad . \quad . \quad . \quad . \quad . \quad (823)$$

where ϕ = flux per pole in C.G.S. lines

Z = conductors in series per phase

f = supply frequency in cycles per second

K = 2·12 for a three-phase motor

= 2·02 for a two-phase motor

= 1·85 for a single-phase machine

ψ = span of the coil in electrical degrees

Owing to saturation of the teeth, even with a sinusoidal spacial distribution of the M.M.F. over the pole pitch, the flux curve becomes flattened.

It is generally assumed that the rotating flux wave is sinusoidal in space variation.

The flattened flux wave can be analysed into its harmonics, and the fundamental is the only one which is really effective in producing counter E.M.F.

We require now to find the shape of the flux wave, and we will proceed to show that the ratio of

$$\frac{\text{flux density at one third along the pole pitch}}{\text{mean flux density}}$$

is nearly constant. If we know the magnetization curve, we can find the ampere-turns at one-third along the pole pitch, which are required to drive the flux density through the magnetic circuit, and hence find the magnetizing current.

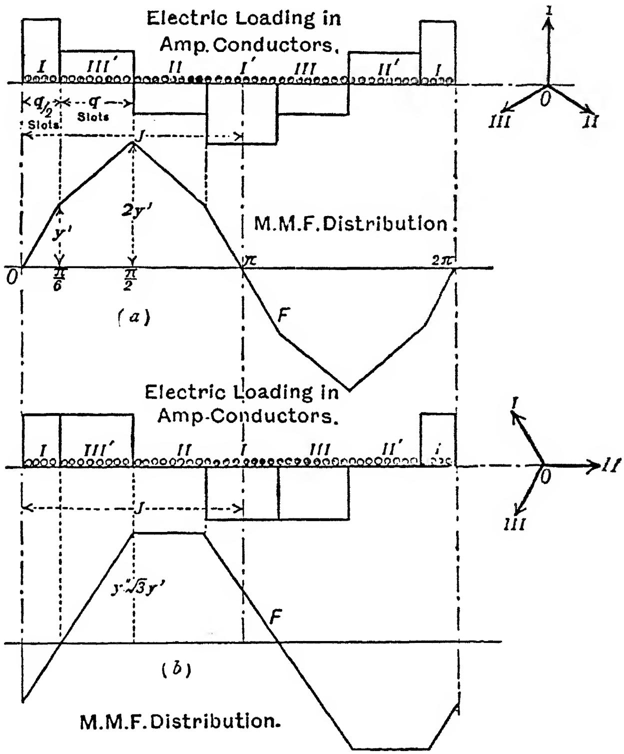


FIG. 114

Assuming the magnetizing current to follow a sine law with respect to time—an assumption involving a slight error—the curve of distribution of electric loading is shown in Fig. 114.

The curves of M.M.F. over the pole pitch when the current in phase I is at its maximum and, secondly, when the current in Phase II is zero, are shown above.

$$\text{It is clear that } y' = \frac{q_1}{2} z_1 I_{max} = \frac{q_1}{\sqrt{2}} \times z_1 I_{eff}. \qquad (824)$$

q_1 = slots per pole per phase

z_1 = conductors per slot

I_{max} = max. current per conductor

I_{eff} = effective or R.M.S. value of current

These are the two limiting cases, between which the M.M.F. fluctuates at a frequency six times that of the fundamental.

The equation to the pointed curve is

$$y = \frac{18}{\pi^2} y' \left(\sin \theta + \frac{1}{25} \sin 5\theta - \frac{1}{49} \sin 7\theta \ldots \right) \quad . \quad (825)$$

The equation for the flat-topped curve is

$$y = \frac{18}{\pi^2} y' \left(\sin \theta - \frac{1}{25} \sin 5\theta + \frac{1}{49} \sin 7\theta \ldots \right) \quad . \quad (826)$$

The fundamental has an amplitude

$$= \frac{18}{\pi^2} y' \text{ in each case} \quad . \quad . \quad . \quad . \quad (827)$$

$$= \frac{18}{\pi^2} \times \frac{1}{\sqrt{2}} q_1 z_1 I_{eff} \quad . \quad . \quad . \quad . \quad (828)$$

$$= 1{\cdot}29 q_1 z_1 I_{eff} \quad . \quad . \quad . \quad . \quad . \quad (829)$$

For a complete magnetic path, the maximum amp.-turns

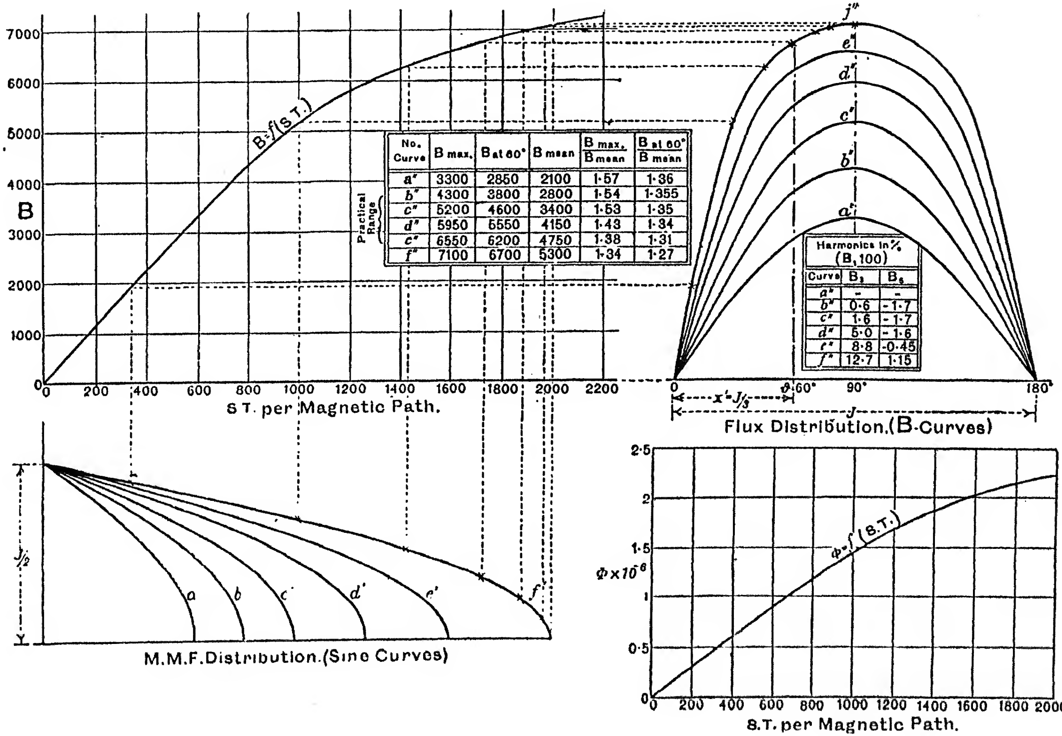
$$= 2{\cdot}58 q_1 z_1 I_{eff}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (830)$$

All harmonics of order 3 and multiples of 3 vanish, and the amplitude of the largest is only 4 per cent of the fundamental. We can therefore take the fundamental as representing the curve of mean M.M.F. distribution.

The curve of flux distribution can now be determined.

Assuming different flux densities in the air-gap, the ampere-turns required to drive the flux, corresponding to these densities, through the magnetic circuit can be found. The curve connecting flux density in the air-gap and ampere-turns is shown in Fig. 115. These curves are from a paper by Dr. S. P. Smith.

Now take different values for the current and calculate the maximum amp.-turns, viz., $2{\cdot}58 q_1 z_1 I_{eff}$. By the simple process of projection, the curve of flux distribution is determined. The area of the B curve can be found and represents the flux per cm. length of core. The maximum flux can then be found and plotted against the maximum amp.-turns, and the magnetizing current can be



No. Curve	B max.	B at 60°	B mean	$\frac{B \max.}{B \text{mean}}$	$\frac{B \text{ at } 60°}{B \text{mean}}$
a"	3300	2850	2100	1·57	1·36
b"	4300	3800	2800	1·54	1·355
c"	5200	4600	3400	1·53	1·35
d"	5950	5550	4150	1·43	1·34
e"	6550	6200	4750	1·38	1·31
f"	7100	6700	5300	1·34	1·27

Harmonics in % (B, 100)

Curve	B_3	B_5
a"	–	–
b"	0·6	- 1·7
c"	1·6	- 1·7
d"	5·0	- 1·6
e"	8·8	-0·45
f"	12·7	1·15

FIG. 115

found thus for any given flux. This is a laborious process and certainly would not be tolerated in a design office.

We can simplify the process by using the relation

$$\frac{B_{60}}{B_{mean}} = \text{constant (very approx.)}$$

The equation to the flux curve is

$$y = B_1 \sin\theta + B_3 \sin 3\theta + B_5 \sin 5\theta + \text{etc.} \quad . \quad . \quad (831)$$

$$B_{mean} = \frac{2}{\pi}\left(B_1 + \frac{1}{3}B_3 + \frac{1}{5}B_5\right) \quad . \quad . \quad . \quad . \quad (832)$$

$$\frac{B_{60}}{B_{mean}} = \frac{\frac{\sqrt{3}}{2}(B_1 - B_5 + B_7)}{\frac{2}{\pi}\left(B_1 + \frac{1}{3}B_3 + \frac{1}{5}B_5\right)} \quad . \quad . \quad . \quad . \quad (833)$$

For a pure sine wave $\dfrac{B_{60}}{B_{mean}} = \dfrac{B_{max}\sin 60}{\frac{2}{\pi}B_{max}}$ . . . (834)

$$= 1{\cdot}36 = K_{60}$$

If we take the more usual case, where

$$B_3 = 5\% \text{ of } B_1 \text{ and } B_5 = -2\% \text{ of } B_1$$

$$K_{60} = 1{\cdot}37 \quad . \quad . \quad . \quad . \quad (835)$$

Therefore K_{60} is practically constant and equal to the value for a sine wave.

K_{90}, on the other hand $= 1{\cdot}365$ for $B_3 = 0{\cdot}1B_1$; while for a sine wave, $K_{90} = 1{\cdot}57$

Dr. Kloss, to whom the method is due, gives the following values for the ratio $\dfrac{B_{60}}{B_{mean}}$

For three-phase machines, $\dfrac{B_{60}}{B_{mean}} = 1{\cdot}28$. . . (836)

,, two-phase machines, $\dfrac{B_{60}}{B_{mean}} = 1{\cdot}18$ . . . (837)

,, single-phase machines, $\dfrac{B_{60}}{B_{mean}} = 1{\cdot}57$ . . . (838)

We shall use this method, and the writer knows from experience that it gives very good results indeed. One point needs to be

of path in the core is taken as two-thirds of the pole-pitch, for we are considering two points, each one-third of a pole pitch from the origin. Our method of procedure is therefore, first, to find the flux per pole; and, secondly, to find the areas and lengths of the various parts of the magnetic circuit, and the flux density in the different parts at 60° along the pole pitch, which is 1·28 times the mean density according to Kloss.

The magnetizing current per phase $I\mu$

$$= \frac{\text{amp.-turns at } 60^\circ}{2\cdot12 \times q_1 z_1} \quad . \quad . \quad . \quad . \quad . \quad (839)$$

for a three-phase machine

$$= \frac{\text{amp.-turns at } 60^\circ}{1\cdot25\, q_1 z_1} \quad . \quad . \quad . \quad . \quad . \quad (840)$$

for two-phase motors

$$= \frac{\text{amp.-turns at } 60^\circ}{0\cdot74\, q_1 z_1} \quad . \quad . \quad . \quad . \quad . \quad (841)$$

for single-phase motors

We can represent the above procedure in tabular form thus—

Part of Path.	Area.	Length.	Flux Density.	Amp.-turns per Cm.	Total Amp.-turns.
Stator core .	A_{sc}	$\frac{2}{3} l_c$	$\frac{\phi}{2 A_{sc}}$		
Stator teeth .	$\left.\begin{matrix} A_{t\,max} \\ A_{t\,min} \end{matrix}\right\}$	$2 l_t$	$\left\{\begin{matrix} 1\cdot28 \frac{\phi}{A_{t\,max}} \\ 1\cdot28 \frac{\phi}{A_{t\,min}} \end{matrix}\right.$		
Air-gap. . .	A_g	$2 l_g$	$1\cdot28 \frac{\phi}{A_g}$		
Rotor teeth .	$\left.\begin{matrix} A_{r\,max} \\ A_{r\,min} \end{matrix}\right\}$	$2 l_{tr}$	$\left\{\begin{matrix} 1\cdot28 \frac{\phi}{A_{r\,max}} \\ 1\cdot28 \frac{\phi}{A_{r\,min}} \end{matrix}\right.$		
Rotor core. .	A_r	$\frac{2}{3} l_r$	$\frac{\phi}{2 A_r}$		

In order to determine the no-load current, it is necessary to know the value of the energy component of current to overcome the no-load losses.

These losses are—

1. Bearing friction loss.
2. Windage loss.
3. Iron loss.
4. Brush friction loss with slip-ring motors.

The bearing loss in watts = (speed in metres per second)$^{1.5}$

$$= 0{\cdot}19 \times d_j \text{ cm.} \times l_j \text{ cm.} \times (\text{peripheral speed in metres per sec.})^{1.5} \quad (842)$$

where d_j = diameter of journal in centimetres

l_j = length of journal in centimetres

The windage loss in watts

$$= 0{\cdot}17 \times 10^{-3} \times \text{rotor barrel surface in sq. cms.} \times (\text{peripheral velocity of rotor in metres per second})^2 \quad (843)$$

The brush friction loss

$$= 9{\cdot}81 \times \text{rubbing velocity in metres per second} \times \text{total brush area in sq. cms.} \times \text{brush pressure in kg. per sq. cm.} \times \rho \text{ watts.} \quad (844)$$

$$\rho = \text{coefficient of friction} = 0{\cdot}3 \text{ or less for soft brushes.} \quad (845)$$

$$\text{Brush pressure} = 0{\cdot}25 \frac{\text{kg.}}{\text{sq. cm.}} \text{ (approx.)} \quad (846)$$

We have already dealt with the iron loss.

It may be pointed out that the iron losses are greatly reduced by using semi-enclosed slots in stator and rotor, and this is usual practice.

With open slots in the stator, which are occasionally used, additional iron losses due to flux pulsation, and which are usually large due to their high frequency, are set up. The frequency of pulsation of flux in the stator teeth = number of rotor teeth × revs. per second, and the frequency of pulsation in the rotor teeth = number of stator slots × revs. per second. These frequencies are of the order of 1000 or more cycles per second. It is doubtful whether any advantage is gained by using open slots. The magnetizing current and short-circuit currents are increased in approximately the same ratio, and the performance is about the same as

regards power factor, but a larger machine is required for a given output and speed. The usual argument is that the coils can be more effectively insulated and removed, but this is little more than a talking point.

The short-circuit current. The ideal short-circuit current per phase

$$= \frac{\text{applied voltage per phase}}{\text{equivalent reactance per phase}} \quad . \quad . \quad . \quad (847)$$

$$= \frac{E}{X_1 + X_2 \times \left(\frac{q_1 z_1}{q_2 z_2}\right)^2 \times \frac{m_1}{m_2}} \quad . \quad . \quad . \quad (848)$$

where X_1 = stator reactance per phase in ohms

X_2 = rotor reactance per phase in ohms

q_1 = stator slots per pole per phase

q_2 = rotor slots per pole per phase

z_1 = conductors in series per slot in stator

z_2 = conductors in series per slot in rotor

m_1 = number of stator phases

m_2 = number of rotor phases

E = applied voltage per phase

It is necessary to calculate X_1 and X_2

The leakage fluxes for primary and secondary are—

1. Slot leakage.
2. Tooth-tip leakage.
3. End-connection leakage.
4. Zigzag leakage.
5. Belt leakage.

Fig. 116 shows part of the stator and rotor in an induction motor and also the leakage fluxes.

We will first calculate the inductance of a coil of T_c turns or z conductors per slot.

This is equal to $\frac{z}{i} \Sigma \phi_x 10^{-8}$

$$= 0{\cdot}4\pi z^2 \Sigma(l_x \lambda_x) 10^{-8} \quad . \quad . \quad . \quad (849)$$

where ϕ_x = leakage flux linking the coil due to a current of i amps.

$$\phi_x = 0{\cdot}4\pi z i \Sigma(l_x \lambda_x) \quad . \quad . \quad . \quad . \quad (850)$$

λ_x is the permeance per unit length of the leakage path and l_x is the length considered.

The inductance of a coil

$$L = 0{\cdot}4\pi z^2 2 l_c \lambda_s 10^{-8} + 0{\cdot}4\pi z^2 2 l_c \lambda_z 10^{-8} + 0{\cdot}4\pi z^2 2 l_e \lambda_T 10^{-8} \quad . \quad . \quad . \quad . \quad (851)$$

where λ_s = specific permeance of the slot per cm. length

λ_z = specific permeance of zigzag path per cm.

λ_T = specific permeance of the overhang

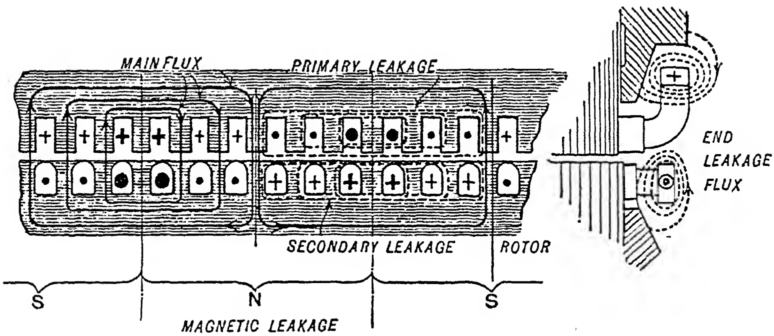


FIG. 116

With q_1 slots per pole per phase, there are $q_1 z_1$ conductors producing leakage flux associated with a pole pair.

This flux has now to cross q_1 slots and

$$\therefore \ \lambda_s \text{ and } \lambda_z \text{ now become } \frac{\lambda_s}{q_1} \text{ and } \frac{\lambda_z}{q_1}$$

Hence the inductance of a group of coils of one phase

$$= L = 0{\cdot}4\pi (q_1 z_1)^2 2 l_c \frac{\lambda_s}{q_1} 10^{-8} + 0{\cdot}4\pi (q_1 z_1)^2 2 l_c \frac{\lambda_z}{q_1} 10^{-8} + 0{\cdot}4\pi (q_1 z_1)^2 2 l_e \,.\, \lambda_T \,.\, 10^{-8} \quad . \quad . \quad . \quad . \quad (852)$$

where l_c = length of core

l_e = length of one end connection

Since there are $\frac{p}{2}$ groups of such coils per phase (where $p =$ number of poles) with the hemi-tropic winding, the inductance per phase of the stator

$$L_s = 0{\cdot}4\pi(q_1 z_1)^2 \frac{p}{2} \times 10^{-8}\left[2l_c \frac{\lambda_s}{q_1} + 2l_c \frac{\lambda_z}{q_1} + 2l_e\lambda_T\right] \quad . \quad . \quad (853)$$

$$= 0{\cdot}4\pi(q_1 z_1)^2 p \times 10^{-8}\left[l_c \frac{\lambda_s}{q_1} + l_c \frac{\lambda_z}{q_1} + l_e\lambda_T\right]. \quad . \quad . \quad (854)$$

$$= 0{\cdot}4\pi(q_1 z_1)^2 p \times 10^{-8}\,[\lambda_L' + \lambda_Z' + \lambda_T'] \quad . \quad . \quad . \quad . \quad (855)$$

where $\lambda_L' = l_c \frac{\lambda_s}{q_1}$; $\lambda_z' = l_c \frac{\lambda_z}{q_1}$; $\lambda_T' = l_e\lambda_T$ . . . . . (856)

In the same way the rotor inductance per phase in henries

$$L_r = 0{\cdot}4\pi(q_2 z_2)^2 p \times 10^{-8}[\lambda_L'' + \lambda_Z'' + \lambda_T''] \quad . \quad . \quad . \quad (857).$$

where $\lambda_L'' = l_c \frac{\lambda_r}{q_1}$; $\lambda_z'' = l_c \frac{\lambda_z}{q_2}$ and $\lambda_T'' = l_e\lambda_T'$ . . . (858)

and referred to the stator, assuming that the rotor has the same number of phases as the stator

$$L_r \text{ (referred)} = 0{\cdot}4\pi(q_1 z_1)^2 p \times 10^{-8}[\lambda_L'' + \lambda_Z'' + \lambda_T''] \quad . \quad (859)$$

Ideal short-circuit current

$$= \frac{E}{2\pi f L_e} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (860)$$

where $L_e =$ equivalent reactance

$$= L_s + L_r \text{ (referred)} \quad . \quad . \quad . \quad (861)$$

$$= \frac{E}{2\pi f 0{\cdot}4\pi(q_1 z_1)^2 \times p \times 10^{-8}\,[\lambda_1 + \lambda_2 + \lambda_3]} \quad . \quad (862)$$

where $\lambda_1 = \lambda_L' + \lambda_L'' = \left(\frac{\lambda_s}{q_1} + \frac{\lambda_r}{q_2}\right)l_c$ . . . . . (863)

$$\lambda_2 = \lambda_z' + \lambda_z'' = \left(\frac{\lambda_z}{q_1} + \frac{\lambda_z}{q_2}\right)l_c \quad . \quad . \quad . \quad . \quad . \quad (864)$$

Now $E = \sqrt{2}\,\pi\,.\,\phi\,.\times q_1 z_1 \frac{p}{2} \times \frac{1}{10^8} f$ . . . . . (865)

$$\therefore I_{sc} \text{ (ideal)} = \frac{\sqrt{2}\,\pi\phi\,.\,q_1 z_1 p \times 10^8}{2\pi f \times 0{\cdot}4\pi\,(q_1 z_1)^2\, 2\, 10^8 \times p \times \lambda} \quad . \quad . \quad (866)$$

$$= \frac{\phi}{0{\cdot}4\pi 2\sqrt{2}\, q_1 z_1 \times \lambda} \quad . \quad . \quad . \quad . \quad . \quad (867)$$

where $\lambda = \lambda_1 + \lambda_2 + \lambda_3$

i.e. the ideal short-circuit current $= \dfrac{\text{working flux per pole}}{\text{leakage flux per pole per amp.}}$

It is now our business to calculate the specific permeances of the different leakage paths.

1. Specific permeance of various types of stator and rotor slots.

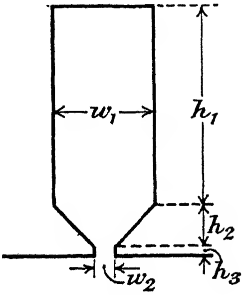


FIG. 117

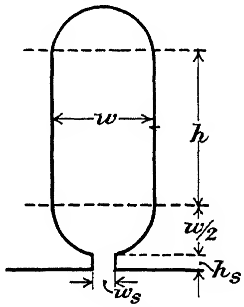


FIG. 118

$$\lambda_s = \frac{h_1}{3\omega_1} + \frac{2h_2}{w_1 + \omega_2} + \frac{h_3}{w_2} \quad . \quad . \quad . \quad . \quad . \quad (868)$$

and a similar expression for λ_r

Slot with semicircular ends with the winding included in the portion marked h. (Fig. 118.)

Equivalent permeance of straight portion $= \dfrac{h}{3w}$

Permeance of semicircular bottom portion. Permeance of elementary strip per cm. length

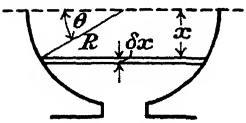


FIG. 119

$$= \frac{\delta x}{2R\cos\theta} \quad (R = \text{radius}) \quad . \quad (869)$$

$$x = R\sin\theta \quad . \quad . \quad . \quad . \quad (870)$$

$$dx = R\cos\theta d\,\theta \quad . \quad . \quad . \quad (871)$$

permeance of strip $= \dfrac{R\cos\theta\, d\theta}{2R\cos\theta} = \dfrac{d\theta}{2}$ . . . (872)

elementary flux in strip per cm. per amp. $= 0{\cdot}4\pi z_1 \dfrac{d\theta}{2}$ (873)

z_1 = conductors per slot

line linkages per amp. per cm. of core length due to flux in strip

$$= 0{\cdot}4\pi z_1^2 \frac{d\theta}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (874)$$

Total line linkages per amp. per cm.

$$= 0{\cdot}4\pi \frac{z_1^2}{2} \int_0^{\frac{\pi}{2}} d\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad (875)$$

$$= 0{\cdot}4\pi \frac{z_1^2}{2} \times \frac{\pi}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (876)$$

$$= 0{\cdot}4\pi z_1^2 \times \frac{\pi}{4} \quad . \quad . \quad . \quad . \quad . \quad . \quad (877)$$

equivalent permeance per cm. $= \dfrac{\pi}{4}$ . . . . . . (878)

$$\therefore \lambda_s = \frac{h}{3w} + \frac{\pi}{4} + \frac{h_s}{\omega_s} \quad . \quad . \quad (879)$$

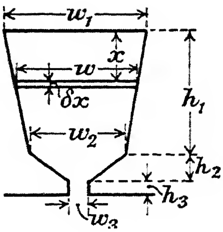


FIG. 120

Type 3 $\quad w = w_1 - (\omega_1 - \omega_2) \dfrac{x}{h_1}$ . . (880)

permeance of strip of width w and thickness δx and unit length $= \dfrac{\delta x}{w}$

let z_1 = wires per slot

wires producing flux in strip $= \dfrac{\dfrac{w'_1 + w'}{2} \times x \times z_1}{\dfrac{w'_1 + \omega_2}{2} \times h_1}$ . . (881)

flux in strip per cm. per amp.

$$= 0{\cdot}4\pi \left[\frac{w'_1 + w'}{w_1 + \omega_2} \times \frac{x}{h_1} \times z_1\right] \times \frac{\delta x}{w} \quad . \quad . \quad . \quad (882)$$

line linkages per amp per cm.

$$= 0{\cdot}4\pi \left[\frac{w_1 + w}{\omega_1 + \omega_2} \times \frac{x}{h_1} \times z_1\right]^2 \times \frac{\delta x}{w'} \quad . \quad . \quad . \quad (883)$$

Total line linkages per amp. per cm. for part h_1

$$h_1 = 0{\cdot}4\pi z_1^2 \int_0^{h_1} \left[\frac{w_1 + \omega}{w_1 + \omega_2} \times \frac{x}{h}\right]^2 \times \frac{\delta x}{w} \quad . \quad . \quad (884)$$

$$= \frac{0{\cdot}4\pi z_1^2}{(w_1 \times w_2)^2} h_1^2 \times \int_o (\omega_1 + \omega)^2 x^2 \frac{dx}{w} \quad . \quad . \quad . \quad (885)$$

$$= \frac{0{\cdot}4\pi z_1^2}{(w_1 + \omega_2)^2 h_1^2} \int_o^{h_1} \left(\frac{\omega_1^2 x^2}{w} + 2\omega_1 x^2 + \omega x^2 \right) dx. \quad . \quad (886)$$

$$= \frac{0{\cdot}4\pi z_1^2}{(w_1 + \omega_2)^2 h_1^2} \int_o^{h_1} \left\{ \frac{w_1^2 x^2}{\omega_1 - (w_1 - \omega_2)\frac{x}{h_1}} \right.$$

$$\left. + 2\omega_1 x^2 + \omega_1 x^2 - (\omega_1 - \omega_2)\frac{x^3}{h_1} \right\} dx \quad . \quad . \quad (887)$$

$$\text{let } \frac{w_1 - \omega_2}{h_1} = a \quad . \quad . \quad . \quad . \quad (888)$$

then the above equation becomes

$$\frac{0{\cdot}4\pi z_1^2}{(w_1 + \omega_2)^2 h_1^2} \int_o^{h_1} \left\{ \frac{w_1^2 x^2}{\omega_1 - ax} + 3\omega_1 x^2 - ax^3 \right\} dx \quad . \quad . \quad . \quad (889)$$

$$= \frac{0{\cdot}4\pi z_1^2}{(\omega_1 + \omega_2)^2 h_1^2} \int_o^{h_1} \left\{ -\frac{\omega_1^2 x}{a} - \frac{\omega_1^3}{a^2} + \frac{\omega_1^4}{a^2(\omega_1 - ax)} \right.$$

$$\left. + 3\omega_1 x^2 - ax^3 \right\} dx \quad . \quad . \quad . \quad (890)$$

$$= \frac{0{\cdot}4\pi z_1^2}{(\omega_1 + \omega_2)^2 h_1^2} \left[-\frac{\omega_1^2 x^2}{2a} - \frac{\omega_1^3 x}{a^2} - \frac{\omega_1^4}{a^3} \log(\omega_1 - ax) \right.$$

$$\left. + \omega_1 x^3 - \frac{ax^4}{4} \right]_o^h \quad . \quad . \quad . \quad (891)$$

$$= \frac{0{\cdot}4\pi z_1^2}{(\omega_1 + \omega_2)^2 h_1^2} \left[-\frac{\omega_1^2 h_1^2}{2a} - \frac{\omega_1^3 h_1}{a^2} - \frac{\omega_1^4}{a^3} \log(\omega_1 - ah) \right.$$

$$\left. + \frac{\omega_1^4}{a^3} \log \omega_1 + \omega_1 h_1^3 - \frac{ah_1^4}{4} \right] . \quad . \quad (892)$$

$$= \frac{0{\cdot}4\pi z_1^2}{(w_1 + \omega_2) h_1^2} \left[-\frac{\omega_1^2 h_1^3}{2(\omega_1 - \omega_2)} - \frac{w_1^3 h_1^3}{(\omega_1 - \omega_2)^2} - \frac{\omega_1^4 h_1^3}{(\omega_1 - \omega_2)^3} \right.$$

$$\log\{\omega_1 - (\omega_1 - \omega_2)\} + \frac{\omega_1^4 h_1^3}{(\omega_1 - \omega_2)^3} \log \omega_1 + \omega_1 h_1^3$$

$$\left. - (\omega_1 - \omega_2)\frac{h_1^3}{4} \right] . \quad . \quad . \quad (893)$$

$\therefore$ equivalent permeance per cm. length of slot for part h_1

$$= \frac{h_1}{(w_1 + \omega_2)^2}\left[-\frac{\omega_1^2}{2(\omega_1 - \omega_2)} - \frac{w_1^3}{(\omega_1 - \omega_2)^2} + \frac{\omega_1^4}{(\omega_1 - \omega_2)^3}\log\frac{w}{\omega_2} + w_1 + \frac{w_1 - \omega_2}{4}\right] \quad . \quad . \quad . \quad (894)$$

To this must be added $\dfrac{2h_2}{\omega_2 + \omega_3} + \dfrac{h_3}{\omega_3}$ . . . . (895)

Rotor slot. In a similar manner the equivalent permeance per cm. length of rotor slot

$$= \frac{h_1}{(\omega_1 + \omega_2)^2}\left[\frac{w_2}{2(\omega_1 - \omega_2)} - \frac{\omega_2^3}{(\omega_1 - \omega_2)^2} + \frac{\omega_2^4}{(\omega_1 - \omega_2)^3}\log\frac{\omega_1}{\omega_2} + \omega_2 + \frac{\omega_1 - \omega_2}{4}\right] \quad (896)$$

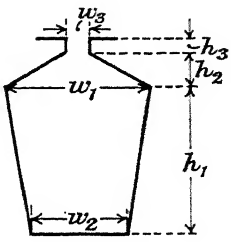


FIG. 121

to this must be added

$$\frac{2h_2}{\omega_1 + \omega_2} + \frac{h_3}{\omega_3} \quad . \quad . \quad . \quad . \quad (897)$$

Rotor circular slot for squirrel-cage machine

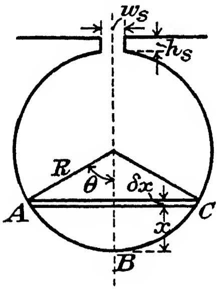


FIG. 122

$$\text{Area } ABC = R^2\theta - R^2 \sin\theta\cos\theta \quad . \quad (898)$$

$$= R^2\theta - \tfrac{1}{2}R^2 \sin 2\theta \quad . \quad . \quad . \quad (899)$$

permeance of elementary strip per cm. length

$$= \frac{\delta x}{2R\sin\theta} \quad . \quad . \quad . \quad (900)$$

$$\text{Now } x = R(1 - \cos\theta) \quad . \quad . \quad (901)$$

$$\therefore \quad dx = R\sin\theta\, d\theta \quad . \quad . \quad (902)$$

$$\text{permeance of elem. strip} = \frac{d\theta}{2} \quad . \quad . \quad . \quad . \quad . \quad (903)$$

portion of conductor producing flux in strip

$$= \frac{R^2\theta - \frac{1}{2}R^2\sin 2\theta}{\pi R^2} \quad . \quad . \quad . \quad . \quad (904)$$

$$= \frac{\theta - \frac{1}{2}\sin 2\theta}{\pi} \quad . \quad . \quad . \quad . \quad (905)$$

$$\therefore \text{ flux in strip per cm.} = 0{\cdot}4\pi \times \frac{\theta - \frac{1}{2}\sin 2\theta}{\pi}\frac{d\theta}{2} \quad . \quad . \quad (906)$$

Total line linkages per cm. length

$$= \frac{0{\cdot}4\pi}{2\pi^2}\int_0^\pi (\theta^2 - \theta\sin 2\theta + \tfrac{1}{4}\sin^2 2\theta)\,d\theta \quad . \quad . \quad . \quad (907)$$

$$= \frac{0{\cdot}4\pi}{2\pi^2}\left[\frac{\pi^3}{3} + \frac{\pi}{2} + \frac{\pi}{8}\right]. \quad . \quad . \quad . \quad (908)$$

$$= \frac{0{\cdot}4\pi}{2}\left[\frac{\pi}{3} + \frac{1}{2\pi} + \frac{1}{8\pi}\right] \quad . \quad . \quad . \quad (909)$$

$$= 0{\cdot}4\pi \times 0{\cdot}62 \quad . \quad . \quad . \quad . \quad . \quad (910)$$

$$\therefore \text{ permeance per cm.} = 0{\cdot}62 + \frac{h_s}{\omega_s} \quad . \quad . \quad (911)$$

Zigzag permeance. This refers to the flux which zigzags from stator to rotor teeth. The following method is approximate only. Indeed, all methods for calculating the leakage flux are only approximate. Assumptions are made that the flux crosses the slots in straight lines. To be strictly correct, one should calculate the distribution of flux in the leakage paths. Life is too short for such refinements! It is possible by the methods outlined to calculate the short-circuit current within 10 to 20 per cent, and this is quite good enough.

Consider the stator; let δ = gap length

permeance of stator zigzag path per cm.

$$P_s = \frac{s_2 - a_1}{2} + \frac{1}{2\delta} \times \frac{1}{0{\cdot}96q_1} \times \frac{1}{2} \quad . \quad . \quad . \quad (912)$$

$$\text{permeance of rotor } P_r = \frac{s_1 - a_2}{2} \times \frac{1}{2\delta} \times \frac{1}{q_2} \times \frac{1}{2} \quad . \quad (913)$$

Average permeance per cm.

$$= \frac{1}{16\delta}\left(\frac{s_2 - a_1}{0{\cdot}96q_1} + \frac{s_1 - a_2}{q_2}\right) \quad . \quad . \quad . \quad (914)$$

Average permeance per cm.

$$= \frac{\tau}{m16q_1q_2\delta}\left[\frac{s_2 - a_1}{0{\cdot}96q_1} \times \frac{mq_1q_2}{\tau} + \frac{s_1 - a_2}{q_2} \times \frac{mq_1q_2}{\tau}\right] \quad . \quad (915)$$

$$= \frac{\tau}{m \times 16 \times q_1q_2 \times \delta} \times \left[\frac{s_2 - a_1}{0{\cdot}96\tau_2} + \frac{s_1 - a_2}{\tau_1}\right] \quad . \quad . \quad (916)$$

where τ = pole pitch

q_1 = stator slots per pole per phase

q_2 = rotor slots per pole per phase

τ_2 = rotor slot pitch in cms. at gap

τ_1 = stator slot pitch in cms. at gap

m = number of phases

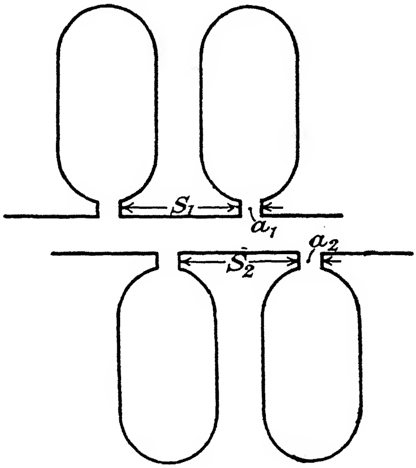


FIG. 123

$$\therefore \quad \lambda_z = \frac{\varepsilon\tau}{m \times 16 \times \delta \times q_1 \times q_2} \text{ per cm.} \qquad (917)$$

$$\text{where } \varepsilon = \frac{s_2 - a_1}{0{\cdot}96\tau_2} + \frac{s_1 - a_2}{\tau_1} \qquad (918)$$

$$\text{Total zigzag permeance } \lambda_2 = \frac{l_c\varepsilon\tau}{m \times 16 \times q_1 \times q_2 \times \delta} \qquad (919)$$

where l_c = core length

$$\text{for three-phase machines, } \lambda_2 = \frac{l_c\varepsilon\tau}{48 \times q_1 \times q_2 \times \delta} \qquad (920)$$

$$\text{two-phase machines, } \lambda_2 = \frac{l_c\varepsilon\tau}{32 \times q_1 \times q_2 \times \delta} \qquad (921)$$

$$\text{single-phase, } \lambda_2 = \frac{l_c\varepsilon\tau}{16 \times q_1 \times q_2 \times \delta} \qquad (922)$$

Calculation of zigzag permeance. The following method is due to Comfort A. Adams.

Let t_1 = primary tooth-tip plus fringe

t_2 = same for secondary

τ_1 = tooth pitch assumed equal in primary and secondary

x = displacement of the centre of the secondary tooth from the centre of primary slot

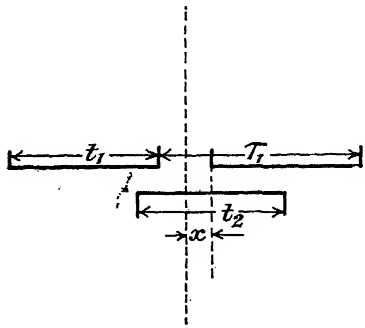


FIG. 124

Consider the series permeance from t_1 to t_2 and back to the next primary tooth-tip, neglecting the iron part of the path.

This permeance takes account of the total tooth-tip leakage for primary and secondary.

Consider 1 cm. of core length parallel to the shaft. The left-hand overlap is—

$$\frac{t_2}{2} - x - \frac{\tau_1 - t_1}{2} = \tau_1\left(\frac{t_1 + t_2}{2\tau_1} - \frac{1}{2}\right) - x \quad . \quad . \quad . \quad (923)$$

$$= m - x \quad . \quad . \quad . \quad . \quad . \quad (924)$$

Right-hand overlap is $\frac{t_2}{2} + x - \frac{\tau_1 - t_1}{2}$ . . . . (925)

$$= \tau_1\left(\frac{t_2 + t_1}{2\tau_1} - \frac{1}{2}\right) + x \quad . \quad . \quad . \quad (926)$$

$$= m + x \quad . \quad . \quad . \quad . \quad . \quad (927)$$

$$\text{where } m = \tau_1\left(\frac{t_1 + t_2}{2\tau_1} - \frac{1}{2}\right) \quad . \quad . \quad . \quad (928)$$

$$\text{The series reluctance is } \frac{\delta}{m-x} + \frac{\delta}{m+x} \quad . \quad . \quad . \quad (929)$$

$$= \frac{2m\delta}{m^2 - x^2} \quad . \quad . \quad . \quad . \quad . \quad (930)$$

where δ = air-gap length

$$\text{Series permeance} = \frac{m^2 - x^2}{2m\delta} \quad . \quad . \quad . \quad . \quad . \quad (931)$$

$$\text{Its average value} = \frac{2}{\tau_1} \times \frac{1}{2m\delta} \int_{x=0}^{x=m} (m^2 - x^2)\, dx \quad . \quad . \quad (932)$$

$$= \frac{1}{m\tau_1\delta} \left[m^2 x - \frac{x^3}{3} \right]_0^m \quad . \quad . \quad . \quad . \quad (933)$$

$$= \frac{1}{m\delta\tau_1} \left[m^3 - \frac{1}{3} m^3 \right] = \frac{2}{3} \frac{m^2}{\delta\tau_1} \quad . \quad (934)$$

$$= \frac{2}{3\tau_1\delta} \left[\left\{ \tau_1 \left(\frac{t_1 + t_2}{2\tau_1} - \frac{1}{2} \right) \right\} \right] \quad . \quad (935)$$

$$= \frac{2}{3} \frac{\tau_1}{\delta} \left(\frac{a_1 + a_2}{2} - \frac{1}{2} \right) \quad . \quad . \quad . \quad (936)$$

$$\text{where } a_1 = \frac{t_1}{\tau_1};\ a_2 = \frac{t_2}{\tau_1} \quad . \quad . \quad . \quad . \quad . \quad (937)$$

Overhang leakage permeance. This subject has been thoroughly investigated by Dr. Max Kloss, and I propose to reproduce his work entirely.[1]

Translation.

THE CALCULATION OF THE OVERHUNG LEAKAGE FLUX IN INDUCTION MOTORS

By Max Kloss

1. Preliminary particulars. The circle diagram of an induction motor is, as is well known, fixed by two quantities—the magnetizing current, and the ideal short-circuit current. The magnetizing current produces in the stator winding such ampere turns as are necessary to induce the normal flux in the main magnetic circuit through the stator, air-gap, and rotor. The short-circuit current produces such ampere turns as are necessary to produce the normal flux (constant E.M.F. assumed) through the leakage field *path*.

[1] By permission of Dr. Kloss.

Now for the connection between flux and ampere turns producing it we have the well-known fundamental expression

$$\phi = 0{\cdot}4\pi \text{ A.W. } \sqcap \quad . \quad . \quad . \quad . \quad (938)$$

where $\sqcap$ denotes the magnetic conductivity of the flux path encircled by the ampere turns, A.W.

For the case of the short-circuit current with q slots per pole and phase, and z conductors in series per slot,

$$\phi = 0{\cdot}4\pi q z i_K \sqrt{2}\ 2(\Lambda_n + \Lambda_z + \Lambda_s) \quad . \quad . \quad . \quad . \quad (939)$$

There the various values of Λ denote the partial magnetic conductivities of the stray flux path encircling a coil side, so that, in order to obtain the total conductivity (magnetic) of the stray flux path, the factor 2 is introduced before the bracket.

Individually,

Λ_n denotes the conductivity of *slot* stray flux path

Λ_z the conductivity of so-called zigzag flux path

Λ_s the conductivity of the path of stray flux round the *overhang*

For the calculation of the short-circuit current, it is consequently necessary, in the next place, to calculate the magnetic conductivity of the stray flux path. The two first portions of this are easily calculated from known formulae.[1] The overhang leakage has formerly been simplified by the assumption that it is small as compared with the slot stray flux. The calculation is not so easily accessible as that, however.[2]

In the following, a simple formula for the overhang stray flux shall be deduced, which has proved itself of great value throughout a lengthy period of use and, therefore, is worthy of a further sphere of interest and value.

If one considers the overhang of a normal induction motor stator winding projecting beyond the iron, with its straight portions, bent portions, and with its repeated subdivisions, and if one attempts to form a picture of the flux paths, taking into account the combined effect of the three phases, the problem appears to be so entangled that it might almost be thought impossible to commence the calculation.

In order to make the complicated portions of the calculation to

[1] For example, Arnold, " Wechselstromtschnik," V. Band, 1, Teil, Seite 51.

[2] " Formeln für die Bercchnung der Sturnstrenung sind u.," a angegeben von Hobart, *ETZ*, 1906, Seite 933.

Wittek, *ETZ*, 1906, Sute 53, Arnold a. a. O, Vgl auch Sumec, *ETZ*, 1906,

a certain extent clearer, we must reduce them to a simpler form by the use of certain assumptions. At the same time, it must be sought to make clear in what direction these simplifications may influence the final result.

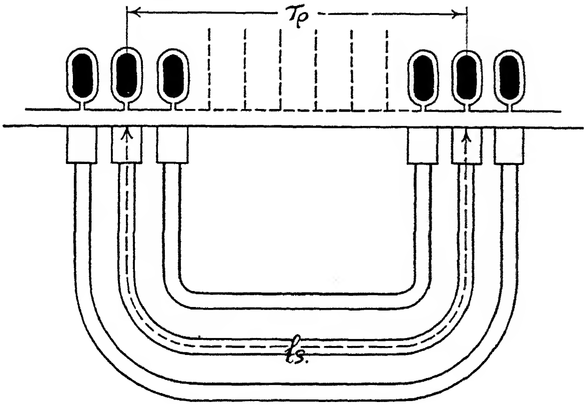


FIG. 125A

In order to be able to check the specified guarantees for the output factor of the overload capacity of an induction motor, we must, as far as possible, avoid the estimation of too large a short-circuit current, or that is to say, too small magnetic conductivity (for stray flux).

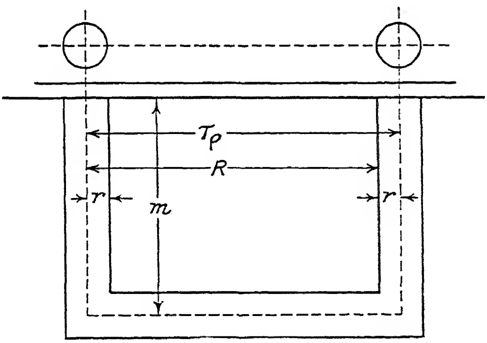


FIG. 125B

If we, therefore, by our estimation make such assumptions that the magnetic conductivity may be obtained larger than it actually is, then we shall be on the safe side.

2. Assumptions for simplification of the calculation. We wish, in the first place, to consider a straight coil overhang (Fig. 125A)

of one phase, without regard to the influence of the other phases. It consists of a group of q individual coils ($q = 3$ is assumed in Fig. 125A), and each of these coils has a rectangular-shaped section corresponding to the type of slot. Further, the plan of the coil top shows a rectangle with considerably rounded corners. For the simplification of the calculation, we shall now make the following assumptions—

(1) The plan of the coil top will be considered as a rectangle of equal area to the actual plan and having sharp corners.

(2) The q separated coils will be considered as combined into one.

(3) The section of this equivalent coil will be taken as of circular form, and of area equal to the sum of the q individual coil sections (including the conductor insulation and the surrounding air spaces).

With z turns per slot and with d as diameter of individual wires, we have—

$$\pi r^2 = qzd^2 \quad . \quad . \quad . \quad . \quad . \quad (940)$$

$$\text{Also } r = d\sqrt{\frac{qz}{\pi}} \quad . \quad . \quad . \quad (941)$$

$$= \text{radius of equivalent coil side bundles}$$

Coil overhang = equivalent rectangle

This simplified equivalent coil end is illustrated in Fig. 125B. The two sides of the rectangle are—

m = in direction of axis; and

T_p' = in direction of circumference, and measured in coil centre

The length T_p' is somewhat greater than the pole pitch measured along the stator bore, and the difference between these two dimensions is greater, the less the diameter of the motor is.

A further assumption is that the path of the flux lines round the coil side (conductor bundle) is circular.

3. The magnetic conductivity for a "conductor bundle" of circular section. The internal and external magnetic conductivity must together be considered.

Since the flux lines in the interior of the conductor group interlink with only a portion of the total number of conductors, so the full effect (of total internal flux and total conductors) must not be taken as producing E.M.F. In consequence thereof the actual magnetic conductivity of the interior stray flux paths must not be calculated, but the equivalent conductivity for the case, where all lines link with all the turns.

Fig. 126 shows the cross-section of the circular conductor group with radius r. The internal flux tube with radius x and section 1 cm. $\times\ \delta x$ interlinks with $(qz)\,\frac{x^2}{r^2}$ conductors. The flux produced in this elementary flux tube has a maximum instantaneous value of

$$\Delta\phi = 0{\cdot}4\pi\left(qz\frac{x^2}{r^2}\right)i\sqrt{2}\,\frac{dx}{2\pi x} \quad . \quad . \quad . \quad . \quad (942)$$

where $\frac{dx}{2\pi x}$ is the magnetic conductivity of the elementary flux tube. This elementary flux produces in the conductors which it embraces an elementary voltage.

$$\Delta_e = 4{\cdot}44c\left(qz\frac{x^2}{r^2}\right)\Delta\phi\,10^{-8}$$

$$= 4{\cdot}44c\,.\,0\,.\,4\pi i\sqrt{2}\,(qz)^2\,\frac{x^3}{2\pi r^4}\,dx10^{-8} \quad . \quad (943)$$

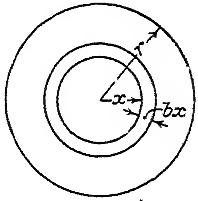


FIG. 126

where c represents the frequency.

By integrating over the whole group, i.e. between the limits $x = 0$. and $x = r$. we get

$$e = 4{\cdot}44c\,.\,0\,.\,4\pi i\sqrt{2}\,(qz)^2\,\frac{1}{2\pi r^4}\int_0^r x^3dx\,10^{-8}$$

$$= 4{\cdot}44cqz(0{\cdot}4\pi qzi\sqrt{2})\left(\frac{1}{2\pi}\,.\,\frac{1}{4}\right)10^{-8} \quad . \quad . \quad . \quad (944)$$

$$\text{Now } e = 4{\cdot}44cqz\phi\,10^{-8} \quad . \quad . \quad . \quad . \quad . \quad (945)$$

$$\text{and } \phi = 0{\cdot}4\pi qzi\sqrt{2}\lambda i \quad . \quad . \quad . \quad . \quad . \quad (946)$$

wherein λ_i is the magnetic conductivity per cm. length. So we find from Equations (944), (945), and (946) that the magnetic conductivity for the interior of the conductor groups is, per cm. length,

$$\lambda_i = \frac{1}{8\pi} = 0{\cdot}040\,\frac{\text{cm.}}{\text{cm.}} \quad . \quad . \quad . \quad . \quad (947)$$

and, therefore, independent of the radius r.

For a flux tube with radius x and breadth dx outside the conductor group, the magnetic conductivity per cm. length of conductor $= \frac{dx}{2\pi x}$. (This is strictly true only for the case, where r is small as compared with the length of conductor. We, therefore, calculate the conductivity greater than it is actually.)

Now all the conductors are embraced by the outside flux, and we have merely to integrate between the limits r and R, where R is the distance from the middle point of the conductor group to the outside point of the opposite groups. (*See* sketch, Fig. 127.)[1]

From this we obtain for the magnetic conductivity of the external path, per cm. conductor length,

$$\lambda_a = \frac{1}{2\pi}\int_r \frac{dx}{x} = \frac{1}{2\pi}\log_e\frac{R}{r} = \frac{2\cdot 3}{2\pi}\log_{10}\frac{R}{r}$$

$$\therefore\ \lambda_a = 0\cdot 367\log_{10}\frac{R}{r}\frac{\text{cm.}}{\text{cm.}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (948)$$

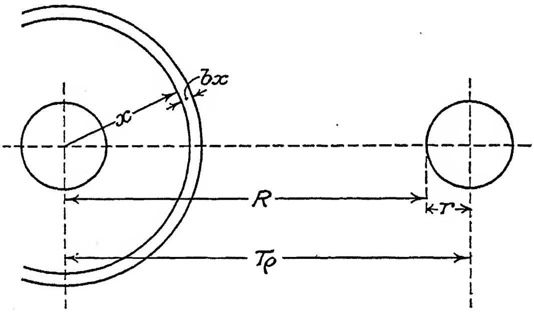


FIG. 127

For the two axial legs of the coil overhang, the length is now $= m$ (*see* Fig. 125B); and for the tangential closing side of the rectangle of length T_p' the upper limit is taken as $= m$ in place of the value R. The assumption is here made that the stray flux from this portion of the coil overhang only stretches in air up to the iron core, whilst the lines of force which enter the iron combine with the "slot" stray flux.

In consequence of this, we arrive at the following expression for the magnetic conductivity of the stray flux path for a coil overhang, the influence of other phases being neglected.

$$\Lambda = \left(0\cdot 040 + 0\cdot 367\log_{10}\frac{R}{r}\right)2m + \left(0\cdot 040 + 0\cdot 367\log_{10}\frac{m}{r}\right)T_p' \quad (949)$$

[1] Had the full pole pitch T_p' been taken as upper limit instead of R, then the areas in Fig. 128, indicated by sloping hatched lines, would have been shown by full lines, whereas they in reality (*tatsachlich*) interlink with only a portion of the conductors. On the other hand, with R as upper limit, we obtain a somewhat small value. However, the error is so small that we may neglect it.

4. The magnetic conductivity of a coil overhang of a normal three-phase winding. We shall yet essentially simplify the formula which has just been arrived at, but we wish yet to find the influence of the two other phases.

Fig. 128 illustrates the distribution of the induction in the coil planes for the instant of maximum current $= i\sqrt{2}$ in Phase I,

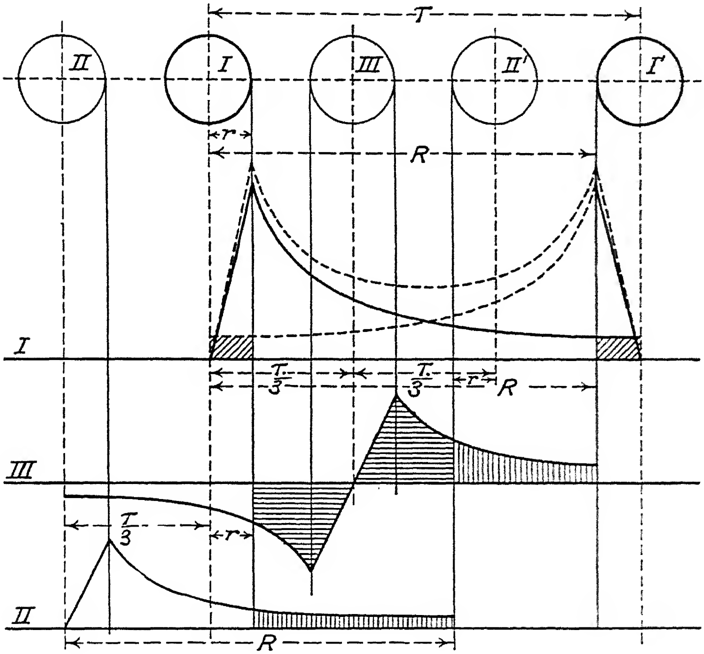


FIG. 128

whilst the current in both other phases II and III is of half this value. From Equation (942) it follows that in the interior of the conductor group the flux density $\frac{\Delta\phi}{dx}$ is directly proportional to the distance x from the centre point. The field distribution will, therefore, for the portion o to r be represented by a straight line. For the outside of the conductor group the flux density is, on the other hand, inversely proportional to the distance x, the distribution will therefore be represented by a rectangular (like sided)

hyperbola. Similarly, the same holds good, also for the flux lines issuing from the second axial leg of the coil overhang, the distribution for this is shown by the dotted line in Fig. 128, so that the total flux produced by both legs will be given by the sum of these two. In the diagram this is represented by the chain-dotted line. Similarly the field distribution diagram for the other two phases may also be plotted, only the amplitude, for the chosen instant, must be halved; and besides, the whole diagram laterally from left to right, or vice versa, are shifted by one-third of the pole pitch.

Since the magnetic conductivity for an axial coil leg is calculated from formula (947) and (948), so we need here also only allow for the influence of the coil sides II and III, which are next to the one under consideration; whereas, the coil sides II′ and III′ will be automatically taken account of if we multiply the magnetic conductivity per cm. length in formula (949) by the length $2m$ of both coil sides.

The simplest method of allowing for the influence of coil sides II and III will be now to add the flux going through coil 1–1′ and produced by these (II and III) to the flux produced by I alone, and then consider this whole flux as produced by I alone; that is to say, we must simply increase the magnetic conductivity of coil side I in the ratio of the total flux to the portion produced by I alone.

On account of their special distribution, the flux produced by II and III must not, relatively to 1, be taken at their full values. The horizontally-hatched portions of the diagram therefore cancel each other, and only the vertically-hatched portions are taken account of. For the estimation of the true values we must, therefore, form the integral between limits chosen from the diagram in place of the values R and r used in formula (948).

The flux produced by 1 is (per cm. length)

$$\phi_{\mathrm{I}} = 0{\cdot}4\pi A W_{\mathrm{I}} 0{\cdot}367 \log_{10} \frac{R}{r} \quad . \quad . \quad . \quad (950)$$

Further, the flux produced by II and effective on I (by half the ampere turns) is

$$\phi_{\mathrm{II}} = 0{\cdot}4\pi \frac{AW_1}{2} 0{\cdot}367 \log_{10} \frac{R}{\frac{T'}{3} + r} \quad . \quad . \quad . \quad (951)$$

and by III, the effective flux on I is

$$\phi_{\mathrm{III}} = 0{\cdot}4\pi \frac{AW_2}{2} 0{\cdot}367 \log_{10} \frac{R - \dfrac{T'}{3}}{\dfrac{T'}{3} - r} \quad . \quad . \quad . \quad (952)$$

We have, therefore, arrived at, in place of the expression in Equation (948), the total expression for the sum $\phi_{\mathrm{I}} + \phi_{\mathrm{II}} + \phi_{\mathrm{III}}$

We obtain, then,

$$\lambda_a = 0{\cdot}367 \left[\log_{10} \frac{R}{r} + \frac{1}{2} \log_{10} \frac{R - \dfrac{T'}{3}}{\dfrac{T'}{3} - r} + \frac{1}{2} \log_{10} \frac{R}{\dfrac{T'}{3} + r}\right] \quad (953)$$

By replacing T' by the equivalent sum $R + r$, the following simplified expression is arrived at—

$$\lambda_a = 0{\cdot}367 \left[\log_{10} \frac{R}{r} + \frac{1}{2} \log_{10} \frac{\dfrac{2R}{r} - 1}{\dfrac{R}{r} - 2} + \frac{1}{2} \log_{10} \frac{3\dfrac{R}{r}}{\dfrac{R}{r} + 4}\right] \quad (954)$$

where R and r have values as given in Figs. 129 or 128.

It is somewhat more difficult to calculate the influence of the two other phases on the tangential portion of the overhang of coil of Phase I.

Let us consider, in the first place, the actually impossible case in which the three coil heads lie on a cylindrical surface; the axis of their tangential portions will then fall together, so that only in the middle third the current group of Phase I would have its complete effect. (Fig. 129.) Whilst in the outer thirds the effect will be halved by reason of the current of half value flowing in the opposite direction in Phases II and III. The true value of the excitation due to the tangential portion would therefore be $\dfrac{1 + \frac{1}{2} + \frac{1}{2}}{3} = \frac{2}{3}$ of that due to the AT_s of Phase I.

In reality, however, the coil heads of Phases II and III are not in the same plane as that of coil I, so that on these grounds the demagnetizing value will be reduced. We may, with some degree of accuracy, assume that the demagnetizing values may be reduced

to about a third of their total value, so that for the parts concerned the reduction on Phase I is $\frac{1}{3} \times \frac{1}{2} \times (0{\cdot}4\pi qzi\sqrt{2})$; and the resulting A.W. therefore are reduced to five-sixths of normal. But this diminution does not truly extend over the two outer thirds, as shown in Fig. 129; but, on account of the rounding of the coil, overhangs at their corners over really a shorter length which shall be taken as a sixth of the pitch.

In place of the current distribution diagram of Fig. 129, we obtain therefore the following distribution—

In the middle four-sixths the full excitation $0{\cdot}4\pi qzi\sqrt{2}$, whilst

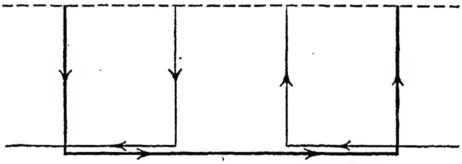


FIG. 129

in both outer sixths only $\frac{5}{6} \times 0{\cdot}4\pi qzi\sqrt{2}$, so that the effective total value is

$$\frac{4 \times 1 + 2 \times \frac{5}{6}}{6}(0{\cdot}4\pi qzi\sqrt{2}) = \frac{17}{18}(0{\cdot}4\pi qzi\sqrt{2}) \quad . \quad (955)$$

Hence the influence of the two other phases is, in effect, to multiply the magnetic conductivity of the tangential part of the coil overhang by approximately 0·94.

Hence, in place of the values found in formula (949), we arrive at the following expression for the magnetic conductivity of a coil overhang.

$$\Lambda_s =$$

$$\left[0{\cdot}040 + 0{\cdot}367\left(\log_{10}\frac{R}{r} + \frac{1}{2}\log_{10}\frac{\frac{2R}{r} - 1}{\frac{R}{r} - 2} + \frac{1}{2}\log_{10}\frac{\frac{3R}{r}}{\frac{R}{r} + 4}\right)\right]$$

$$2m + 0{\cdot}94\left[0{\cdot}040 + 0{\cdot}367\log_{10}\frac{m}{r}\right]T'_p \quad . \quad . \quad . \quad (956)$$

And now to simplify the formula for practical use, we may graph the values r and R/r.

These values should be ascertained for a line of motors. In Fig. 130 these are plotted as functions of T_p'.

It is evident that we can put, with sufficient accuracy for pract cal purposes,

$$r = 0{\cdot}50 \text{ cm.} + 0{\cdot}052 T_p' \quad . \quad . \quad . \quad (957)$$

$$\text{and } \frac{R}{r} = 10 + 0{\cdot}10 T_p' \quad . \quad . \quad . \quad . \quad (958)$$

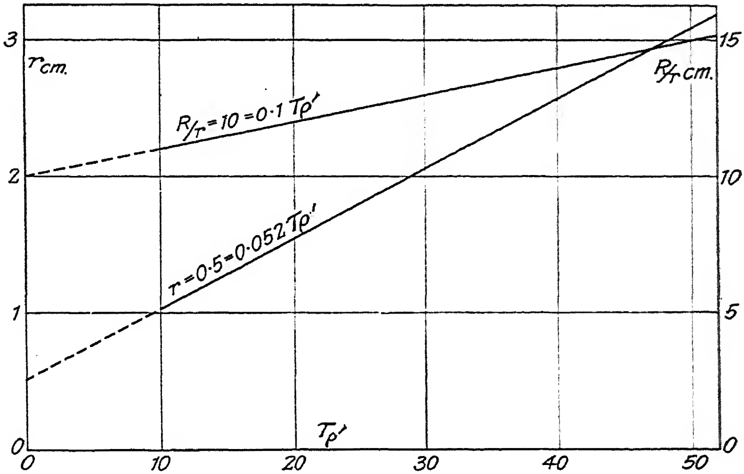


FIG. 130

Putting these values in formula (956), we obtain

$$\Pi_s = \left[0{\cdot}040 + 0{\cdot}367\left\{\log_{10}(10 + 0{\cdot}1T_p') + \frac{1}{2}\log_{10}\frac{19 + 0{\cdot}2T_p'}{8 + 0{\cdot}1T_p'} + \frac{1}{2}\log\frac{30 + 0{\cdot}3T_p'}{14 + 0{\cdot}1T_p}\right\}\right]2m + \left[0{\cdot}038 + 0{\cdot}345\log_{10}\frac{m}{0{\cdot}5 + 0{\cdot}052T_p'}\right]T_p' \quad . \quad . \quad . \quad (959)$$

We have now reduced the formula to a function of two independent quantities, T_p' and m. To a certain extent, m is of course, dependent on T_p', since the combined section of the copper in the overhang will increase as T_p' is increased, whereby the overhang

m will also be increased, however; clearance and such dimensions as are dependent on the voltage of the m/c are, of course, independent of T_p'.

The formula (959) just deduced is not suitable for practical use; it has the form

$$\Lambda_s = x2m + yT_p' \quad . \quad . \quad . \quad . \quad (960)$$

where, however, X and Y are not constants, but are functions of T_p' and of m and T_p'.

It remains to be shown that for the usual practical range of T_p' and m, the effective values of x and y may be assumed as constants with a fair degree of accuracy. To arrive at these constants,

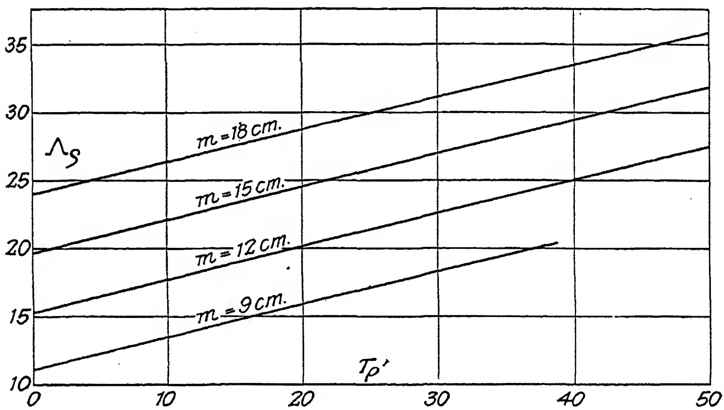


FIG. 131

we shall plot Λ_s as a function of T_p' with m as "parameter," so that, therefore, we obtain a series of curves, wherein each individual curve corresponds to a certain value of m. (*See* Fig. 131.)

In Fig. 131, calculated values are given in graphical form for $m = 9$; $m = 12$, $m = 15$ and $m = 18$ cm., and for $T_p' = 10, 20, 30, 40, 50$ cm., which correspond to the most frequently-occurring cases in practice.

If we wish to replace the somewhat intricate formula (959) by a simpler one, we must draw, with the greatest possible accuracy, straight lines through the points in Fig. 131; and, moreover, these straight lines must be parallel to one another if the factor y is to be constant throughout the range of T_p'. Similarly, the distance between the neighbouring lines must be equal if the factor x

depending on m is to be a constant. It turns out that for practical cases these conditions are fairly well fulfilled. From Fig. 131 the equations of the series of straight lines may easily be derived. It is evident that the first term of this $\begin{Bmatrix}\text{substitute}\\ \text{equivalent}\end{Bmatrix}$ equation is not exactly proportional to m, but to $(m-1)$.

The equation, therefore, is of the simple form,

$$\Lambda_s = 1{\cdot}41(m-1) + 0{\cdot}24T_p' \qquad . \qquad . \quad (961)$$

The quantity in this equation is still inconvenient in so far as it must be estimated from the actual measurements of the coil overhang. For practical purposes, it is advantageous to arrive at this from a quantity which is readily obtainable from constructional particulars of the windings. It is now the general custom to express the length of mean turn of the winding in the form $lm = 2(L + T_p' + A)$, where L is the active iron length of core, T_p' the pole pitch measured at centre of slots, and A a quantity which is dependent on the voltage of the stator and in general practice fixed from standard tables. It may be shown that a simple relationship exists between A and m, which obtained empirically may be expressed as in the following equation,

$$m = 1{\cdot}0 + 0{\cdot}5A + 0{\cdot}04T_p' \qquad . \qquad . \quad (962)$$

By putting this in Equation (961), we have

$$\begin{aligned}\Lambda_s &= 1{\cdot}41(0{\cdot}5A + 0{\cdot}04T_p') + 0{\cdot}24T_p' \\ &= 0{\cdot}705A + 0{\cdot}296T_p' = 0{\cdot}7A + 0{\cdot}3T_p' \text{ approx.} \qquad .. \quad (963)\end{aligned}$$

If one is accustomed to use the mean length l_s of the coil overhang (*see* Fig. 125A), then Equation (963) may be altered to suit, since $l_s = A + T_p'$,

$$\text{so we have } \Lambda_s = 0{\cdot}7l_s - 0{\cdot}4T_p' \, . \qquad . \qquad . \qquad . \qquad . \quad (964)$$

This gives therefore the magnetic conductivity of stator coil overhang, taking into consideration the influence of the other two phases, but still, however, without consideration of the influence of the rotor.

5. The influence of the rotor winding. The flux produced by the rotor ampere-turns takes effect on the magnetic conductivity of the stator winding overhang in different ways, according as the direction or the position of these ampere-turns is considered. Because of their direction, they assist in the magnetization of the air space between the stator and rotor windings, since the rotor

currents are in the opposite direction to the stator currents, and so form a winding with an M.M.F. in the same direction. Their effect is, therefore, to increase the magnetic conductivity. On the other hand, because of their position, they limit the section available for the stray flux, in so far as they crowd the stray lines interlinking with the stator current groups into a comparatively small section between the stator and rotor conductors, instead of allowing the free circular path for the stray flux, as before assumed. From this point of view they therefore act as a limiting factor on the magnetic conductivity for the stray flux.

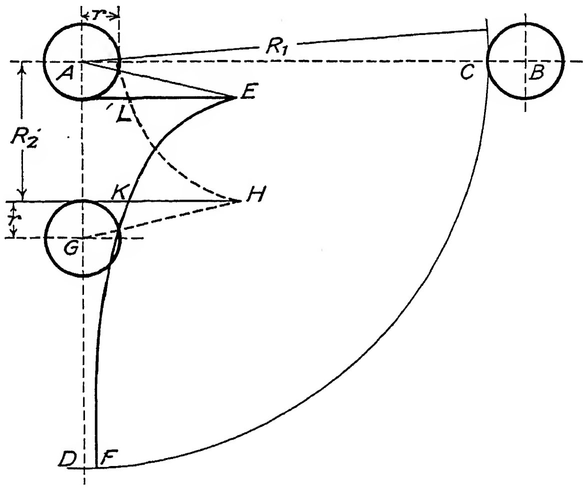


FIG. 132.—INFLUENCE OF ROTOR WINDING

It is, therefore, necessary to settle which of the two influences will be the greater, that is to say, in what manner we have to correct our formula.

Fig. 132 illustrates further (as in Fig. 127) the section through the two axial "coil legs" A and B. In the derivation of our formula, we had assumed that the lines round leg A take a circular path. The outermost line with radius R is shown in the figure.

The assumption made by us of a circular path would be correct if the lines could spread themselves not merely between the bundles A and B lying on the stator periphery, but in a like manner also in the direction of the stator radius, that is to say, towards D in Fig. 132, whereby then the induction over the distance AD would be distributed accordingly to the curve AEF.

In reality, however, there is in the direction AD a second current group G parallel to A, and with opposing current direction. By this the available section for the stray flux will be reduced from the limit R to the limit R_2. Simultaneously, however, the group G magnetizes the intervening space in the same sense as A, according to the diagram HI, which, when the radius of the second group is r, becomes, as it were, a reflected maze of the portion EK of the diagram of flux issuing from A. The resultant flux between the two groups now becomes the sum of the areas bounded by EK and HL.

In the case where the second group G is at the same distance as the group B, then $R_2 = R_1$, then the stray flux calculated from $0{\cdot}4\pi \times AW_1 \times 0{\cdot}367 \log_{10} \dfrac{R_1}{r}$ will be doubled. Should, however, the second group fall together with the first in one axis, then the stray flux issuing from A would be completely nullified. It is now clear that there must be between these two limiting values a position for G, for which the resulting flux will be exactly as great as that produced by A alone across a distance R_1. We shall take this as the "neutral" distance of G. For this condition the expression becomes (equal ampere turns in both groups assumed),

$$2 \times 0{\cdot}4\pi AW_1 \times 0{\cdot}367 \log_{10} \frac{R_2}{r} = 0{\cdot}4\pi AW_1 \times 0{\cdot}367 \log_{10} \frac{R_1}{r} \quad (965)$$

$$\text{or } 2 \log_{10} \frac{R_2}{r} = \log \frac{R_1}{r} \quad . \quad . \quad . \quad . \quad . \quad (966)$$

$$\text{so that } \frac{R_2}{r} = \sqrt{\frac{R_1}{r}} \quad . \quad . \quad . \quad . \quad . \quad (967)$$

When the second group is brought nearer to the first, the resultant flux between the two, on account of the diminished value of the section, is smaller than that for the first group alone, under the assumption of circular propagation of lines up to the limit R_1. When the second group is removed farther from the first, so will the resultant flux between the two be greater than the value calculated for the first group alone to the limiting distance R_1.

Now applying this to the case of the induction motor, we find that neutral distance of the equivalent rotor conductor group is

$$R_2 = r\sqrt{\frac{R_1}{r}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (968)$$

From Fig. 130 the value of $\frac{R_1}{r}$ varies between 11 and 15, according to the pole pitch, and r between 1·0 and 3·0. We obtain, therefore, with these limiting values

$$R_2 = 1\sqrt{11} = 3\cdot3 \text{ cm. and } 3 \times \sqrt{15} = 11\cdot6 \text{ cm.}$$

In reality, however, the rotor winding is considerably nearer to the stator winding. We have, therefore, to deal with the case that the assumption of circular leakage lines, which does not truly hold, produces too great values for the magnetic conductivity of a stator winding overhang.

But this requires still another correction. We have, in the above investigation, assumed that the second group of conductors is parallel to the first. It happens, however, that this is not so in the usual case of basket-wound (drum-wound) slip-ring rotors. Here the rotor currents do not run parallel to the stator currents, but approximately at an angle of 45° to these. The effect of this will be that the influence of rotor winding will be further reduced; that is to say, the reduction in the value of the flux, just deduced, will not be so great as was to be expected. We shall, therefore, make no great error if, for slip-ring motors with drum winding in the rotor, we leave unaltered the formula already found, though it must be kept in mind that herein lies a source of deviation between calculation and test results.

If in two motors of similar type, the rotor winding of one is cylindrical, that is to say, lying close to the stator winding, and that of the other, on the other hand, is more radial and arched towards the centre, and therefore more cone shaped, then the first motor will show less stray flux than the second. Consequently, under certain circumstances, it may be necessary to modify the constants in our formula.

In the case of squirrel-cage rotor, one must differentiate between the case where the short circuiting ring lies close to the iron core and the case where, in order to increase the ventilation in an axial direction, the bars project considerably from the core. When the rings lie close to the core, it may be assumed that the influence of the rotor on the formula for the magnetic conductivity of the stator winding overhang (calculated for circular path) may be neglected, so that the formula may be used without alteration.

On the other hand, for short-circuited rotors, with long bars and rings considerably removed from the core, the rotor currents flow

sensibly parallel to those of the stator, consequently it must be expected that for this case the field weakening effect of the rotor must make itself apparent. For such motors, therefore, the constant of our stray formula must therefore be diminished.

All of these considerations will be confirmed by trial. For short-circuited rotors with long bars, the constants of formula (963) must be multiplied by a constant varying from 0·8 to 0·9.

6. Split-phase windings—Three-plane windings. Up to this period the inquiry into the stray flux has been for a winding according to that shown in Fig. 125, that is to say, all the end windings for one pole and phase are grouped together to form one coil group. For long pole pitches, the split phase winding is, however, often made use of, whereby the coil overhangs of

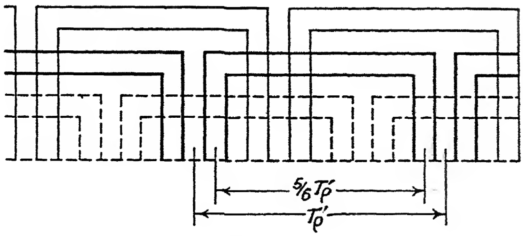


FIG. 133

the several phases lie in different winding planes perpendicular to the axis. This winding is diagrammatically illustrated in Fig. 133.

No alteration occurs in the portion of the coil which projects from the core in an axial direction. It remains therefore to leave the portion with $2m$—in formula (956) unaltered.

It is, however, a different matter in the case of the part of the formula dealing with the magnetic conductivity of the portion of the coil overhang lying along the periphery. This portion carries only one-half of the ampere turns and produces therefore only one-half of the stray flux. To obtain all fluxes for cases of equal ampere turns per pole and phase, we must therefore halve the magnetic conductivity obtained from this part of the formula. In this connection, it must still be noticed also that the equivalent radius r must be reduced approximately in the ratio of 1 to 2. Further, the shortened coil pitch $\frac{5}{6}T_p'$ (*see* Fig. 133) must be substituted in place of the pole pitch T_p'. Finally the influence of the two other phases may be arrived at by an investigation similar to that in connection with Fig. 129. It will be found that this

influence is slightly greater and may be expressed by replacing the factor 0·94 of formula (956) by the factor 1·06.

We arrive, therefore, at the following expression for the portion of the overhang lying along the periphery—

$$\frac{1}{2} \times 1{\cdot}06 \left[0{\cdot}040 + 0{\cdot}367 \log_{10}\frac{m}{r}\right] \frac{5}{6} T_p'$$

$$= \left[0{\cdot}0176 + 0{\cdot}162 \log_{10}\frac{m}{r}\right] T_p' \quad . \quad . \quad (969)$$

Further, the value of Λ_s for several values of m and T_p' may now be calculated and plotted as functions of T_p' with m as parameter in similar manner to Fig. 131. It will then be found that one may use the following expression with sufficient accuracy,

$$\Lambda_s = 0{\cdot}65A + 0{\cdot}20T_p' \quad . \quad . \quad . \quad (970)$$

$$\text{where } A = l_s - (\tfrac{5}{6}T_p') \quad . \quad . \quad . \quad . \quad (971)$$

When using length of the coil overhang as l_s, the formula becomes

$$\Lambda_s = 0{\cdot}65l_s - 0{\cdot}34T_p' \quad . \quad . \quad . \quad (972)$$

where T_p' is the pole pitch measured at the middle of the slots (not the coil pitch). The length l_s has naturally a different value for each of the three phases.

7. Review of the assumptions made in the derivation of formulae. The assumptions of Figs. 125A and 125B that we may take the coil sides as equivalent to a circular conductor group of equal area, is somewhat arbitrary. If greater accuracy is required, it must be noted that the coils proceeding from the several slots do not lie quite close together. A sort of "winding factor" must, therefore, be introduced. If this be neglected, then presumably the magnetic conductivity is calculated too large and therefore on the safe side.

Further, no difference has been made between straight and bent coils, but the neglect of the point gives again a result on the safe side, since the area enclosed by the overhang of the bent coils is less than that of the straight coils.

A contrary effect is exercised by the vicinity of the iron of the end shields, whereby the effect of the first two approximations will be somewhat nullified.

An exact calculation must naturally include all these points. Now, in practice, it is not necessary to obtain a great accuracy; instead, it is much more important to arrive at a handy formula,

by making simplifying assumptions, which will give an estimation of the stray flux in the majority of cases.

The formula agrees fully with this condition, especially since, for the calculation, they replace the inconvenient logarithmic terms by linear functions.

On the ground of experience in a given type of machine, the designer may slightly alter at his discretion the coefficients of the formulae.

8. Application to generators. It need scarcely be mentioned that the formulae may, without much ado, be applied also to generators.

9. Summary. The short-circuit current of an induction motor is (formula 939)

$$i_k = \frac{\phi}{0{\cdot}4\pi 2\sqrt{2}\, qz\,(\Lambda_n + \Lambda_z + \Lambda_s)}$$

where Λ_n, Λ_z, Λ_s are the magnetic conductivities of a coil side for slot, zigzag, and overhang stray flux.

A simple formula is deduced, for the magnetic conductivity Λ_s of a coil overhang, under consideration of the influence of the other phases.

For the usual three-phase winding,

$$\Lambda_s = 0{\cdot}7A + 0{\cdot}3T_p', \text{ where } A = l_s - T_p' \quad (973)$$

or $$\Lambda_s = 0{\cdot}7l_s - 0{\cdot}4T_p' \quad (974)$$

and for split-phase winding (three-plane winding)

$$\Lambda_s = 0{\cdot}65A + 0{\cdot}2T_p', \text{ where } A = l_s - \tfrac{5}{6}T_p' \quad (975)$$

or $$\Lambda_s = 0{\cdot}65l_s - 0{\cdot}34T_p' \quad (976)$$

These formulae are valid for motors with slip-ring drum windings, and for squirrel-cage motors with rings close up to core.

For squirrel-cage machines with rotor bars projecting far from the core, and for slip-ring machines with the rotor winding lying particularly close to the stator winding, the corresponding coefficients require to be somewhat reduced (about 10–20 per cent).

CHAPTER XVI

DESIGN (CONTD.)

THE method we have just outlined for calculating the ideal short-circuit current is fairly long, and in getting out a preliminary design it is advisable to have a quick method which will give us a rough approximate value for this quantity. The following method gives fairly good values, but is very rough. Nevertheless it is used in several design offices to advantage, and I have found it myself sufficiently accurate. The method is as follows—

Short-circuit current per phase

$$= \frac{1000}{z_1} \times \frac{\text{short-circuit volts per conductor}}{\text{volts per conductor at 50 cycles}}$$

z_1 = conductors in series per slot

$$\text{Volts per conductor at } 50 \sim\; = \frac{\text{volts per phase}}{\text{conductors in series per phase}}$$

The short-circuit volts per conductor is grouped in two parts: (*a*) for the slots, and (*b*) for the end connections.

(*a*) Short-circuit volts per conductor for the slots

$$= 0{\cdot}04 \times \text{length of core in inches,}$$

where the number of rotor slots is greater than the number of stator slots

$$= 0{\cdot}05 \times \text{length of core in inches,}$$

where the number of rotor slots is less than the number of stator slots.

(*b*) Short-circuit volts per conductor for the end connections

$$= 0{\cdot}012 \times \frac{(\text{pole pitch in inches})^{1{\cdot}5}}{\text{slot pitch in inches}}$$

Total short-circuit volts per conductor $= a + b$

This is merely another form for the short-circuit current, for volts per conductor at 50 is proportional to the working flux per pole, and the short-circuit volts per conductor is proportional to the leakage flux; and we have seen that the short-circuit current

$$= \frac{\text{working flux per pole}}{\text{leakage flux per ampere per pole}}$$

It is obvious that this cannot give us accurate values for the slot leakage, and zigzag leakage are bunched together and no

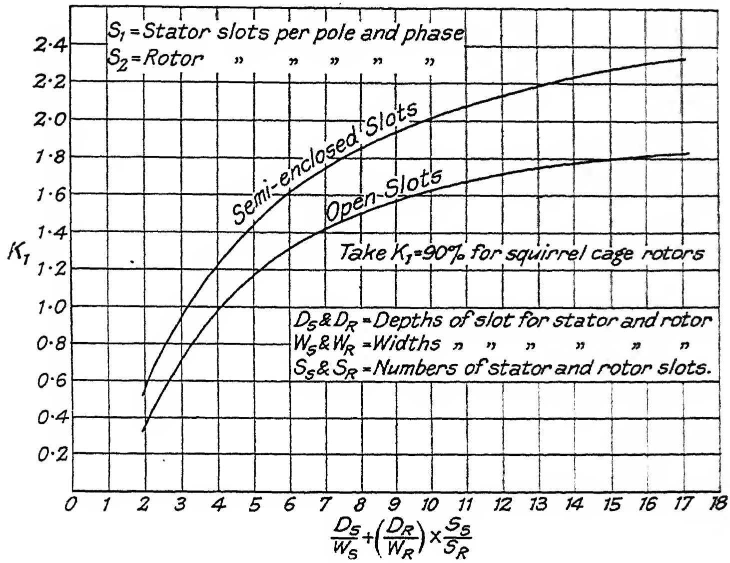


FIG. 134

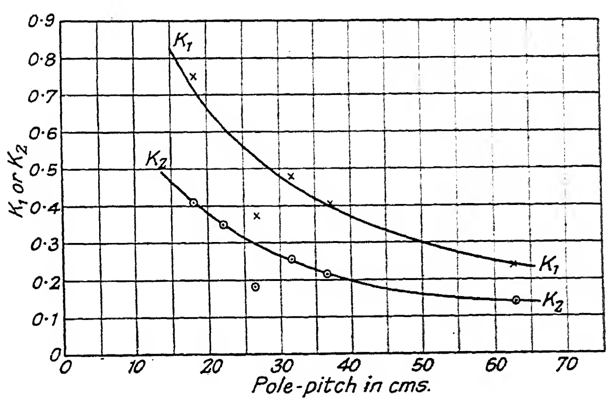


FIG. 135

account is taken of the widely varying shapes of slot. In spite of this, I have found that it gives wonderfully good values for the short-circuit current. It is frankly empirical, of course.

Still another method, due to Mr. Fletcher of the Metropolitan Vickers Co., is given in Fig. 134.

The curve is self-explanatory. This is also empirical, but the constants are taken from test results and cover a variety of windings.

The Short-Circuit Current per phase in amperes by Mr. Fletcher's method is—

Amperes per phase at $\cos\phi = 0{\cdot}5$ at short-circuit

$$= \frac{\text{Diameter of Rotor in inches} \times \text{Mean Gap density in lines per sq. cm}}{\text{Conductors per phase}\left[K_1\left(\frac{1}{S_1}+\frac{1}{S_2}\right)+K_2\frac{\text{length of end connection}}{\text{length of embedded conductor}}\right]}$$

End Winding Constants

Type of Winding

Stator.	Rotor.	K_2
Mush	Mush	0·24
Mush	Squirrel Cage	0·17
Mush	Diamond	0·22
Mush	Concentric	0·25
Concentric	Mush	0·25
Concentric	Squirrel Cage	0·175
Concentric	Concentric	0·26
Concentric	Diamond	0·24
Diamond	Mush	0·21
Diamond	Squirrel Cage	0·14
Diamond	Diamond	0·20
Diamond	Concentric	0·215

Another useful curve for the leakage permeance of the overhang of bar windings is given in Fig. 135.

To calculate the actual short-circuit current per phase, we need to know the stator and rotor resistances per phase.

We have already calculated the ideal short-circuit current per phase, viz., OB.

If we set up from B a vertical line and make

$$DE = \frac{OB^2 \times R_s}{V} \quad . \quad . \quad . \quad . \quad . \quad (977)$$

where R_s = stator resistance per phase (hot)

V = applied volts per phase

and in the case of a slip-ring type motor,

$$\text{make } EF = \frac{\left(AB \cdot \frac{T_s}{T_r}\right)^2 \times R_r}{V} \quad . \quad . \quad . \quad (978)$$

where AB = ideal short-circuit current per phase of the rotor referred to the stator

The ideal *rotor* short-circuit current per phase

$$= AB \times \frac{\text{effective turns per phase of stator}}{\text{effective turns per phase of rotor}}$$

$$= AB \times \frac{T_s}{T_r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (979)$$

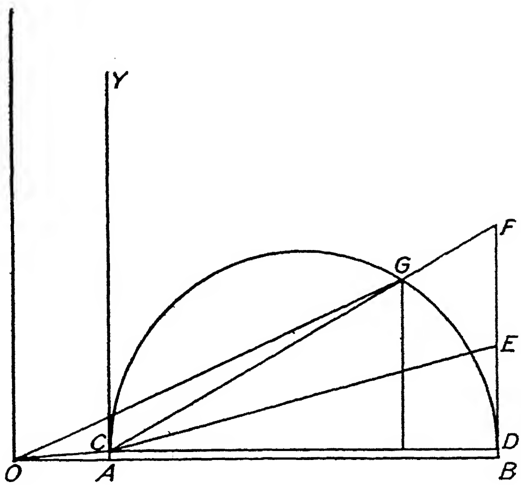


FIG. 136

Join OF, then OG is the actual short-circuit current per phase in the stator.

In the case of a squirrel-cage machine, one finds the loss in the rotor with the ideal rotor short-circuit current,

$$\text{then } EF = \frac{\text{loss in rotor with ideal short-circuit rotor current}}{\text{number of stator phases} \times V} \quad (980)$$

Calculation of loss in squirrel-cage rotors. Current per bar in the squirrel cage at short circuit

$$= AB \times \frac{\text{total conductors in series on stator}}{\text{total number of bars on rotor}} \quad . \quad (981)$$

$$= AB \times \frac{mZ}{T_R} \quad . \quad . \quad . \quad . \quad . \quad . \quad (982)$$

where m = number of phases on stator

Z = conductors in series per phase

T_R = number of rotor bars

Loss in bars

$$= \left(\frac{AB \times mZ}{T_R}\right)^2 \times \text{resis. of rotor bars hot} \quad . \quad (983)$$

Resistance of rotor bars (hot) in ohms

$$= \frac{0{\cdot}017 \times T_R \times l_r \times 1{\cdot}2}{a_r} \quad . \quad . \quad . \quad . \quad (984)$$

l_r = length of rotor bar in metres

a_r = area of rotor bar in square millimetres

Current and loss in end rings.

$$\text{Average current in each bar} = \frac{\text{effective current per bar}}{1{\cdot}11} \quad . \quad (985)$$

Max. current in end rings

$$= \text{average current per bar} \times \frac{\text{bars per pole}}{2} \quad . \quad . \quad (986)$$

Effective or R.M.S. value of current per ring

$$= \frac{\text{effective current per bar}}{1{\cdot}11} \times \frac{\text{bars}}{2 \text{ poles}} \times \frac{1}{\sqrt{2}} \quad . \quad . \quad (987)$$

$\therefore$ loss in 2 rings with ideal rotor short-circuit current

$$= \left(AB \times \frac{mZ}{T_r} \times \frac{1}{1{\cdot}11} \times \frac{T_r}{2p} \times \frac{1}{\sqrt{2}}\right)^2 \times \text{resis. per ring} \times 2 \quad (988)$$

$$\text{Resis. per ring (hot)} = \frac{0{\cdot}017 \times \text{mean perimeter of ring in metres}}{\text{area of ring in sq. mm.}} \times 1{\cdot}2 \quad . \quad . \quad (989)$$

CHAPTER XVII

TYPICAL DESIGN OF SLIP-RING MOTOR

OUTPUT = 250 b.h.p.; 3-phase; 50 cycles; 440 volts; 1500 r.p.m. synchronous.

Temperature rise not greater than 40° C. by thermometer after 6 hours full-load run.

$$\text{Number of poles} = \frac{120 \times \text{frequency}}{\text{revs. per min.}} \quad . \quad . \quad . \quad (990)$$

$$= \frac{120 \times 50}{1500} = 4 \quad . \quad . \quad . \quad (991)$$

$$D^2L \text{ cu. cm.} = \frac{4\cdot06 \times 10^{11} \times \text{B.H.P.}}{B \times q \times \text{R.P.M.} \times \cos\theta \times \eta} \quad . \quad (992)$$

$$= \frac{4\cdot06 \times 10^{11} \times 250}{4500 \times 300 \times 1450 \times 0\cdot9 \times 0\cdot9} \quad . \quad (993)$$

B assumed $= 4500$ $\quad q = 300$ $\quad \cos\phi = 0\cdot9$ $\quad \eta = 0\cdot9$

$$D^2L \text{ cu. cm.} = 64{,}000 \text{ cu. cm.} \quad . \quad . \quad (994)$$

$$\text{for best power factor, } \frac{\tau}{L} = \frac{18}{\tau} \quad . \quad . \quad . \quad . \quad (995)$$

$$\tau = \frac{\pi D}{p} \quad \therefore \tau^2 = \frac{\pi^2 D^2}{p^2} \quad . \quad . \quad . \quad . \quad (996)$$

$$\therefore \frac{\pi^2 D^2}{p^2 L} = 18 \quad . \quad . \quad . \quad . \quad . \quad (997)$$

$$\therefore D^2 = \frac{18p^2L}{\pi^2} = 1\cdot82p^2L \quad . \quad . \quad (998)$$

$$\therefore D = 1\cdot35p\sqrt{L} \quad . \quad . \quad . \quad (999)$$

Now $p = 4$

$$\therefore D = 5\cdot4\sqrt{L} \quad . \quad . \quad . \quad . \quad (1000)$$

$$\therefore 29\cdot2L^2 = 64{,}000 \quad . \quad . \quad . \quad . \quad (1001)$$

$$\therefore L^2 = 2190 \quad \therefore L = 46\cdot7 \text{ cm.} \quad . \quad (1002)$$

$$\text{and } D = 37 \text{ cm.} \quad . \quad . \quad . \quad . \quad (1003)$$

We see that a small diameter machine of relatively great core length is needed for best power factor. Such a machine could be built, but the cooling qualities would be better were we to adopt a shorter machine of greater diameter. This we can do on a 4-pole machine without sacrificing power factor to any great extent.

We will decide to build this machine with a diameter of 45 cm.

$$\text{Then } L = \frac{64{,}000}{45 \times 45} = 31{\cdot}6 \text{ cm.} \qquad . \qquad . \quad (1004)$$

There will be a ventilating duct every 7 cm., so our gross core length will be (approx.) 34 cm., with 4 ducts each 1 cm. wide.

The apparent air-gap area per pole = pole pitch $\times$ core length

$$\text{the pole pitch} = \frac{\pi \times 45}{4} = 35{\cdot}4 \text{ cm.}$$

$\therefore$ apparent gap area per pole $= 35{\cdot}4 \times 34 = 1200$ sq. cm.

Assuming an apparent air-gap density of 4500—which may need to be modified—we have the flux per pole

$$= 4500 \times 1200 \text{ C.G.S. hr.}$$

$$= 5{\cdot}4 \times 10^6 \text{ lines}$$

We have now to choose the number of stator slots.

On a machine having a pole pitch of 35·4 cm., we should expect to use 5 or 6 slots per pole per phase.

If we choose 18 slots per pole, we get a slot pitch of roughly 2 cm., and this is very suitable.

The total number of stator slots $= 18 \times 4 = 72$

From our E.M.F. equation, we get the conductors in series per phase required, viz., Z,

$$\text{thus } E = 2{\cdot}12 \times \phi \times Z \times f \times 10^{-8} \qquad . \qquad . \qquad . \quad (1005)$$

Assuming the machine is delta-connected, the applied volts per phase = 440

Allowing 2% for resis. drop, $E = 430$

$$\therefore Z = \frac{430 \times 10^8}{2{\cdot}12 \times 5{\cdot}4 \times 10^6 \times 50} = 75 \qquad . \qquad . \quad (1006)$$

Slots per phase = 24

We may, therefore, try 3 conductors per slot, which will give us 72 conductors in series per phase.

With $Z = 72$ $\qquad\qquad \phi = 5{\cdot}62 \times 10^6$

Before proceeding farther, we will check the overload capacity with this winding. Here the great advantage of the rough approx. formula for the short-circuit current comes in. We will save ourselves a great deal of work by its use.

$$\text{Volts per conductor at } 50\sim = \frac{440}{24 \times 3} = 6{\cdot}12$$

Short-circuit volts per conductor

$$(a) \quad = 0{\cdot}04 \times 13{\cdot}4 = 0{\cdot}536$$

$$(b) \quad 0{\cdot}012 \times \frac{(13{\cdot}9)^{1\cdot5}}{0{\cdot}756} = 0{\cdot}795$$

$$\text{total short-circuit volts per conductor} = a + b = 1{\cdot}331$$

$$\text{Ideal short-circuit current per phase} = \frac{1000}{z_1} \times \frac{6{\cdot}12}{1{\cdot}331}$$

$$z_1 = \text{conductors per slot}$$

Since the machine is delta-connected,

$$\text{the short-circuit current per line} = \sqrt{3} \times \frac{1000}{3} \times \frac{6{\cdot}12}{1{\cdot}331} = 2650$$

$$\text{Full-load current (line)} = \frac{250 \times 746}{\sqrt{3} \times 440 \times 0{\cdot}92 \times 0{\cdot}92} = 290$$

Max. amps. about 73

$$\text{Actual short-circuit current} = (2650 - \cancel{290}\ 73) \times {\cdot}866 = 2320$$

$$\text{Max. H.P.} = \frac{\sqrt{3} \times 440 \times 2320}{2{\cdot}8 \times 746} = 845 = 3{\cdot}4 \text{ full load}$$

The power factor at short circuit has been taken as 0·4. It would be lower, probably 0·25 to 0·3.

$$\text{Now } \sigma = \frac{73}{2650} = 0{\cdot}0276 \quad . \quad . \quad . \quad (1007)$$

A glance at the power-factor curves already given will show that this machine would have a good power factor at full load with this value of σ, but at light loads it would fall off more rapidly than it ought.

It is quite clear therefore that the overload capacity is too great.

Since the reactance is proportional to the square of the number of turns, if we increase the number of turns per coil from 3 to 3·5,

$$\text{we shall reduce the max. h.p. to } 845 \times \left(\frac{3}{3{\cdot}5}\right)^2 = 620$$

$$= 2{\cdot}48 \text{ F.L.}$$

This will be quite suitable.

We will use, therefore, 7 turns per coil and connect each phase in two parallel circuits delta. This will give us the equivalent of 3·5 turns per coil in series.

With this winding, $\phi = \dfrac{430 \times 10^8}{2{\cdot}12 \times 24 \times 3{\cdot}5 \times 50} = 4{\cdot}84 \times 10^6$

(1008)

Now the full-load current (line) = 290 amp. (line) approx.

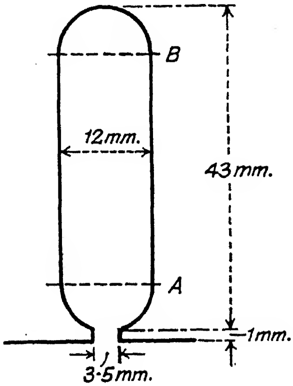


FIG. 137

Full-load amps. per phase = 167·5 amp.

Current per circuit = 83·75 amp.

Assuming a current density in the conductors of 3·3 amp. per sq. mm.—a conservative value—we have section of conductor

$$= \frac{83{\cdot}75}{3{\cdot}3} = 25{\cdot}4 \text{ sq. mms.}$$

If we use 8 wires in parallel and each wire of 2·0 mm. diameter bare d.c.c. to 2·3 mm., the section of the 8 wires constituting one effective conductor = 25·12 sq. mm.

Wires per slot = 56

Space taken up by wires $= 56 \times 2{\cdot}3 \times 2{\cdot}3 = 297$

Allowing a space factor of 0·75, space required for wires insulated

$$= \frac{297}{0{\cdot}75} = 396 \text{ sq. mm.}$$

Insulation for the slot will consist of presspahn and leatheroid troughs of total thickness per side = 0·7 mm.

After trial, the slot shown is found suitable. It has semi-circular ends.

Diameter at $A = 450 + 14$ mm. = 464 mm.

Pitch of slot at $A = \dfrac{\pi \times 464}{72} = 20{\cdot}3$ mm.

Width of tooth at $A = 20{\cdot}3 - 12 = 8{\cdot}3$ mm.

$$\text{Area of teeth per pole at } A = 0{\cdot}83 \times 18 \times 30 \times 0{\cdot}91$$
$$= 409 \text{ sq. cm.}$$

$$B_{60} \text{ at } A = \frac{4{\cdot}84 \times 10^6}{409} \times 1{\cdot}28 = 15{,}100 \text{ lines per sq. cm.}$$

$$\text{Area of teeth at } B \text{ per pole} = 537$$
$$B_{60} \text{ at } B = 11{,}500$$

These are suitable densities, and the slot will suit quite well.

Assuming a flux density in the stator core of 7500 lines per sq. cm.

$$\text{Area of core in sq. cm.} = \frac{\phi}{2 \times 7500} = \frac{2{\cdot}42 \times 10^6}{7500} = 322 \text{ sq. cm.}$$

Let R = radial depth of core behind the stator teeth,

$$R \times 30 \times 0{\cdot}91 = 322$$
$$R = 11{\cdot}75 \text{ cm.}$$

[0·91 is the iron factor for core]

$$\text{Outside diameter of stator core} = 450 + 88 + 235 \text{ mm.}$$
$$= 773 \text{ mm.}$$

Rotor. A rotor winding with 2 bars per slot gives the soundest mechanical job. We might use a wave winding with a fractional number of slots per pole per phase opened out in 6 places and the opposite phases connected in series. The voltage between rings should, if possible, be kept below 500 volts to minimize danger from shock and risk of breakdown of insulation; and, on the other hand, the current per ring must not be excessive on account of loss and cost and space of slip-rings, short-circuiting gear, and brush-lifting devices.

On very large machines it is difficult to keep down the voltage between rings; and one finds, in practice, voltages of 1000 or more with correspondingly large rotor currents. Resort must be made to the use of parallel circuits in the rotor to keep down the volts between rings and to deal with the large rotor currents.

In the present case we will use 7 slots per pole per phase in the rotor,

i.e. 84 rotor slots and 2 bars per slot

The rotor current will be

$$\frac{250 \times 746}{3 \times 430 \times 0{\cdot}9} \times \frac{72 \times 3{\cdot}5}{84 \times 2} = 241 \text{ amp.}$$

and the volts between rings at start (approx.)

$$= \frac{250 \times 746}{\sqrt{3} \times 241 \times 0{\cdot}9} = 498$$

practically 500 volts.

We may safely allow a current density of 4 amp. per sq. mm.

$$\text{Area of rotor bar} = \frac{241}{4} = 60{\cdot}25 \text{ sq. mm.}$$

The slot shown is suitable, and the rotor bar will be of the type shown below.

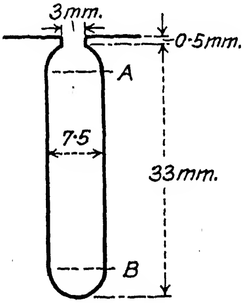


FIG. 138

The slot has semicircular ends, and the bar has one semicircular end as shown in Fig. 139.

Area of bar = 60·37 sq. mm.

There will be a slot lining of 0·15 mm. thick, and the insulation on the bars will be of paper 1·0 mm. thick ironed on to coil.

Area of teeth at A per pole = 520 sq.cm.

Area at B per pole = 405sq. cm.

$$B_{60} \text{ at } A = 11{,}900 = \frac{4{\cdot}84 \times 10^6}{520} \times 1{\cdot}28$$

$$B_{60} \text{ at } B = 15{,}300$$

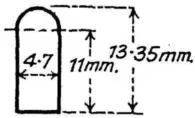


FIG. 139

$$\text{Area of rotor core} = \frac{2{\cdot}42 \times 10^6}{9000} = 268 \text{ sq. cm.}$$

$$\text{Depth of iron below teeth} = \frac{268}{30 \times 0{\cdot}91} = 9{\cdot}8 \text{ cm.}$$

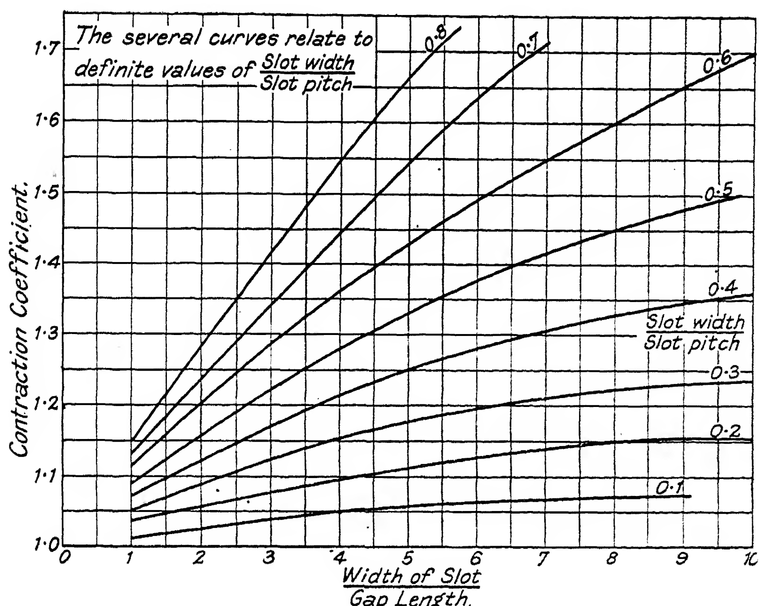
Internal diameter of rotor = 447·4 − 67 − 196 = 184 mm.

$$\text{Diameter of rotor shaft in inches} = 9\sqrt[3]{\frac{\text{H.P.}}{\text{R.P.M.}}}$$

$$= 9\sqrt[3]{\frac{250}{1450}} = 5''$$

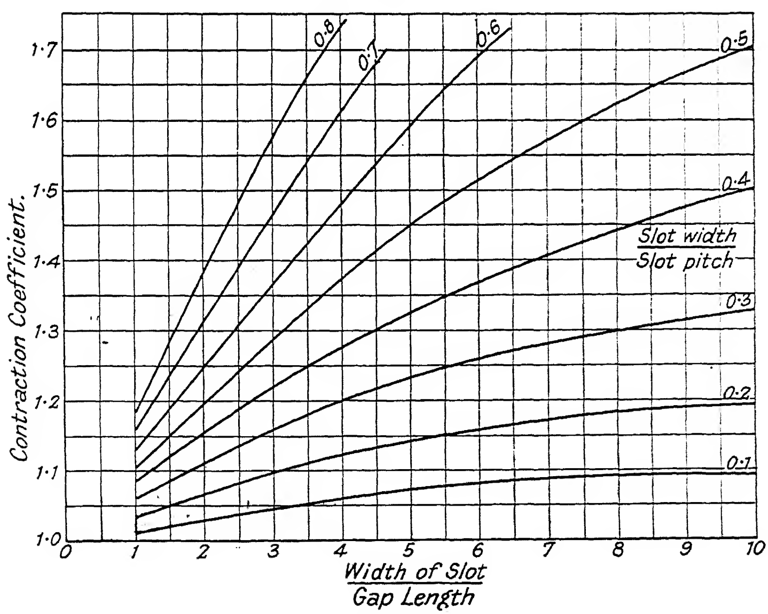
$$= 127 \text{ mm.}$$

Calculation of magnetizing current. We will first find the air-gap area per pole, corrected for slots and ventilating ducts. For this purpose, the sets of curves shown on next page, and which



FOR SEMI-ENCLOSED SLOTS (٢١·٦)

FIG. 140A



FOR OPEN SLOTS

FIG. 140B

are taken from Prof. Miles Walker's book on Design, may be conveniently used.

$$\frac{\text{width of stator slot opening at the gap}}{\text{air-gap length}} = \frac{3\cdot5}{1\cdot3} = 2\cdot69$$

$$\frac{\text{width of stator slot}}{\text{pitch of stator slot}} = \frac{3\cdot5}{19\cdot6} = 0\cdot178$$

Contraction coefficient for stator slots = 1·085

$$\frac{\text{width of rotor slot opening at the gap}}{\text{air-gap length}} = \frac{3\cdot0}{1\cdot3} = 2\cdot3$$

$$\frac{\text{width of rotor slot opening}}{\text{pitch of rotor slots at the air-gap}} = \frac{3\cdot0}{16\cdot75} = 0\cdot179$$

Contraction coefficient for rotor slots = 1·075

$$\frac{\text{width of ventilating duct}}{\text{gap length}} = \frac{10\cdot0}{1\cdot3} = 7\cdot7$$

$$\frac{\text{width of ventilating duct}}{\text{pitch of ventilating ducts}} = \frac{1\cdot0}{6\cdot8} = 0\cdot147$$

Contraction coefficient for ventilating ducts = 1·1

Actual air-gap area per pole

$$= \frac{\text{apparent gap area per pole}}{\text{product of contraction coefficients}} = \frac{1200}{1\cdot085 \times 1\cdot075 \times 1\cdot1}$$
$$= 935 \text{ sq. cm.}$$

In estimating the amp.-turns required for the stator and rotor teeth, the curve shown in Fig. 141 is very useful.

Part.	Area sq. cm.	Length of Path cm.	Flux per Pole.	B_{60}	A_{60} per Pole-pair.
Stator core .	322	34·4 ($\frac{2}{3}$ of pole-pitch)	$2\cdot42 \times 10^6$	7,500	113
Stator teeth A / B	409 / 537	8·8	$4\cdot84 \times 10^6$	15,[illegible]00 / 11,400	136
Air-gap. . .	935	0·26	$4\cdot84 \times 10^6$	6,625	1370
Rotor teeth A / B	520 / 405	6·7	$4\cdot84 \times 10^6$	11,900 / 15,300	113
Rotor core. .	268	14·8	$2\cdot42 \times 10^6$	9,000	78
					1810

Total amp.-turns at 60° (A_{60}) per pair of poles = 1810

Magnetizing current per line wire

$$= \frac{\sqrt{3} \times 1810}{2{\cdot}12 \times 6 \times 3{\cdot}5} = 70{\cdot}5 \text{ amp.}$$

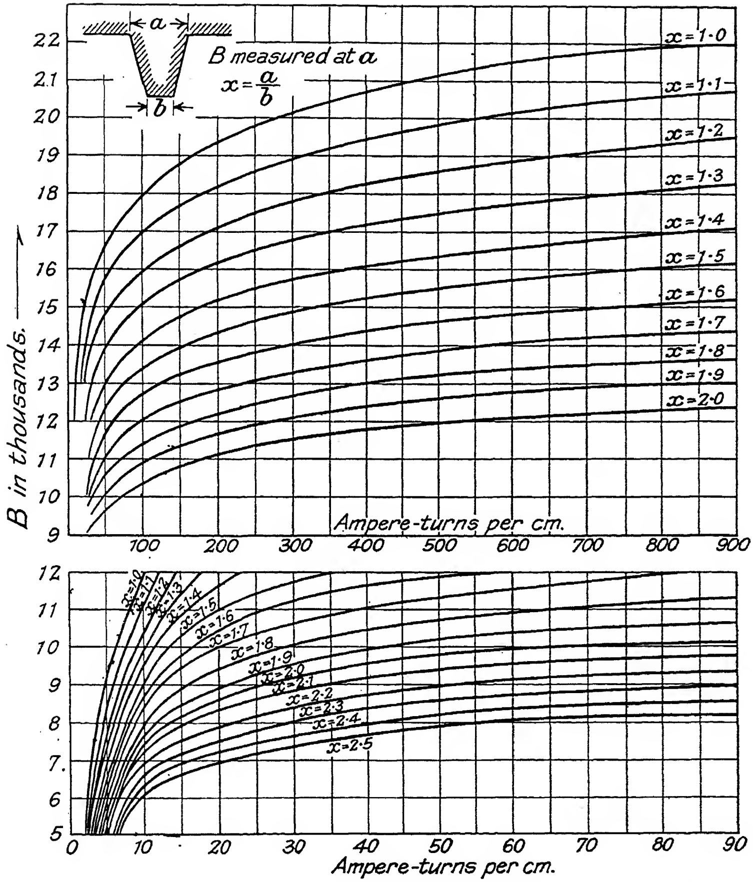


FIG. 141.—MAGNETIZATION CURVES FOR TAPERED TEETH 21·9
Based on Leysaght's, Ltd., samples

The ideal short-circuit current. Stator slot permeance per cm. length of core

$$= \frac{31}{36} + 0{\cdot}7854 + \frac{1{\cdot}0}{3{\cdot}5} = 0{\cdot}86 + 0{\cdot}7854 + 0{\cdot}286 = 1{\cdot}9314 = \lambda_s$$

Rotor slot permeance per cm. length of core

$$= \frac{25\cdot5}{22\cdot5} + 0\cdot7854 + \frac{0\cdot5}{2\cdot5} = 1\cdot13 + 0\cdot7854 + 0\cdot2 = 2\cdot115 = \lambda_r$$

$$\lambda_1 = \left[\frac{1\cdot93}{0\cdot96 \times 6} + \frac{2\cdot115}{7}\right] \times 30 = 19\cdot1$$

$$\lambda_2 = 0\cdot73(0\cdot36\tau + A) \qquad [A = 22 \text{ cm.}$$

$$= 0\cdot73(0\cdot36 \times 35\cdot4 + 22) = 25\cdot3$$

$$\lambda_3 = \frac{L\varepsilon\tau}{48 \times \delta \times q_1 \times q_2} = \frac{34 \times 1\cdot715 \times 35\cdot4}{48 \times 0\cdot13 \times 6 \times 7}$$

$$= 7\cdot9$$

$$\therefore\ \lambda = \lambda_1 + \lambda_2 + \lambda_3$$

$$= 19\cdot1 + 25\cdot3 + 7\cdot9 = 52\cdot3$$

$$\therefore\quad I_{sc} \text{ ideal (line)} = \frac{\sqrt{3} \times 4\cdot84 \times 10^6}{3\cdot55 \times 52\cdot3 \times 6 \times 3\cdot5} = 2145 \text{ amp.}$$

Our rough method gives us the following value—

$$\text{Volts per conductor at } 50\sim = \frac{440}{24 \times 3\cdot5} = 5\cdot24$$

$$\text{Short-circuit volts per conductor} = 1\cdot331$$

$$\text{Short-circuit current per line} = \sqrt{3} \times \frac{1000}{3\cdot5} \times \frac{5\cdot24}{1\cdot331}$$

$$= 1950 \text{ amp.}$$

The agreement is fairly good.

$$\text{The dispersion coefficient} = \frac{70\cdot5}{2145} = 0\cdot0329$$

Rotor winding. Length of mean turn of rotor coil

$$= 2(34 + 50) = 168 \text{ cm.}$$

$$\text{Resistance per phase at } 15^\circ \text{ C.} = \frac{0\cdot017 \times 28 \times 1\cdot68}{60\cdot37}$$

$$= 0\cdot0132 \text{ ohms}$$

$$\text{Resistance (hot) per phase} = 0\cdot0156 \text{ ohms}$$

$$\text{Weight of rotor copper} = 0\cdot0197 \times 84 \times 1\cdot68 \times 60\cdot37 = 168 \text{ lb.}$$

Stator winding.

$$\text{L.M.T.} = 2(34 + 70\cdot5) = 209 \text{ cm.}$$

$$\text{Resistance per phase at } 15^\circ \text{ C.} = \frac{0{\cdot}017 \times 12 \times 7 \times 2{\cdot}09}{25{\cdot}12 \times 4}$$

$$= 0{\cdot}0296 \text{ ohms}$$

$$\text{Resistance (hot) per phase} = 0{\cdot}035 \text{ ohms}$$

$$\text{Weight of stator copper (total)} = 0{\cdot}0197 \times 36 \times 2{\cdot}09 \times 7 \times 25{\cdot}12$$

$$= 261 \text{ lb.}$$

Power factor at short circuit. Stator loss per phase with ideal short-circuit current

$$= \left(\frac{2145}{\sqrt{3}}\right)^2 \times 0{\cdot}035 = 53{,}830 \text{ watts}$$

Watt component of current corresponding to this $= DE$ in diagram

$$= \frac{53{,}830}{440} = 122{\cdot}5 \text{ amp.}$$

$$\text{Rotor ideal short-circuit current per phase} = 1195 \times \frac{72 \times 3{\cdot}5}{84 \times 2}$$

$$= 1800$$

Rotor loss per phase with ideal short-circuit current

$$= 1800^2 \times 0{\cdot}0156 = 50{,}600 \text{ watts}$$

Watt component of current corresponding to this loss per phase

$$= \frac{50{,}600}{440} = 115 = FE \text{ in our diagram}$$

$$\therefore \quad FD = 237{\cdot}5$$

$$\text{and } AB \text{ per phase} = \frac{2145 - 70{\cdot}5}{\sqrt{3}} = 1195$$

$$\therefore \tan \phi' = \tan FCD = \frac{237{\cdot}5}{1195} = 0{\cdot}198$$

$$\therefore \phi' = 11^\circ\, 35'$$

$$\therefore \cos \phi \text{ at short circuit} = 0{\cdot}2$$

$$\therefore \text{max. h.p.} = \frac{\sqrt{3} \times 440 \times 2010}{2(1 + 0{\cdot}2) \times 746} = \frac{\sqrt{3} \times 440 \times 2010}{2{\cdot}4 \times 746}$$

$$= 860 \text{ h.p.}$$

$$\frac{\text{full load}}{\text{max. load}} = \frac{250}{860} = 0{\cdot}29$$

$$\cos \phi \text{ at full load from the curves} = 93{\cdot}5\%$$

The overload capacity is rather great, and the performance at light loads would be improved in this case by working with a smaller flux per pole. In fact, 4 turns per coil would have given us ample overload capacity.

We have here an illustration of the fact that it is easy to get both overload capacity and high power factor on a fairly large machine of high speed. We should find matters very different were we to attempt to design a small machine running at low speed.

Efficiency at full load—

$$\text{Stator copper loss (3 phases)} = 3 \times 164^2 \times 0{\cdot}0035$$
$$= 2830 \text{ watts}$$
$$\text{Rotor copper loss (3 phases)} = 3 \times 241^2 \times 0{\cdot}0156$$
$$= 2720 \text{ watts}$$
$$\text{Iron loss (teeth)} = 1050 \text{ watts}$$
$$\text{,, (core)} = 2130 \text{ ,,}$$
$$\text{Total iron loss} = 3180 \text{ ,,}$$

Bearings. Journal of 4 in. diameter and 10 in. long.

$$\text{Rubbing velocity} = 1515 \text{ ft. per minute}$$
$$\text{Bearing loss} = 0{\cdot}81 \times 4 \times 10 \times 15{\cdot}15^{1{\cdot}5} \times 2$$
$$= 3{,}770 \text{ watts}$$
$$\text{Total losses} = 12{,}500 \text{ ,,}$$
$$\text{Output} = 186{,}500 \text{ ,,}$$
$$\text{Input} = 199{,}000 \text{ ,,}$$
$$\text{Eff.} = 93{\cdot}5\%$$

$$\text{Slip} = \frac{2720}{192{,}990} = 0{\cdot}0141 = 1{\cdot}41\% \text{ at full load}$$

$$\text{Full-load speed} = 1480 \text{ r.p.m.}$$

The stator will be connected in two parallel circuits delta, and will have the usual hemi-tropic concentric type of winding, with 1 coil per pair of poles.

The rotor winding will be of the non-academic wave type, split in six places and opposite phases connected in series.

Rotor winding. Two bars per slot, 84 slots wave type. Bars numbered in clockwise direction at the slip-ring end of rotor.

Left-hand winding. Upper bars numbered 1, 2, 3, etc.
lower bars numbered 1′, 2′, 3′, etc.

Number of slots	=	84
Number of live bars	=	168
Slots per pole per phase	=	7
Step of winding—front	=	21
back	=	21
Abnormal step	=	20
Total abnormal lower bars	=	36

Lower bars with abnormal step on front—

Phase I	Phase II	Phase III
22′–27′	78′–83′	50′–55′
43′–48′	15′–20′	71′–76′

Connections to slip-rings on upper bars—

Phase	I . . .	Bar 1
,,	II . . .	,, 57
,,	III . . .	,, 29

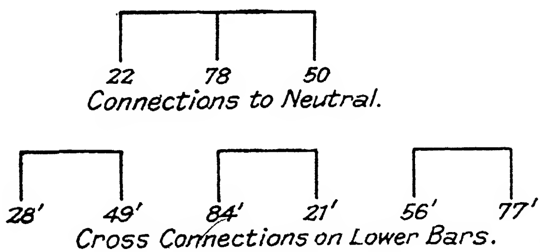


FIG. 142

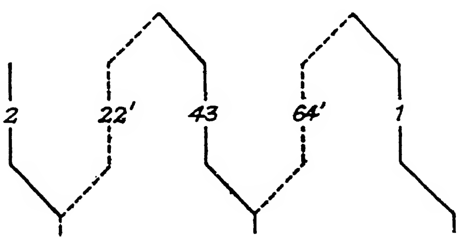


FIG. 143

March of rotor winding shown above.

CHAPTER XVIII

DESIGN FOR SYNCHRONOUS SLIP-RING MOTOR 3750 B.H.P.

Design for 3750 b.h.p.; 40 cycles, 2750 volts; 400 r.p.m. synchronous, slip-ring type of induction motor.

$$\text{Poles} = \frac{120 \times \text{frequency}}{\text{R.P.M.}} = \frac{120 \times 40}{400} = 12$$

$$D^2L \text{ cu. cm.} = \frac{4{\cdot}06 \times 10^{11} \times \text{B.H.P.}}{B \times q \times \text{R.P.M.} \times \cos\theta \times \eta}$$

$$B = 4100 \text{ average}$$

$$q = 310 \text{ amp. conductors per cm.}$$

$$\text{R.P.M.} = 396 \text{ at full load}$$

$$\cos\theta = 0{\cdot}86$$

$$\eta = 95{\cdot}5\%$$

$$\therefore D^2L = \frac{4{\cdot}06 \times 10^{11} \times 3750}{4100 \times 310 \times 396 \times 0{\cdot}86 \times 0{\cdot}955}$$

$$= 3{\cdot}68 \times 10^6 \text{ cu. cm.}$$

For best power factor $\dfrac{\tau}{L} = \dfrac{18}{\tau}$ $\quad \therefore D = 1{\cdot}35p\sqrt{L}$

$$= 16{\cdot}2\sqrt{L}$$

$$\therefore 262L^2 = 3{\cdot}68 \times 10^6$$

$$L^2 = 1{\cdot}42 \times 10^4$$

$$\therefore L = 1{\cdot}19 \times 10^2 = 119 \text{ cm.}$$

$$\text{and } D = 16{\cdot}2 \times 10{\cdot}9 = 177 \text{ cm.}$$

Actually this machine was built with $D = 205{\cdot}74$ cm.

and $L = 94$ cm.

giving $D^2L = 205{\cdot}74^2 \times 94 = 3{\cdot}98 \times 10^6$

No doubt from a cooling standpoint it is better to use the larger diameter and shorter core length, although the machine could well be built on the smaller diameter and would give better power factor.

We will chose $D = 205{\cdot}74$ cm. and $L = 94$ cm.

There will be 14 ventilating ducts, each of 1 cm. wide.

Net iron length $= 80 \times 0{\cdot}91 = 72$ cm.

Apparent air-gap area per pole

$$= \frac{\pi \times 205{\cdot}74}{12} \times 94 = 5050 \text{ sq. cm.}$$

Flux per pole $= 5050 \times 3760 = 19 \times 10^6$ lines

$$\text{Pole pitch} = \frac{\pi \times 205{\cdot}74}{12} = 53{\cdot}8 \text{ cm.}$$

We will use 12 slots per pole, giving a slot pitch of

$$\frac{53{\cdot}8}{12} = 4{\cdot}48 \text{ cm.}$$

Number of stator slots $= 144$

We will adopt a star-connected winding for this voltage.

$$\text{Volts per phase} = \frac{2750}{\sqrt{3}} = 1585$$

Now $E = 2{\cdot}12 \times Z \times \phi \times f \times 10^{-8}$

$$\therefore\ Z = \frac{1585 \times 10^8}{2{\cdot}12 \times 19 \times 10^6 \times 40} = 98$$

i.e. approx. 98 conductors in series per phase are required.

Now the number of slots per phase $= 48$

$\therefore$ this suggests 2 conductors per slot.

With 2 conductors per slot, or 96 conductors in series per phase, the flux per pole

$$= \frac{1585 \times 10^8}{2{\cdot}12 \times 96 \times 40} = 19{\cdot}4 \times 10^6$$

Assuming a current density of 3·1 amp. per sq. mm.,

$$\text{Full-load stator current} = \frac{3750 \times 746}{\sqrt{3} \times 2750 \times 0{\cdot}86 \times 0{\cdot}955}$$

$$= 710 \text{ amps.}$$

$$\text{Area of stator conductor} = \frac{710}{3{\cdot}1} = 229 \text{ sq. mm.}$$

Limiting the flux density in the teeth at minimum section to 10,500 lines per sq. cm. average, we have

$$\text{Width of tooth at minimum section} = \frac{19{\cdot}4 \times 10^6}{10{,}500 \times 12 \times 72}$$

$$= 2{\cdot}1 \text{ cm. approx.}$$

Pitch of teeth at minimum section

$$= \frac{\pi \times 213}{144} \; 4{\cdot}6 \text{ cm.}$$

This gives a width of slot = 2·5 cm.

We will make the slot = 2·4 cm. wide

Now the slot insulation will consist of micanite troughs of 2 mm. thickness for this voltage, so the width of copper will be

$$24 - 4 - 1 \text{ mm.} = 19 \text{ mm.}$$

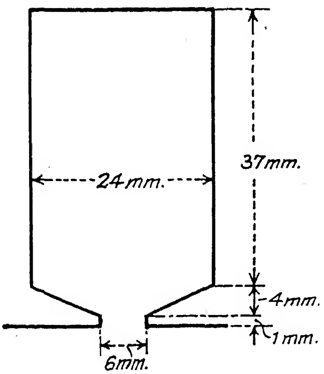


FIG. 144

and the depth of the copper will be 12 mm., giving an area of 228 sq. mm.

The depth of slot will be, allowing for insulation and slack,

$$24 + 8 + 5 = 37 \text{ mm.}$$

and our stator slot is shown in Fig. 144.

Assuming a flux density in the core of 9200 lines per sq. cm.,

$$\text{Area of core} = \frac{\phi}{2 \times 9200}$$

$$= \frac{19{\cdot}4 \times 10^6}{18{,}400} = 1{\cdot}05 \times 10^3 \text{ sq. cm.}$$

Radial depth of iron behind the teeth

$$= \frac{1050}{72} = 14{\cdot}6 \text{ cm.}$$

$$\text{External dia. of stator core} = 205{\cdot}74 + 8{\cdot}4 + 29{\cdot}2$$

$$= 243{\cdot}34 \text{ cm.}$$

say, 244 cm.

$$\text{L.M.T. of stator winding} = 2(122 + 78) \text{ cm.}$$

$$= 400 \text{ cm.}$$

$$\text{Resis. per phase at } 15^\circ \text{ C.} = \frac{0{\cdot}017 \times 48 \times 4{\cdot}0}{228}$$

$$= 0{\cdot}0143$$

$$\text{Resis. (hot)} = 0{\cdot}0169\ \omega$$

$$\text{Total weight} = 0{\cdot}0197 \times 144 \times 1 \times 4{\cdot}0 \times 228 = 2600 \text{ lb.}$$

Rough check of overload capacity. Volts per conductor at 50 ~

$$= \frac{1585}{48 \times 2} \times \frac{50}{40} = 20\cdot6$$

Short-circuit volts per conductor

$$(a) \text{ for slots} = 0\cdot04 \times 37\cdot0'' = 1\cdot48$$

$$(b) \text{ end connection} = 0\cdot012 \times \frac{(21\cdot2)^{1\cdot5}}{1\cdot76} = \frac{0\cdot683}{\underline{\underline{2\cdot163}}}$$

$$\text{Ideal short-circuit current} = \frac{1000}{2} \times \frac{20\cdot6}{2\cdot163} = 4770 \text{ amp.}$$

$$\text{Max. h.p. (approx.)} = \frac{\sqrt{3} \times 2750 \times 4270}{2\cdot4 \times 746} = 11{,}400$$

$$= 3 \text{ times full load}$$

This machine was intended for work requiring a large overload capacity, and so the above winding will suit. From the standpoint of sound mechanical construction and reliability, a 2-bar per slot winding is best for the rotor.

We will use 7 slots per pole per phase for the rotor, which, with a three-phase winding, will give us 252 rotor slots.

The rotor current with all the conductors per phase in series

$$= \frac{3750 \times 746}{\sqrt{3} \times 2750 \times 0\cdot95} \times \frac{144 \times 2}{252 \times 2} = 373 \text{ amp.}$$

and the volts between rings at the start

$$= \frac{3750 \times 746}{\sqrt{3} \times 373 \times 0\cdot95} = 4550 \text{ approx.}$$

Obviously it would be better to use 4 parallel circuits per phase in the rotor, giving a rotor current of 1500 amp. and volts between rings at start = 1137.

This will be adopted. Since this machine has to be capable of dealing with heavy overloads, the current densities at normal full load will be kept lower than usual, and a density of 3·75 amp. per sq. mm. will be used for the rotor conductors.

$$\text{Current per conductor} = \frac{1500}{4} = 375 \text{ amp.}$$

$$\text{Section of rotor conductor} = \frac{375}{3\cdot75} = 100 \text{ sq. mm.}$$

We will use a copper bar of 100 sq. mm. section.

A bar 10 mm. deep and divided into 5 strips, each 2 mm. wide in parallel, will suit.

This bar will be insulated with micanite 1·5 mm. thick. This leads to a rotor slot 14 mm. wide (allowing for slack) and 28 mm. in depth. The rotor slot is shown in Fig. 145.

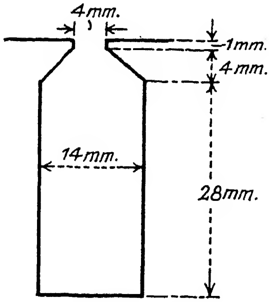


FIG. 145

L.M.T. of rotor coil = 3·76 metres

Resis. of rotor winding per phase at 15° C.

$$= \frac{0{\cdot}017 \times 84 \times 1 \times 3{\cdot}76}{100 \times 16} = 0{\cdot}00335$$

Resis. per phase (hot) = 0·00396 ω

Wt. of rotor copper = 0·0197 × 252 × 3·76 × 100 = 1870 lb.

MAGNETIZING CURRENT

Part.	Area (sq. cm.)	Length of Path (cm.)	$\phi \times 10^6$	B_{60}	A_{60}
Stator core	1050	49·6	9·66	9,200	248
Teeth	1965 1815 3360	7·4 1·0	19·32 19·0	12,600 13,400 7,150	111 6·5
Gap	4540	0·9	18·75	5,280	3810
Rotor teeth	3700 1725 1620	1·0 5·6	18·75 18·5	6,500 12,700 14,400	7 112
Rotor core	885	40·8	9·1	10,900	306

Total amp.-turns at 60° = 4600

$$\text{Magnetizing current} = \frac{4600}{2{\cdot}12 \times 4 \times 2} = 271 \text{ amp.}$$

The ideal short-circuit current.

$\lambda_s = 1{\cdot}27$ $\qquad\qquad$ $\lambda_r = 4{\cdot}29$

$$\lambda_1 = \left[\frac{1{\cdot}27}{0{\cdot}96 \times 4} + \frac{4{\cdot}29}{7}\right] \times 87 = 82 \text{ cm.}$$

$$\lambda_2 = 0{\cdot}6 \times 100 = 60$$

$$\lambda_3 = \frac{L\varepsilon\tau}{48 \times \delta \times q_1 \times q_2} = \frac{87 \times 1{\cdot}6 \times 53{\cdot}8}{48 \times 0{\cdot}45 \times 4 \times 7} = 12{\cdot}4$$

$$\lambda = \lambda_1 + \lambda_2 + \lambda_3 = 154{\cdot}4$$

$$\text{Ideal short-circuit current} = \frac{19{\cdot}32 \times 10^6}{3{\cdot}55 \times 4 \times 2 \times 154{\cdot}4}$$

$$= 4420$$

This also agrees very closely with our rough method.

Rotor connection diagram for 3750 h.p. slip-ring motor. *Left-hand winding. Bars numbered in clockwise direction looking at slip-ring end.*

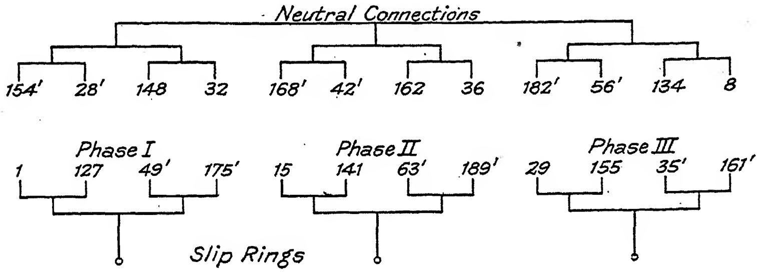


FIG. 146

Number of poles = 12

Number of slots = 252

Upper bars = 252 in number, numbered 1, 2, 3, etc.

Lower bars = 252 ,, ,, 1′, 2′, 3′, etc.

Rotor star-connected, with 4 circuits in parallel per phase.

Step of winding = 21 back

= 21 front

Abnormal step of 20 on 84 lower bars.

$$\sigma = \frac{271}{4420} = 0{\cdot}061 \quad \cos\phi = 86\%$$

Losses. Full load.

Stator copper = 31·2 kW

Iron loss = 34·03 ,,

Rotor copper loss = 29·3 ,,

Friction and windage loss = 25 kW
Total losses = 119·53 ,,
Output kW = 2800·0
Input kW = 2919·53
Efficiency = 95·8%
Slip = 1·0% at full load

The stator conductor will be suitably laminated to reduce loss due to eddy currents.

CHAPTER XIX

DESIGN OF SQUIRREL-CAGE INDUCTION MOTOR

OUTPUT 15 b.h.p., 3-phase; 50 cycles, 440 volts, 1000 r.p.m. synchronous; $\cos\phi = 87\cdot5\%$; eff. $= 86\%$

Starting torque at full volts = full-load torque

Temp. rise 40° C. after 6-hours full-load run

$$D^2L \text{ cu. cm.} = \frac{4\cdot06 \times 10^{11} \times \text{B.H.P.}}{B \times q \times \text{R.P.M.} \times \cos\phi \times \eta}$$

$$= \frac{4\cdot06 \times 10^{11} \times 15}{4500 \times 215 \times 960 \times 0\cdot86 \times 0\cdot875}$$

$$= 8700 \text{ cu. cms.}$$

For best power factor, $\frac{\tau}{L} = \frac{18}{\tau}$

$$D = 1\cdot35 p\sqrt{L}$$

$$p = 6 \qquad \therefore D = 8\cdot1\sqrt{L}$$

$$\therefore 65\cdot5L^2 = 8700 \qquad \therefore L^2 = \frac{8700}{65\cdot5} = 133$$

$$\text{and } D = 27\cdot5 \text{ cm.} \qquad \therefore L = 11\cdot5 \text{ cm.}$$

This machine was built with $D = 25$ cm. and $L = 14$ cm., which is quite close to the best proportions.

$$\text{Pole pitch} = \frac{\pi \times 25}{6} \text{ cm.} = 13\cdot1 \text{ cm.}$$

Apparent air-gap area per pole $= 13\cdot1 \times 14\cdot0 = 183$ sq. cm.

$$\text{Flux per pole} = 183 \times 4500 = 0\cdot825 \times 10^6$$

$$\text{Conductors in series per phase} = \frac{430 \times 10^8}{2\cdot12 \times 0\cdot825 \times 10^6 \times 50} = 486$$

(allowing 10 volts drop in stator winding at full load).

Using 3 slots per pole per phase in the stator, we have 54 stator slots, which gives us a suitable pitch of slot.

$$\text{Conductors per slot} = \frac{486}{18} = 27$$

The stator winding will be delta-connected.

Full-load current (line) $= 19{\cdot}4$ amp.

„ „ per phase $= 11{\cdot}2$ „

Actually 28 conductors per slot were used in this case.

Check on overload capacity with 28 conductors per slot:

$$\text{Volts per conductor at } 50\sim = \frac{440}{18 \times 28} = 0{\cdot}87$$

Short-circuit volts per conductor:

(*a*) for slot $= 0{\cdot}04 \times 5{\cdot}5'' = 0{\cdot}22$

(*b*) end connection $= 0{\cdot}012 \times \dfrac{(5{\cdot}15'')^{1.5}}{0{\cdot}572} = 0{\cdot}258$

$$a + b = 0{\cdot}478$$

$$I_{sc} \text{ line} = \sqrt{3} \times \frac{1000}{28} \times \frac{0{\cdot}87}{0{\cdot}478} = 113 \text{ amp.}$$

$$\text{Max. h.p.} = \frac{\sqrt{3} \times 440 \times 93}{3{\cdot}0 \times 746} = 31{\cdot}8$$

$$= 2{\cdot}12 (\text{full-load H.P})$$

Current per stator conductor at full load $= 11{\cdot}2$ amp.

Current density $= 3{\cdot}2$ amp. per sq. mm.

$$\text{Area of conductor} = \frac{11{\cdot}2}{3{\cdot}2} = 3{\cdot}51 \text{ sq. mm.}$$

We will use 3 wires in parallel of 18 S.W.G. diameter bare

$= 1{\cdot}22$ mm. d.c.c. to $1{\cdot}42$ mm.

Space required in slot for conductors $= 28 \times 3 \times 1{\cdot}42 \times 1{\cdot}42$

$= 170$ sq. mm.

Allowing a space factor of 0·75 for slot space required

$$= \frac{170}{0{\cdot}75} = 228 \text{ sq. mm.}$$

The slot shown in Fig. 147 is suitable.

Slot insulation of presspahn and leatheroid troughs of 0·7 mm. thickness per side will be used.

The stator will have the usual type of concentric winding of the hemi-tropic type.

L.M.T. of stator coil $= 101$ cm.

$$\text{Resis. per phase at } 15^\circ \text{ C.} = \frac{0{\cdot}017 \times 9 \times 28 \times 1{\cdot}01}{3{\cdot}51}$$

$$= 1{\cdot}23 \text{ ohms}$$

$$\text{Weight of copper} = 52\tfrac{1}{2} \text{ lb.}$$

$$\text{The length of air-gap} = 0{\cdot}5 \text{ mm.}$$

Number of rotor slots. The number of rotor slots will be chosen such that there will be 1 slot more or 1 slot less per pair of poles than in the stator. If we use 57 rotor slots, this condition will be satisfied. Much trouble is frequently experienced due to incorrect choice of slot ratio in squirrel-cage machines. The resultant torque-slip curve is compounded of the torque-slip curves due to the fundamental and those of the higher harmonics. In consequence of this, depressions in the torque-slip curve occur; or, in other words, the torque drops at some speed to a lower value than is necessary to continue the acceleration of rotor under the load or even when running unloaded. The torque may actually become negative. There may be more than one such point of low torque. These points are called saddle points. The trouble from saddle points are apt to be due to those harmonic fields which rotate in the same direction as the fundamental field, viz., the 7th, 13th, and higher harmonics. These forward harmonics will increase the torque at standstill, but reduce it seriously beyond the synchronous speed of these harmonics.

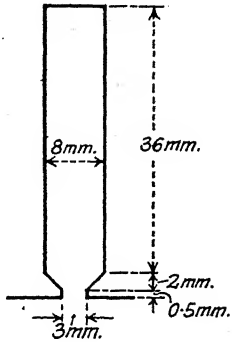


FIG. 147

Again at standstill, locking points may occur when the numbers of stator and rotor slots have a common factor; and the higher this common factor, the poorer the starting torque. Numbers of slots which are prime to one another are best for this purpose. The trouble is chiefly due to zigzag leakage flux. In cases where motors refuse to start up due to this cause, increasing the air-gap length will sometimes effect a cure, but there is a limit to this set by power-factor considerations.

Smooth starting and accelerating conditions will be secured by adopting one slot less or one slot more per pole pair in the rotor than in the stator.[1]

[1] See discussion on this point in *The Electrician* by Dr. Chapman.

With 57 rotor slots, the current per bar at full load

$$= \frac{15 \times 746}{3 \times 440 \times 0{\cdot}9} \times \frac{54 \times 28}{57 \times 1}$$

$$= 256 \text{ amp.}$$

$$\text{Current per ring} = \frac{256}{1{\cdot}11} \times \frac{\text{bars}}{2 \text{ poles}} \times \frac{1}{1{\cdot}414}$$

$$= \frac{256}{1{\cdot}11} \times \frac{57}{12} \times \frac{1}{1{\cdot}414}$$

$$= 770 \text{ amp.}$$

Assuming a rotor current density of 3·5 amp. per sq. mm. in the bars,

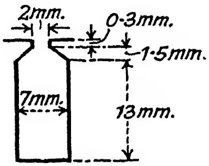


FIG. 148

$$\text{Area of bar} = \frac{256}{3{\cdot}5} = 72 \text{ sq. mm.}$$

The rotor slot is shown in Fig. 148.

Rotor bar of copper 6·0 mm. wide by 12 mm. deep.

$$\text{Resis. of rotor bars at } 15^\circ \text{ C.} = \frac{0{\cdot}017 \times 57 \times 0{\cdot}19}{72}$$

$$= 0{\cdot}00256 \text{ ohms}$$

$$\text{Resis. (hot)} = 0{\cdot}00307\ \omega$$

In order to get the desired starting torque, end rings of brass 20 mm. × 20 mm. will be used.

$$\text{Resis. of each ring} = 0{\cdot}000163\ \omega \text{ hot}$$

MAGNETIZING CURRENT

Part.	Area (sq. cm.)	Length (cm.)	ϕ	B_{60}	A_{60}
Stator core . .	67·5	13·3	0·4025	5,950	33
Stator teeth . .	126	7·2	0·805	{8,170 / 13,150}	62
Gap	145·5	0·1	0·805	7,070	563
Rotor teeth . .	{79 / 62}	2·6	0·805	{13,050 / 16,600}	78
Rotor core . .	89	5·22	0·4025	4,520	10

Total amp.-turns at 60° = A_{60} = 746

Magnetizing current per line wire $= \dfrac{\sqrt{3} \times 746}{2\cdot12 \times 3 \times 28} = 7\cdot27$

Ideal Short-circuit current.

$$\lambda_s = \frac{36}{24} + \frac{4}{11} + \frac{0\cdot5}{3\cdot0} = 2\cdot03$$

$$\lambda_r = \frac{13}{14} + \frac{3\cdot0}{9\cdot0} + \frac{0\cdot3}{2\cdot0} = 1\cdot413$$

$$\lambda_1 = \left[\frac{2\cdot03}{0\cdot96 \times 3} + \frac{1\cdot413}{3\cdot16}\right] \times 14 = 16\cdot2$$

$$\lambda_2 = 0\cdot73(\cdot36\tau + A) = 0\cdot73(\cdot36 \times 13\cdot1 + 14) = \underline{13\cdot7}$$

$$\lambda_3 = \frac{L\varepsilon\tau}{48 \times \delta \times q_1 \times q_2} = \frac{14 \times 13\cdot1 \times \underline{1\cdot337}}{48 \times \cdot05 \times 3 \times 3\cdot16} = 10\cdot75$$

Note $\delta = 0\cdot5$ mm. $= 0\cdot05$ cm.

$$\lambda = \lambda_1 + \lambda_2 + \lambda_3 = 40\cdot65$$

$$I_{sc} \text{ line (ideal)} = \frac{\sqrt{3} \times 0\cdot805 \times 10^6}{3\cdot55 \times 3 \times 28 \times 40\cdot65} = 115 \text{ amp.}$$

Our rough method gave 113 amp.

Max. h.p. $= 31\cdot8$ Actual short-circuit current $= 93$ amp.

$\sigma = \dfrac{7\cdot27}{115} = 0\cdot063$ $\cos\phi$ F.L. $= 87\cdot5\%$

Efficiency.

Stator copper loss $= 3 \times 11\cdot2^2 \times 1\cdot45 = 545$

Iron loss $\left\{\begin{array}{l}\text{Teeth} = 204 \\ \text{Core} = 178\end{array}\right\} = 382$

Rotor copper loss $= 394$

F.L. and windage $= 140$

Total losses $= 1{,}461$

Output $= 11{,}200$

Input $= 12{,}661$

Eff. $= 88\frac{1}{2}\%$

$$\begin{array}{l}\text{Starting torque as \%} \\ \text{of full-load torque}\end{array} = \left(\frac{93}{\sqrt{3}} \begin{array}{c}\times \\ \times\end{array} \frac{54 \times 28}{256 \times 57}\right)^2 \times \text{slip \%}$$

$$= \left(\frac{1420}{256}\right)^2 \times 3{\cdot}4 = 105\%$$

$$= \left(\frac{\text{rotor current at start}}{\text{rotor current at full load}}\right)^2 \times \%\ \text{slip}$$

$$\text{Slip} = \frac{394}{11{,}594} = 0{\cdot}034$$

$$\text{and in } \% = 3{\cdot}4$$

This machine was built and tested. The results of the test are as follows—

Temperatures after 6-hours full-load run—

Part.	Temperature.	Atmosphere.	Rise ° F.
Stator core	116° F.	66° F.	50° F.
,, winding (back) . . .	117° F.	66° F.	51° F.
,, ,, (front) . .	120° F.	66° F.	54° F.
Rotor end ring	128° F.	66° F.	62° F.

Magnetizing current line = 6·7 amp.

(Air-gap a little under 0·5 mm.)

Short-circuit current = 93 amp.

CHAPTER XX

HOW TO ADAPT WINDINGS TO A GIVEN FRAME

It frequently happens in a design office that a winding is required for a given frame for a different horse-power and frequency and speed than the standard.

Let z_1 = turns per coil for standard motor of horse-power H.P.$_1$ and frequency f_1 and voltage V_1

z_2 = turns per stator coil for motor of horse-power H.P.$_2$ and frequency f_2 and volts V_2

Then for the same number of phases and the same overload capacity

$$z_2 = z_1 \times \frac{V_2}{V_1} \times \sqrt{\frac{\text{H.P.}_1}{\text{H.P.}_2}} \times \sqrt{\frac{f_1}{f_2}} \quad . \quad . \quad . \quad (1009)$$

This may be shown as follows—

$$\frac{I_{d1}}{I_1} = \frac{I_{d2}}{I_2} \text{ for same overload capacity} \quad . \quad . \quad (1010)$$

I_{d1} = ideal short-circuit current per phase of motor 1
I_{d2} = ideal short-circuit current per phase of motor 2
I_1 = full-load current of motor 1
I_2 = full-load current of motor 2
V_1 = applied volts per phase of motor 1
V_2 = applied volts per phase of motor 2
f_1 = frequency of motor 1
f_2 = frequency of motor 2

$$\therefore \frac{I_{d1}}{I_1} = \frac{V_1}{Kq_1^2z_1^2f_1I_1} \quad . \quad . \quad . \quad . \quad . \quad (1011)$$

since $I_{d1} = \dfrac{V}{Kq_1^2z_1^2f_1}$

and K = constant

q_1 = slots per pole per phase

z_1 = conductors per slot in stator

$$\frac{I_{d2}}{I_2} = \frac{V_2}{Kq_2^2z_2^2f_2I_2} = \frac{I_{d1}}{I_1} \quad . \quad . \quad . \quad (1012)$$

$$\therefore\ q_2{}^2 z_2{}^2 = \frac{V_2 \times}{V_1 \times} \frac{q_1{}^2 z_1{}^2 \times f_1 \times I_1}{f_2 \times I_2} \quad . \quad . \quad . \quad . \quad (1013)$$

$$\text{Now } \frac{\text{H.P.}_1}{\text{H.P.}_2} = \frac{V_1 I_1}{V_2 I_2} \text{ for same power factor and efficiency} \quad (1014)$$

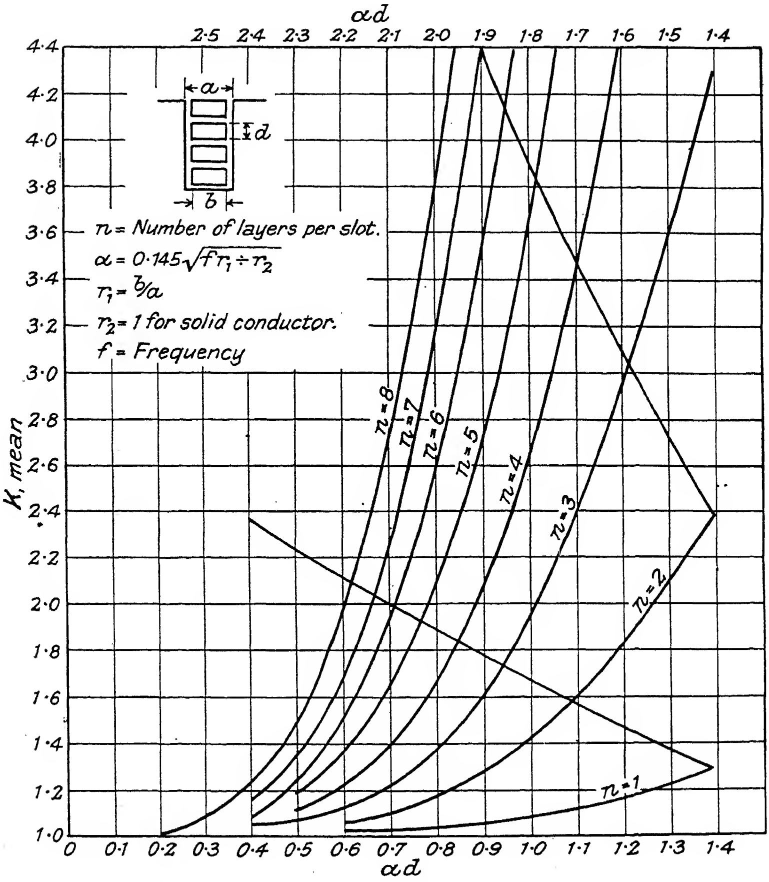


FIG. 149.—INCREASE IN LOSS DUE TO EDDY CURRENTS IN SLOT COPPER
(Mean value of Field's curves)

$$\therefore\ \frac{I_1}{I_2} = \frac{V_2}{V_1} \cdot \frac{\text{H.P.}_1}{\text{H.P.}_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1015)$$

$$\therefore\ q_2{}^2 z_2{}^2 = \frac{V_2{}^2}{V_1{}^2} \times \frac{\text{H.P.}_1}{\text{H.P.}_2} \times q_1{}^2 z_1{}^2 \times \frac{f_1}{f_2} \quad . \quad . \quad . \quad (1016)$$

$$\therefore\ q_2 z_2 = \frac{V_2}{V_1} \times \sqrt{\frac{\text{H.P.}_{\cdot 1}}{\text{H.P.}_{\cdot 2}}} \times q_1 z_1 \times \sqrt{\frac{f_1}{f_2}} \quad . \quad . \quad (1017)$$

and if $q_2 = q_1$

$$\therefore\ z_2 = z_1 \times \frac{V_2}{V_1} \times \sqrt{\frac{\text{H.P.}_{\cdot 1}}{\text{H.P.}_{\cdot 2}}} \times \sqrt{\frac{f_1}{f_2}} \quad . \quad . \quad (1018)$$

To illustrate this. In the previous example, we had a 15 h.p., three-phase squirrel cage, 50 ~, 1000 r.p.m. synchronous induction motor for 440 volts.

The turns per stator coil were 28.

It is required to find the turns per stator coil for the same motor running on 40 cycles and giving 12 h.p. at 800 r.p.m. synchronous. The voltage is 500 volts. The overload capacity is the same in the two cases.

Turns per coil for 12 h.p. 40 ~ motor

$$= 28 \times \sqrt{\frac{\text{H.P.}_{\cdot 1}}{\text{H.P.}_{\cdot 2}}} \times \frac{V_2}{V_1} \times \sqrt{\frac{f_1}{f_2}}$$

$$= 28 \times \sqrt{\frac{15}{12}} \times \frac{500}{440} \times \sqrt{\frac{50}{40}}$$

$$= 40 \text{ turns per coil}$$

Increase in copper losses due to eddy currents. In estimating the copper losses in the conductors, it is necessary to take into account the increased losses due to eddy currents. This is conveniently done by the aid of Field's curves, which are redrawn to represent the mean increases in loss with different number of layers in the slot.

CHAPTER XXI

SHORT PITCH AND FRACTIONAL WINDINGS

SHORT-PITCH windings are sometimes used for induction motors, and by their use many advantages are secured. In the first place, a considerable reduction in the weight of copper required is effected; and, secondly, by correct chording of the winding, it is possible to eliminate harmonics from the flux wave which are so troublesome in producing crawling and other troubles. This is more fully dealt with in the section dealing with the nature of the rotating field.

The following investigation deals with the pitch of the coils which will give minimum copper weight for all values of the pole pitch and core lengths. It is taken from an article by the author published in *The Electrician* for 29th October, 1920.

In making the calculation, it is assumed that the current density is constant, and that there is room to deepen the slots to accommodate the increased numbers of conductors due to chording. In most cases of importance, this is possible.

The Electrician, 29th October, 1920—

SHORT-PITCH WINDINGS

Let E = E.M.F. generated per phase in volts

Z = number of conductors in series per phase

f = frequency of supply current

$\beta = \dfrac{\text{span of coil}}{\text{full pitch}}$ k = distribution factor $\times$ form factor

τ = pole pitch

a = area of conductor

ϕ = flux per pole

C.G.S. units are used throughout.

Then for a distributed winding,

$$E = 2 \times k \times Z \times \phi \times f \times \sin\frac{\beta\pi}{2} \times 10^{-8} \text{ volts} \qquad (1019)$$

From this equation,

$$Z = \frac{p}{\sin\beta\dfrac{\pi}{2}} \text{ where } p = \frac{E \times 10^8}{2 \times k \times \phi \times f} \qquad (1020)$$

p is constant for a given rating.

The mean length per turn with full pitch coils

$$= 2(l + \alpha\tau) \quad . \quad . \quad . \quad (1021)$$

where l = length of core
and $\alpha\tau$ = length of overhang at one end of the coil

In any given case the value of α may be easily found, for the inclination of the coil end to the core with diamond-shaped coils is given by

$$\sin^{-1}\theta = \frac{\text{width of slot copper + clearance}}{\text{width of slot + width of tooth}}$$

The volume of copper with full-pitch coils, with m phases, is thus

$$= m\frac{Z'}{2} \times 2(l + \alpha\tau) \times a. \quad . \quad . \quad (1022)$$

$$= mZ' \times a \times (l + \alpha\tau) \quad . \quad . \quad . \quad (1023)$$

where Z' = number of conductors in series per phase with full-pitch coils.

The volume of copper required with fractional pitch coils

$$= mZ \times a \times (l + \beta\tau\alpha) = \frac{mZ'}{\sin\beta\frac{\pi}{2}} \times a \times (l + \alpha\beta\tau). \quad (1024)$$

$$\text{Since } Z\sin\beta\frac{\pi}{2} = Z' \quad . \quad . \quad . \quad . \quad . \quad . \quad (1025)$$

$$\text{Therefore } \frac{\text{volume of copper with full-pitch coils}}{\text{volume of copper with fractional pitch coils}}$$

$$= \sin\beta\frac{\pi}{2} \times \frac{l + \alpha\tau}{l + \alpha\beta\tau} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1026)$$

It is clear that this ratio will have a maximum value for minimum weight of copper ; in other words, the most economical span of the coil will be that which gives the above ratio a maximum value. This span can be determined by differentiating the above expression with respect to β

Calling this ratio A, we have

$$\frac{dA}{d\beta} = \frac{(l + \alpha\tau)\left(\frac{\pi}{2}\cos\beta\frac{\pi}{2}\right)(l + \alpha\beta\tau) - \alpha\tau\left(\sin\beta\frac{\pi}{2}\right)(l + \alpha\tau)}{(l + \alpha\beta\tau)^2} \quad (1027)$$

for a minimum, $\dfrac{dA}{d\beta} = 0$

$$\therefore \frac{\pi}{2} \cos \beta \frac{\pi}{2} \times (l + \alpha\beta\tau) = \alpha\tau \sin \beta \frac{\pi}{2} \quad . \quad . \quad . \quad (1028)$$

$$\text{Therefore } \tan \beta \frac{\pi}{2} = \frac{\pi}{2}\left(\frac{l + \alpha\beta\tau}{\alpha\tau}\right) \quad . \quad . \quad . \quad . \quad (1029)$$

$$= \beta \frac{\pi}{2} + \frac{\pi}{2} \frac{l}{\alpha\tau} \quad . \quad . \quad . \quad . \quad (1030)$$

$$\text{Let } \beta \frac{\pi}{2} = \theta, \text{ then } \tan \theta = \theta + \frac{\pi}{2} \frac{l}{\alpha\tau} \quad . \quad . \quad . \quad (1031)$$

Its solution is shown in the following curve, Fig. 150, which gives the most economical span of the coal for different ratios of core-length to pole-pitch.

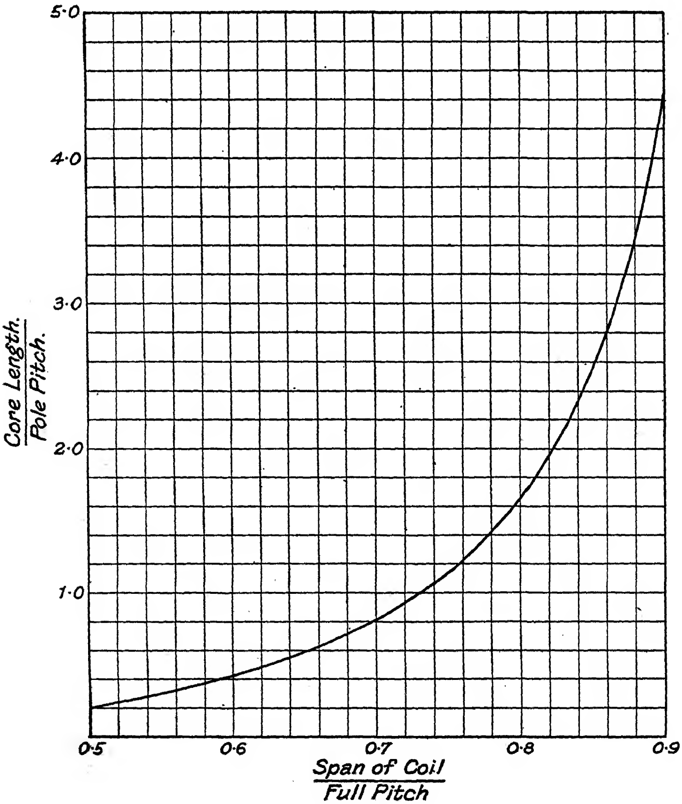


FIG. 150

In drawing this curve, a value of $\alpha = 1{\cdot}4$ has been taken, but in any given case its correct value can be substituted in the equation.

Inductance of fractional pitch windings. Messrs. Adams, Cabot, and Irving have investigated this subject, and the following is an abstract of their work.

Slot leakage. The reactance due to that part of the leakage flux which crosses a slot, linking with more or less of the conductors in the slot, may be expressed as follows

$$X_s = 2\pi f \times \phi_s q_1^2 \times l \times \frac{N_t}{m} \times 10^{-8} \quad . \quad . \quad . \quad (1032)$$

where f = frequency

ϕ_s = flux per ampere conductor per unit length of slot in C.G.S. lines

q_1 = conductors per slot

l = core length

N_t = total number of slots

m = number of phases

Only ϕ_s in the above is affected by fractional pitch.

Let θ = pitch of the coil in electrical degrees

β = angle of pitch deficiency = $180 - \theta$

If the machine has a large number of phases, the two coil sides located in any given slot will, in general, carry currents differing in phase by β, and the component of one of these currents in phase with the other will be proportional to $\cos \beta$.

If the product of the inductances of these two coil sides is equal to the square of the mutual inductance, i.e. if there is no relative leakage flux between them, the ratio of the average leakage flux linked with one to what it would be with a full pitch winding and no phase difference

$$= \frac{1 + \cos \beta}{2} = \cos^2 \frac{\beta}{2} = \sin \frac{^2\theta}{2} = K_p \quad . \quad . \quad . \quad (1033)$$

This may be called the slot pitch factor under the above-mentioned conditions.

Neither of these conditions exists in practice.

Consider, first, the relative leakage between the two coil sides in the same slot.

The flux linked with coil b at the bottom of the slot per cm. length of slot for 1 amp. in b is

$$\phi_{bb} = 0{\cdot}4\pi \left(\frac{d_1}{6w} + \frac{d_1}{2w} + \frac{d_2}{\omega}\right) = \frac{0{\cdot}4\pi}{\omega}\left(\frac{d_1}{6} + \frac{d_1}{2} + d_2\right) \quad (1034)$$

The flux linked with b per cm. of slot for 1 amp. in a is

$$\phi_{ab} = \frac{0{\cdot}4\pi}{w}\left(\frac{d_1}{4} + d_2\right) \quad . \quad . \quad . \quad . \quad (1035)$$

But the current in a differs in phase from that in b by an angle β, and the component of ϕ_{ab} in phase with ϕ_{bb} is $\phi_{ab} \cos \beta$.

Then the total in-phase flux linked with b per cm. length of slot, and for 1 amp. distributed uniformly over the whole copper section of the slot, is

$$\phi_b = \frac{\phi_{bb} + \phi_{ab} \cos \beta}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1036)$$

$$= \frac{0{\cdot}4\pi}{w}\left[\frac{d_1}{3} + \frac{d_2}{2} + \cos \beta \left(\frac{d_1}{8} + \frac{d_2}{2}\right)\right] \quad . \quad . \quad (1037)$$

Similarly, $$\phi_{aa} = \frac{0{\cdot}4\pi}{w}\left(\frac{d_1}{6} + d_2\right). \quad . \quad . \quad . \quad . \quad (1038)$$

$$\phi_{ba} = \frac{0{\cdot}4\pi}{w}\left(\frac{d_1}{4} + d_2\right). \quad . \quad . \quad . \quad . \quad (1039)$$

and $$\phi_a = \frac{\phi_{aa} + \phi_{ba} \cos \beta}{2} \quad . \quad . \quad . \quad . \quad . \quad (1040)$$

$$= \frac{0{\cdot}4\pi}{w}\left[\left(\frac{d_1}{12} + \frac{d_2}{2}\right) + \cos \beta \left(\frac{d_1}{8} + \frac{d_2}{2}\right)\right] \quad (1041)$$

Then since each coil has one side in the bottom and the other in the top of a slot, the average flux linkage per amp. per cm. of slot will be

$$\phi_s = \frac{\phi_a + \phi_b}{2}$$

$$= \frac{0{\cdot}4\pi}{w}\left[\left(\frac{5}{24} d_1 + \frac{d_2}{2}\right) + \cos \beta \left(\frac{d_1}{8} + \frac{d_2}{2}\right)\right] \quad . \quad (1042)$$

If the winding were full pitch,

$$\phi_b = \frac{0{\cdot}4\pi}{w}\left[\frac{d_1}{3} + \frac{d_2}{2} + \frac{d_1}{8} + \frac{d_2}{2}\right]$$

$$= \frac{0{\cdot}4\pi}{w}\left[\frac{11}{24} d_1 + d_2\right] \quad . \quad . \quad (1043)$$

and $$\phi_a = \frac{0{\cdot}4\pi}{w}\left[\frac{d_1}{12} + \frac{d_2}{2} + \frac{d_1}{8} + \frac{d_2}{2}\right]$$

$$= \frac{0{\cdot}4\pi}{w}\left[\frac{5}{24} d_1 + d_2\right] \quad . \quad . \quad (1044)$$

$$\text{and } \phi_s = \frac{\phi_a + \phi_x}{2} = \frac{0{\cdot}4\pi}{w}\left[\frac{d_1}{3} + d_2\right] \quad . \quad . \quad . \quad (1045)$$

The slot-pitch factor

$$K_{ps} = \frac{\frac{5}{24}\,d_1 + \frac{d_2}{2} + \cos\beta\left(\frac{d_1}{8} + \frac{d_2}{2}\right)}{\frac{d_1}{3} + d_2} \quad . \quad . \quad . \quad (1046)$$

$$K_{ps} = \frac{\frac{5}{24}\,d_1 - \frac{1}{8}\,d_1 + \left(\frac{d_1}{4} + d_2\right)\left(\frac{1 + \cos\beta}{2}\right)}{\frac{d_1}{3} + d_2}$$

$$= \frac{\frac{d_1}{12} + \left(\frac{d_1}{4} + d_2\right)\left(\frac{1 + \cos\beta}{2}\right)}{\frac{d_1}{3} + d_2}$$

$$= \frac{\frac{d_1}{12} + \left(\frac{d_1}{4} + d_2\right)\left(\frac{1 + \cos\beta}{2}\right)}{\frac{d_1}{12} + \left(\frac{d_1}{4} + d_2\right)} \quad . \quad . \quad . \quad (1047)$$ [1]

Tooth-tip or zigzag leakage. The tooth-tip leakage reactance

$$x_{tt} = 2\pi f \phi_{tt} q_1^2 \times l \times \frac{N_t}{m} \times 10^{-8} \quad . \quad . \quad . \quad . \quad (1048)$$

ϕ_{tt} = tooth-tip flux per amp. conductor per cm. length of slot for stator and rotor

As far as the fractional pitch effect is concerned, this element is exactly on a par with that part of the slot leakage which crosses the slots above the conductors, since it is wholly common to both coil sides.

Coil-end leakage. The coil-end reactance may be expressed as follows—

$$x_f = 2\pi f \times \phi_f \times \frac{p}{2} \times \left(\frac{N}{p}\right)^2 \times l_e \times 10^{-8} \quad . \quad . \quad (1049)$$

where ϕ_f = the flux per amp.-conductor per cm. length of the whole phase bundle of coil ends

[1] d_1 = distance from the bottom of the slot to top of upper conductor.
d_2 = distance from top of upper conductor to top of slot.
w = width of slot; b = lower conductor.
a = upper conductor.

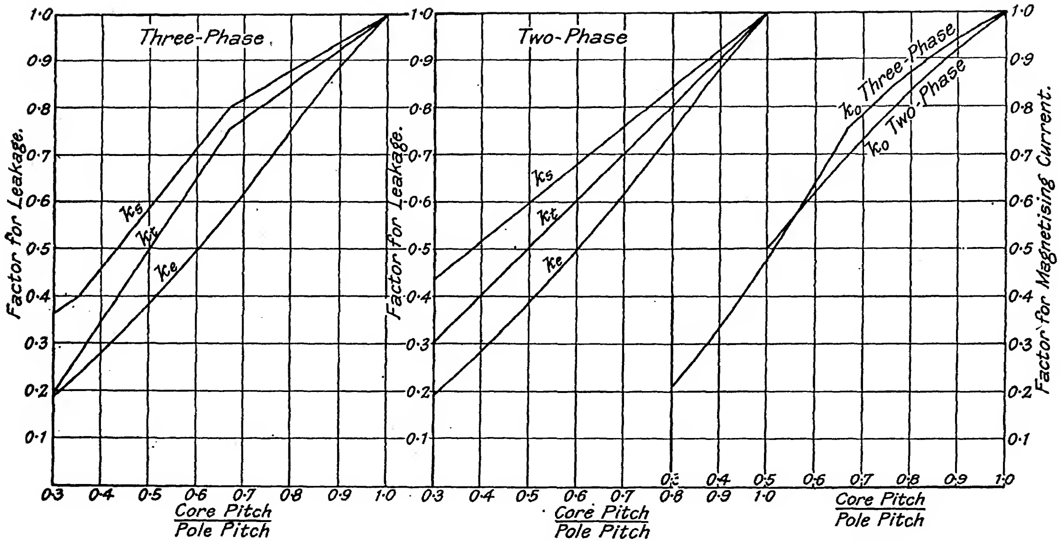


FIG. 151.—CONSTANTS FOR FRACTIONAL PITCH WINDINGS (*Prof. Adams*)

k_s = constant for slot leakage
k_t = constant for tooth-tip leakage
k_e = constant for end-winding leakage
k_o = constant for magnetizing current

p = number of poles
N = conductors per phase
l_e = length of two ends of one coil

l_e varies with the coil pitch in a simple manner and ϕ_f in a manner not so obvious.

For a circular coil in air, ϕ_f the flux linked with 1 cm. of the coil as a whole per ampere distributed uniformly throughout the section of the coil is proportional to the log of the ratio of coil diameter to the diagonal of its cross-section, provided this ratio is large.

For coils, such as are used on induction motors, this relation holds in a general way only, especially when the mutual inductive effect of neighbouring coils is taken into account and the pitch is fractional.

Within practical limits, ϕ_f would increase in approximate logarithmic relation to the coil pitch, were it not for mutually inductive effects of adjacent phases, and the curve representing this relation would tend to a zero which is *not* that of the pitch.

For pitches less than unity, the mutually inductive effect decreases in about the same ratio as the self-inductive effect, and this does not change much the general shape of the curve.

Thus the curve showing the relation between ϕ_f and the coil-pitch should be logarithmic in its general character, tending towards zero at some small (not zero) pitch and falling increasingly below the log curve for pitches greater than unity.

Belt leakage. The belt-leakage reactance

$$X_b = 2\pi f \phi_b p \left(\frac{N}{p}\right)^2 . l \times 10^{-8} \quad . \quad . \quad . \quad . \quad (1050)$$

ϕ_b = flux per amp. conductor per cm. of belt, and is inversely proportional to the reluctance of the belt magnetic circuit, and proportional to $\sin^2$ of $\frac{1}{4}$ of the angle of phase difference between the currents in the two opposing belts.

ϕ_b is proportional to the belt pitch and inversely proportional to the air-gap length.

The authors give, as the results of tests, a value for ϕ_f, the coil-end leakage flux, when all charged to the primary a value lying between 0·45 and 0·53 per cm. length. The first value corresponds more nearly to a normal induction motor.

For squirrel-cage motors with bars extending well out from the core, the value of ϕ_f (all charged to the primary) is given as 0· per cm.

The curves on page 314 (due to the authors mentioned) show the reduction factors due to chording.

CHAPTER XXII

RATINGS OF MOTORS

FOR protected type motors, the full-load rating is that which will give a temperature rise not exceeding 40° C. by thermometer after a continuous run. The time taken to reach a steady temperature will depend on the size of machine. Small machines may take 2 to 3 hours. Large machines seldom longer than 6 hours. Usually a 6-hour run at full load is specified. For machines which have the openings covered with $\frac{1}{16}$ in. mesh gauze covers, the outputs will be approximately 85 per cent of the protected type outputs for 40° C. rise.

For totally-enclosed machines for continuous rating, the

$$\frac{\text{total enclosed output}}{\text{protected type output}}$$

will vary with the size of machine. For machines up to about 10 b.h.p. as protected type machine, the totally enclosed output will be roughly 50 per cent. For protected type outputs between 10 and 30 b.h.p., the totally enclosed output for continuous rating will be about 40 per cent of the protected type output. Between 30 h.p. and 50 b.h.p., the ratio is about 33 per cent, and for larger machines the ratio decreases to 25 per cent or lower.

Generally it may be said that, for 50° C. use by thermometer after 6 hour full-load run as totally enclosed type, the watts per sq. cm. of case area (excluding the bearings) should be roughly about 0·078 or $\frac{1}{2}$ watt per square inch of case. One calculates the total losses, excluding the bearing losses, and computes the radiating area of the frame excluding the bearing areas, and the ratio

$$\frac{\text{total losses} - \text{bearing losses}}{\text{radiating area of case}} = \frac{1}{2} \text{ watt per sq. inch}$$

for 50° C. by thermometer after 6-hours run.

Intermittent ratings. The following are the relative ratings for protected type, enclosed ventilated, and totally-enclosed machines for 1 hour and $\frac{1}{2}$ hour respectively.

$$\frac{\text{rating as intermittent type}}{\text{continuous protected type rating}} = K$$

Protected type	1 hr. 40° C. use	$K = 1 \cdot 35$
	½ hr. 40° C. use	$K = 1 \cdot 45$
Enclosed vent	1 hr. 50° C.	$K = 1 \cdot 13$
	½ hr. 50° C.	$K = 1 \cdot 25$
Totally enclosed . . .	1 hr. 50° C.	$K = 100\%$
	½ hr. 50° C.	$K = 125\%$
Enclosed vent	1 hr. 40° C.	$K = 1 \cdot 1$
	½ hr. 40° C.	$K = 1 \cdot 2$
Totally enclosed . . .	1 hr. 40° C.	$K = 0 \cdot 7$
	½ hr. 40° C.	$K = 1 \cdot 1$

The author found, after analysis of a large number of totally-enclosed motor tests, that the time constant in minutes of the motor

$$= \frac{1310 \times \text{total weight in lbs.}}{\text{area of frame in sq. cms.}}$$

Area of frame considered is the sum of the areas of yoke + end shields.

The temperature rise at any time t sec. after the commencement of the run θ

$$\theta = \theta_f\left(1 - e^{-\frac{t}{T_c}}\right)$$

where θ = temp. use at time t

θ_f = final temp. rise

t = time from commencement of run

T_c = time constant in the same units, either seconds or minutes, whichever is used

Sparking and leakage distances. The following table shows the sparking and leakage distances required, and thickness of insulation for various voltages—

Limit Pressure Volts.	*Test Volts.* To Earthed Metal Volts.	Between Insulated Phases (Volts).	*Distance in mm.* Earthed Metal.	Between Phase Windings.	Thickness of Slot Insulation Tubes.	Test Pressure for Slot Insulation.
650	2,000	1,300	10 mm.	5 mm.	0·7–1·0 mm. pphn.	2,000
1,200	3,000	2,500	15 ,,	10 ,,	2·0 mm. pphn.	4,000
2,200	5,500	4,400	25 ,,	20 ,,	2·0 mm. micanite	16,000
3,300	8,200	6,600	30 ,,	25 ,,	2·0 ,, ,,	16,000
6,600	14,000	13,000	40 ,,	35 ,,	2·5 ,, ,,	20,000
8,000	16,000	16,000	50 ,,	45 ,,	3·5 ,, ,,	30,000
10,000	20,000	20,000	70 ,,	60 ,,	4·0 ,, ,,	35,000
13,000	26,000	26,000	90 ,,	80 ,,	5·0 ,, ,,	45,000

SPARKING AND LEAKAGE DISTANCES

Limit Voltage.	Test Pressure Volts.	Sparking Distance to Earthed Metal Mm.	Leakage Distance to Earthed Metal. Exposed Surface. To Bare Conductor.	Leakage Distance to Earthed Metal. Exposed Surface. To Taped Conductor.	Leakage Distance to Earthed Metal. Enclosed Surface to Taped Conductor.
650	2,000	5 mm.	10 mm.	5 mm.	5 mm.
1,200	3,000	10 ,,	12 ,,	10 ,,	8 ,,
2,200	5,500	15 ,,	30 ,,	25 ,,	20 ,,
3,300	8,200	20 ,,	35 ,,	30 ,,	25 ,,
6,600	14,000	30 ,,	70 ,,	60 ,,	45 ,,
8,000	16,000	40 ,,	95 ,,	80 ,,	60 ,,
10,000	20,000	50 ,,	130 ,,	110 ,,	80 ,,
13,000	26,000	65 ,,	190 ,,	160 ,,	120 ,,

The above are minimum limits and give a safety factor of about 2 on test voltage, assuming the surface damp and dusty.

INSULATION FOR INDUCTION MOTOR WINDINGS

Concentric Coils. The voltage between adjacent conductors should preferably not exceed 40.

Single section coils should be used wherever possible, but the ratio of width to depth of copper should not exceed 5. Two copper strips, side by side in parallel, may be used when the ratio of width to depth of the whole conductor is $>$ 5, and are generally preferable when the ratio exceeds 4. Such coils are more expensive than multiple section coils in series. The latter type should be given preference on voltages greater than 3500.

Insulation between conductors must be either *dcc* or *tcc*. *Dcc* or *tcc* wire should not be used when either the width or depth of wire exceeds 0·25 in.

Taped conductors of 2 or more wires in parallel may be used when neither the width nor depth of each single wire is less than $\frac{1}{8}$ in. When neither dimension of the whole conductor exceeds $\frac{1}{4}$ in., separate *dcc* or *tcc* wires should be used.

Mica separators between turns are necessary with *dcc* conductors above 3500 volts, and with *tcc* or taped conductors above 3500 volts.

Mica separators cannot be used if the width of the conductor is less than $\frac{1}{8}$ in.

INSULATION BETWEEN SECTIONS IN SERIES

Insulation between sections in series. For 3500 volts and below, alternate sections will be taped with empire cloth. For higher voltages, mica separators will be used.

INTERNAL INSULATION. Thicknesses (inch units)

Around conductors—*dcc* or *tcc* 0·015″

		Width.	Depth.
Mica between *dcc* conductors	3500–6600 volts	$0{\cdot}02'' \times s$	$0{\cdot}01'' \times n - 1$
,, ,,	above 6600 volts	$0{\cdot}04'' \times s$	$0{\cdot}02'' \times (n - 1)$
tcc or taped . .	,, 6600 ,,	$0{\cdot}02'' \times s$	$0{\cdot}01'' \times n - 1$
Empire tape on alternate sections .	3500 ,, and below	$0{\cdot}028'' \times (s - 1)$	0·028″
Mica between sections of coils	above 3500 volts	$0{\cdot}03'' \times (s - 1)$	0·03″

s = number of sections in series

n = number of conductors in depth

EXTERNAL INSULATION. Thicknesses in inches

Voltage.	Width.	Depth.
0–600	0·10″	0·22″
601–2,200	0·15″	0·27″
2,201–3,500	0·17″	0·29″
350–6,600	0·21″	0·33″
6,601–9,000	0·26″	0·39″
9,001–11,000	0·31″	0·42″
11,001–13,000	0·38″	0·47″

The overhang will be taped with white tape ½ in. lap.

Insulation for rotor windings, with 2 bars per slot. A slot lining of 0·15 mm. paper is used. On the straight portion of the coil the following thicknesses of paper (ironed on to the coil) are used—

Volts between rings at start.	Thickness of paper.
$\leqq$ 300 volts	0·5 mm. paper
301–600 volts	1·0 ,, ,,
Above 600 volts	1·5 ,, ,,

The insulation projects a distance of 7 to 15 mm., according to the voltage from the core.

Double thickness of 0·2 mm. tape on overhang.

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